Collisions in 2D

Specifications

Collisions in two dimensions

Momentum as a vector.

Impulse as a vector. $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ and $\mathbf{I}=\mathbf{F}t$ will be required.

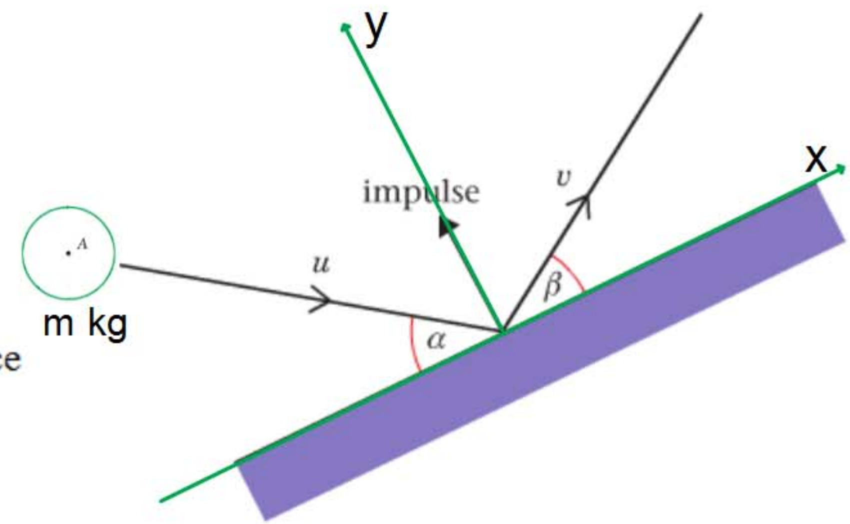
Conservation of momentum in two dimensions. $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

Coefficient of restitution and Newton's experimental law.

Impacts with a fixed surface. The impact may be at any angle to the surface. Candidates may be asked to find the impulse on the body. Questions that require the use of trigonometric identities will not be set.

Oblique Collisions Collisions between two smooth spheres. Candidates will be expected to consider components of velocities parallel and perpendicular to the line of centres.

Collision on a smooth flat surface



The impulse applied by the surface on the particle A of mass m is

the VECTOR $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

The impulse is perpendicular to the surface

We consider the set of axes such that the x-*axis* is parallel to the surface and the y-*axis* is perpendicular to the surface.

The components of the impulse are: $\mathbf{I} = \begin{pmatrix} 0 \\ I \end{pmatrix} = m\begin{pmatrix} v_x \\ v_y \end{pmatrix} - m\begin{pmatrix} u_x \\ u_y \end{pmatrix}$

This gives : $v_x = u_x$

- The component of the velocity parallel to the surface is UNCHANGED

And : $I = mv_y - mu_y$

- Newton's law of restitution applies to the component of the velocity perpendicular to surface

ie: $v_y = -eu_y$ where e is the coefficient of restitution

With the notations used in the sketch above: $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$$\begin{pmatrix} 0 \\ I \end{pmatrix} = m\begin{pmatrix} v\cos\beta \\ v\sin\beta \end{pmatrix} - m\begin{pmatrix} uCos\alpha \\ -u\sin\alpha \end{pmatrix}$$

This gives:

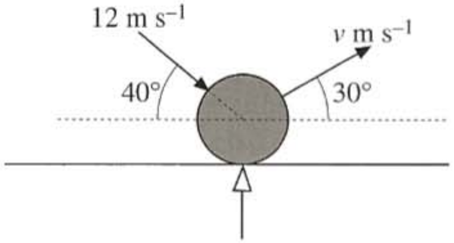
- $vCos\beta = uCos\alpha$
- $I = vSin\beta + uSin\alpha$
- $e = \dfrac{vSin\beta}{uSin\alpha}$

Also: Show that $e = \dfrac{Tan\beta}{Tan\alpha}$.

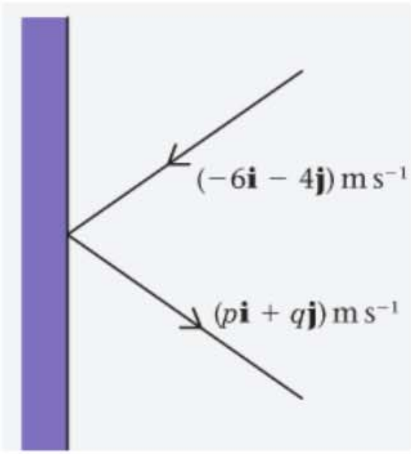
a) 10.61 m/s
b) 0.688
c) 0.911 Ns

Examples:

- In the Basque game of pelota, a ball of mass 70 grams hits the floor of the court with a speed of $12\,\text{m s}^{-1}$ at 40° to the horizontal, and rebounds at 30° to the horizontal, as shown in Fig. 7.10. Assuming that the contact between the ball and the floor is smooth, calculate
  - (a) the speed of the ball as it rebounds,
  - (b) the coefficient of restitution, e,
  - (c) the impulse given to the ball from the floor.



