First order differential equations – exam questions

Question 1: Jan 2010 Q3

(a) A differential equation is given by

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=3x$$

Show that the substitution

$$u=\frac{dy}{dx}$$

transforms this differential equation into

$$\frac{du}{dx}+\frac{2}{x}u=3$$ *(2 marks)*

(b) Find the general solution of

$$\frac{du}{dx}+\frac{2}{x}u=3$$

giving your answer in the form $u=f(x)$. *(5 marks)*

(c) Hence find the general solution of the differential equation

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=3x$$

giving your answer in the form $y=g(x)$. *(2 marks)*

Question 2: Jun 2007 Q3

By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx}+(\tan x)y=\sec x$$

given that $y=3$ when $x=0$. *(8 marks)*

Question 3: Jan 2008

By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx}+\frac{4x}{x^2+1}y=x$$

given that $y=1$ when $x=0$. Give your answer in the form $y=f(x)$. *(9 marks)*

Question 4: Jun 2009

By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx}-y\tan x=2\sin x$$

given that $y=2$ when $x=0$. *(9 marks)*

Question 5: Jun 2007 Q5

(a) A differential equation is given by

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = x^2 + 1$$

Show that the substitution

$$u = \frac{dy}{dx} + x$$

transforms this differential equation into

$$\frac{du}{dx} = \frac{2xu}{x^2 - 1}$$

(4 marks)

(b) Find the general solution of

$$\frac{du}{dx} = \frac{2xu}{x^2 - 1}$$

giving your answer in the form $u = f(x)$. *(5 marks)*

(c) Hence find the general solution of the differential equation

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = x^2 + 1$$

giving your answer in the form $y = g(x)$. *(3 marks)*

Question 6: Jun 2008 Q4

(a) A differential equation is given by

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$$

Show that the substitution

$$u = \frac{dy}{dx}$$

transforms this differential equation into

$$\frac{du}{dx} - \frac{1}{x}u = 3x$$

(2 marks)

(b) By using an integrating factor, find the general solution of

$$\frac{du}{dx} - \frac{1}{x}u = 3x$$

giving your answer in the form $u = f(x)$. *(6 marks)*

(c) Hence find the general solution of the differential equation

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$$

giving your answer in the form $y = g(x)$. *(2 marks)*

Question 7: Jun 2006 Q3

(a) Show that $\sin x$ is an integrating factor for the differential equation

$$\frac{dy}{dx} + (\cot x)y = 2\cos x$$ *(3 marks)*

(b) Solve this differential equation, given that $y = 2$ when $x = \frac{\pi}{2}$. *(6 marks)*

Question 8: Jun 2006 Q6

(a) Show that the substitution

$$u = \frac{dy}{dx} + 2y$$

transforms the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

into

$$\frac{du}{dx} + 2u = e^{-2x}$$ *(4 marks)*

(b) By using an integrating factor, or otherwise, find the general solution of

$$\frac{du}{dx} + 2u = e^{-2x}$$

giving your answer in the form $u = f(x)$. *(5 marks)*

(c) Hence find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

giving your answer in the form $y = g(x)$. *(5 marks)*

Question 9: Jan 2007

(a) Show that x^2 is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = 3(x^3 + 1)^{\frac{1}{2}}$$ *(3 marks)*

(b) Solve this differential equation, given that $y = 1$ when $x = 2$. *(6 marks)*

Question 10: Jan 2009 Q2

(a) Show that $\frac{1}{x^2}$ is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = x$$ *(3 marks)*

(b) Hence find the general solution of this differential equation, giving your answer in the form $y = f(x)$. *(4 marks)*

