Differential equations

Generalities and definitions

Definitions

- A differential equation is an equation involving the derivatives of a function.

- The ORDER of a differential equation is the same as the highest order of derivation occuring in the equation.

- A differential equation is linear if it is LINEAR in y and the derivative of y.

  (Any equation containing powers of y and/or its derivative
  or products of y and/or its derivatives are non-linear)

Solving differential equations

- To solve a differential equation is to find all the functions satisfying the equation.

  All these solutions constitue a FAMILY of solutions.
- Solutions that involve ARBITRARY constants are called GENERAL SOLUTIONS.
- A solution which contains NO arbitrary CONSTANT is called a PARTICULAR SOLUTION.
- To work out a particular solution, you need initial/boundary conditions: $y(x_0) = y_0$

Methods to solve *first order* differential equations

- *Method* 1: Direct integration

  This method can be used if the differential equation can be written as

  $$\frac{dy}{dx} = f(x). \qquad \text{By integrating both sides, you obtain}$$

  $$y = \int f(x)dx$$

- *Method* 2: Separating variables

  This method can be used if the differential equation can be written as

  $$g(y)\frac{dy}{dx} = f(x). \qquad \text{By integrating both sides, you obtain}$$

  $$\int g(y)dy = \int f(x)dx$$

- *Method* 3: Recognising the derivative of a product function

  This method can be used if the differential equation can be written as

  $$u\frac{dy}{dx} + \frac{du}{dx}y = f(x), \; where\ u\ is\ a\ function\ of\ x.$$

  Re-*write as* $\frac{d}{dx}(u \times y) = f(x)$ *and* integrate both sides:

  $$u \times y = \int f(x)dx \;\; so \;\; y = \frac{1}{u}\int f(x)dx$$