Constant Acceleration Equations

For an object that has an initial velocity u and that is moving in a straight line with constant acceleration a, the following equations connect the final velocity v and displacement s in a given time t.

$$v = u + at \tag{1}$$

$$s = \frac{1}{2}(u + v)t \tag{2}$$

$$s = ut + \frac{1}{2}at^2 \tag{3}$$

$$s = vt - \frac{1}{2}at^2 \tag{4}$$

$$v^2 = u^2 + 2as \tag{5}$$

Note: These equations cannot be used if the acceleration is not constant.

Worked Example 1.

A motorbike joins a motorway traveling at 10 m s^{-1}, and increases speed to 30 m s^{-1} with a constant acceleration of 1.25 m s^{-2} along the straight road. How much time does this take, and how far does the bike travel in this time?

Solution.

Firstly consider what information has been given, namely $u = 10$ m s^{-1}, $v = 30$ m s^{-1} and $a = 1.25$ m s^{-2}.

The question asks for the values of t and then s.

The equation that connects u, v, a and t is (1). Inserting the known values into (1) gives:

$$\begin{aligned} 30 &= 10 + 1.25t \\ 20 &= 1.25t \\ \Rightarrow t &= 16 \text{ s} \end{aligned}$$

Now either equation (2), (3), (4) or (5) can be used to calculate s. For example, using (2):

$$s = \tfrac{1}{2}(u + v)t = \tfrac{1}{2}(10 + 30) \times 16 = 320 \text{ m}.$$

Worked Example 2.

The driver of a car traveling along a straight road sees that the traffic lights, 40 metres away, have turned to red. Given that after 4 seconds the car stops exactly at the traffic lights, what is the deceleration of the car?

Solution.

Again, consider what information has been given, namely $s = 40$ m and $t = 4$ s.

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It can also be deduced that because the car was at rest when it reached the traffic lights, $v = 0$ m s $^{-1}$. The question asks for the deceleration and so involves a.
The equation that connects s, t, v and a is (4). Inserting the known values into (4) gives:

$$\begin{aligned} s &= vt - \tfrac{1}{2}at^2 \\ 40 &= 0 \times 4 - \tfrac{1}{2} \times a \times 4^2 \\ 40 &= -8a \\ \Rightarrow a &= -5.0 \text{ m s}^{-2} \text{ (to 2 s.f.)} \end{aligned}$$

Therefore, the car decelerates at a rate of 5 m s^{-2}

Worked Example 3.

A child throws a tennis ball vertically upwards at 7.7 m s^{-1} from ground level. Assuming that no resistance forces act on the ball, so that it moves only under the influence of gravity (g = 9.81 m s^{-2}), what is the maximum height the tennis ball reaches?
Solution.
Here, consider what information is already known and what can be used.
It is known that $u = 7.7$ m s^{-1} and $a = -9.81$ m s^{-2} as gravity acts downwards and the positive direction is upwards. It can also be deduced that at the maximum height $v = 0$ m s^{-1}.

Therefore, using (5):

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0 &= 7.7^2 + 2 \times (-9.81) \times s \\ 0 &= 59.29 - 19.62 \times s \\ \Rightarrow s &= 3.0 \text{ m (to 2 s.f.)} \end{aligned}$$

Exercises

1. A rally car accelerates from 10 m s^{-1} to 58 m s^{-1} in 8 seconds as it moves along a straight road. Given that the acceleration is constant, what is the acceleration of the car?

2. A bus traveling along a straight road accelerates at 2 m s^{-2}, for 4 seconds, covering a distance of 44 metres. After the 4 seconds what velocity is the bus traveling at?

3. A rowing boat crosses the finish line at 12 m s^{-1} and carries on in a straight line. If it immediately decelerates at 4 m s^{-2} until it comes to rest, how far past the finish line will the rowing boat come to a stop?

4. During the middle of an 800 metre race an athlete running at 6.8 m s^{-1} constantly accelerates, along part of the straight, to 8 m s $^{-1}$ in order to get in a better position for the final lap. Given this took 2 seconds, what distance did the athlete cover in this time?

5. A train leaves a station from rest and travels along a straight track. If after 20 seconds the train is 500 metres from the station, what is the acceleration of the train?

6. A lift at the ground floor rises vertically from rest with constant a acceleration of 0.6 m s^{-2}. If it passes the first floor at 1.8 m s^{-1}, how high is the first floor?

Answers (all to 2 s.f.)

1. 6 m s $^{-2}$ 2. 15 m s^{-1} 3. 18 m 4. 15 m 5. 2.5 m s^{-2} 6. 2.7 m

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