First order differential equations – exam questions MS

Question 1: Jan 2010 Q3

(a)	$u=\frac{dy}{dx} \Rightarrow \frac{du}{dx}=\frac{d^2y}{dx^2}$	M1	
	$x\frac{du}{dx}+2u=3x \Rightarrow \frac{du}{dx}+\frac{2}{x}u=3$	A1	2
(b)	IF is $\exp\left(\int \frac{2}{x}dx\right)$	M1	
	$=e^{2\ln x}$; $=x^2$	A1;A1	
	$\frac{d}{dx}\left(ux^2\right)=3x^2$	M1	
	$ux^2=x^3+A \Rightarrow u=x+Ax^{-2}$	A1	5
(c)	$\frac{dy}{dx}=x+Ax^{-2}$	M1	
	$\frac{dy}{dx}=x+Ax^{-2} \Rightarrow y=\frac{1}{2}x^2-\frac{A}{x}+B$	A1F	2
	Total		9

Question 2: Jun 2007 Q3

IF is $e^{\int \tan x\,dx}$	M1	
$=e^{-\ln\cos x}=e^{\ln\sec x}$	A1	
$=\sec x$	A1ft	
$\frac{d}{dx}\left(y\sec x\right)=\sec^2 x$	M1A1	
$y\sec x=\int \sec^2 x\,dx$		
$y\sec x=\tan x+c$	A1	
$y=3$ when $x=0 \Rightarrow 3\sec 0=0+c$	m1	
$c=3 \Rightarrow y\sec x=\tan x+3$	A1	8
Total		8

Question 3: Jan 2008 Q5

IF is $e^{\int \frac{4x}{x^2+1}dx}$	M1	
$=e^{2\ln\left(x^2+1\right)}$	A1	
$=e^{\ln\left(x^2+1\right)2}=\left(x^2+1\right)^2$	A1√	
$\frac{d}{dx}\left(y(x^2+1)^2\right)=x(x^2+1)^2$	M1	
	A1√	
$y(x^2+1)^2=\int x(x^2+1)^2\,dx$		
$y(x^2+1)^2=\frac{1}{6}\left(x^2+1\right)^3+c$	M1 A1	
$y(0)=1 \Rightarrow c=\frac{5}{6}$	m1	
$y=\frac{1}{6}\left(x^2+1\right)+\frac{5}{6(x^2+1)^2}$	A1	9
Total		9

Question 4: Jun 2009

IF is $e^{\int -\tan x\,dx}$	M1	
$=e^{\ln(\cos x)\ (+c)}$	A1	
$=(k)\cos x$	A1F	
$\cos x\frac{dy}{dx}-y\tan x\cos x=2\sin x\cos x$		
$\frac{d}{dx}\left(y\cos x\right)=2\sin x\cos x$	M1	
$y\cos x=\int 2\sin x\cos x\,dx\,dx$	A1F	
$y\cos x=\int \sin 2x\,dx$	m1	
$y\cos x=-\frac{1}{2}\cos 2x(+c)$	A1	
$2=-\frac{1}{2}+c$	m1	
$c=\frac{5}{2}$		
$y\cos x=-\frac{1}{2}\cos 2x+\frac{5}{2}$	A1	9
Total		9

Question 5: Jun 2007 Q5

(a)	$u=\frac{dy}{dx}+x \Rightarrow \frac{du}{dx}=\frac{d^2y}{dx^2}+1$	M1A1	
	$\left(x^2-1\right)\left(\frac{du}{dx}-1\right)-2x(u-x)=x^2+1$	M1	
	DE $\Rightarrow (x^2-1)\frac{du}{dx}-2xu=0$		
	$\Rightarrow \frac{du}{dx}=\frac{2xu}{x^2-1}$	A1	4
(b)	$\int \frac{1}{u}\,du=\int \frac{2x}{x^2-1}\,dx$	M1 A1	
	$\ln u=\ln\lvert x^2-1\rvert+\ln A$	A1A1	
	$u=A(x^2-1)$	A1	5
(c)	$\frac{dy}{dx}+x=A(x^2-1)$	M1	
	$\frac{dy}{dx}=A(x^2-1)-x$		
	$y=A\left(\frac{x^3}{3}-x\right)-\frac{x^2}{2}+B$	M1	
		A1ft	3
	Total		12

Question 6: Jun 2008 Q4

a)	$u=\frac{dy}{dx}\Rightarrow\frac{du}{dx}=\frac{d^2y}{dx^2}$	M1	
	$x\frac{du}{dx}-u=3x^2\Rightarrow\frac{du}{dx}-\frac{1}{x}u=3x$	A1	2
b)	IF is $\exp\left(\int-\frac{1}{x}\,dx\right)$	M1	
	$=e^{-\ln x}$	A1	
	$=x^{-1}$ or $\frac{1}{x}$	A1	
	$\frac{d}{dx}\left[ux^{-1}\right]=3$	M1	
	$\Rightarrow ux^{-1}=3x+A$	m1	
	$u=3x^2+Ax$	A1	6
(c)	$\frac{dy}{dx}=3x^2+Ax$	M1	
	$y=x^3+\frac{Ax^2}{2}+B$	A1F	2
	Total		**10**

Question 7: Jun 2006 Q3

(a)	IF is $e^{\int\cot x\,dx}$	M1	
	$=e^{\ln\sin x}$	A1	
	$=\sin x$	A1	3
(b)	$\frac{d}{dx}(y\sin x)=2\sin x\cos x$	M1 A1	
	$y\sin x=\int\sin 2x\,dx$	M1	
	$y\sin x=-\frac{1}{2}\cos 2x+c$	A1	
	$y=2$ when $x=\frac{\pi}{2}\Rightarrow$		
	$2\sin\frac{\pi}{2}=-\frac{1}{2}\cos\pi+c$	m1	
	$c=\frac{3}{2}\Rightarrow y\sin x=\frac{1}{2}(3-\cos 2x)$	A1	6
	Total		**9**

Question 8: Jun 2006 Q6

(a)	$u=\frac{dy}{dx}+2y\Rightarrow\frac{du}{dx}=\frac{d^2y}{dx^2}+2\frac{dy}{dx}$	M1 A1	
	LHS of DE$\Rightarrow\frac{du}{dx}-2\frac{dy}{dx}+4\frac{dy}{dx}+4y$		
	LHS: $\frac{du}{dx}+2(u-2y)+4y$	M1	
	$\Rightarrow\frac{du}{dx}+2u=e^{-2x}$	A1	4
(b)	IF is $e^{\int 2dx}=e^{2x}$	B1	
	$\frac{d}{dx}\left[ue^{2x}\right]=1$	M1 A1	
	$\Rightarrow ue^{2x}=x+A$	A1	
	$\Rightarrow u=xe^{-2x}+Ae^{-2x}$	A1	5

Question 9: Jan 2007 Q3

(a)	IF is $\exp\left(\int\frac{2}{x}dx\right)$	M1	
	$=e^{2\ln x}$	A1	
	$=x^2$	A1	3
(b)	$\frac{d}{dx}\left[yx^2\right]=3x^2(x^3+1)^{\frac{1}{2}}$	M1A1	
	$\Rightarrow yx^2=\frac{2}{3}(x^3+1)^{\frac{3}{2}}+A$	m1	
		A1	
	$\Rightarrow 4=\frac{2}{3}(9)^{\frac{3}{2}}+A$	m1	
	$\Rightarrow A=-14$		
	$\Rightarrow y=x^{-2}\left\{\frac{2}{3}(x^3+1)^{\frac{3}{2}}-14\right\}$	A1	6
	Total		**9**

Question 10: Jan 2009 Q2

(a)	IF is $e^{\int-\frac{2}{x}dx}$	M1	
	$=e^{-2\ln x}$	A1	
	$=e^{\ln x^{-2}}=x^{-2}=\frac{1}{x^2}$	A1	3
(b)	$\frac{d}{dx}\left(\frac{y}{x^2}\right)=\frac{1}{x^2}x$	M1	
		A1	
	$\frac{y}{x^2}=\int\frac{1}{x}dx=\ln x+c$	A1	
	$y=x^2\ln x+cx^2$	A1	4
	Total		**7**