- A small smooth ball of mass 2 kg is moving in the xy-plane and collides with a smooth fixed vertical wall which contains the y-axis. The velocity of the ball just before impact is $(-6\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$. Find the speed of the ball immediately after the impact,



$2\sqrt{5}ms^{-1} = 4.47ms^{-1}$

Exercises:

- A smooth sphere S is moving on a smooth horizontal plane with speed u when it collides with a smooth fixed vertical wall. At the instant of collision the direction of motion of S makes an angle of $\tan^{-1}\frac{3}{4}$ with the wall. The coefficient of restitution between S and the wall is $\frac{1}{3}$.

  Find the speed of S immediately after the collision.

- A smooth sphere S is moving on a smooth horizontal plane with speed u when it collides with a smooth fixed vertical wall. At the instant of collision the direction of motion of S makes an angle of 30° with the wall. Immediately after the collision the speed of S is $\frac{7}{8}u$.

  Find the coefficient of restitution between S and the wall.

- A small smooth ball is falling vertically. The ball strikes a smooth plane which is inclined at an angle of 30° to the horizontal. Immediately before striking the plane the ball has speed $8\,\text{m s}^{-1}$. The coefficient of restitution between the ball and the plane is $\frac{1}{4}$.

  Find the exact value of the speed of the ball immediately after the impact.

- A small smooth ball of mass 800 g is moving in the xy-plane and collides with a smooth fixed vertical wall which contains the y-axis. The velocity of the ball just before impact is $(5\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$. The coefficient of restitution between the sphere and the wall is $\frac{1}{2}$. Find

  the velocity of the ball immediately after the impact, as a vector.

- A smooth snooker ball strikes a smooth cushion with speed $8\,\text{m s}^{-1}$ at an angle of 45° to the cushion. Given that the coefficient of restitution between the ball and the cushion is $\frac{2}{5}$, find the magnitude and direction of the velocity of the ball after the impact.

Answers

$v = \frac{u\sqrt{17}}{5}$

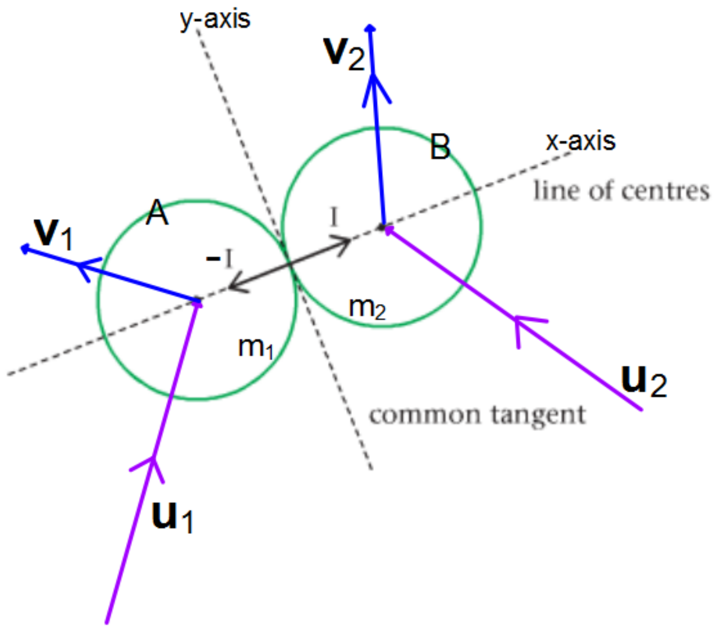
$e = \frac{1}{4}$

$\sqrt{19}\,\text{m s}^{-1}$

$v = -2.5\mathbf{i} - 3\mathbf{j}$

$6.09\,\text{m s}^{-1}$ at 21.8° to the cushion

Two spheres/particles colliding in 2D



- **The reaction between the two spheres acts along the line of centres, so the impulse affecting each sphere also acts along the line of centres.**

The impulse applied by A onto B is the vector $\mathbf{I} = m_2\mathbf{v}_2 - m_2\mathbf{u}_2$

The impulse applied by A onto B is the vector $-\mathbf{I} = m_1\mathbf{v}_1 - m_1\mathbf{u}_1$

The impulses act along the line of centres

We consider the set of axes such that the x - *axis* is parallel to the line of centres and the y - *axis* is perpendicular to the line of centres.

The components of the impulse are : $\begin{pmatrix} I \\ 0 \end{pmatrix} = m_2\begin{pmatrix} v_{2x} \\ v_{2y} \end{pmatrix} - m_2\begin{pmatrix} u_{2x} \\ u_{2y} \end{pmatrix}$

and $\begin{pmatrix} -I \\ 0 \end{pmatrix} = m_1\begin{pmatrix} v_{1x} \\ v_{1y} \end{pmatrix} - m_1\begin{pmatrix} u_{1x} \\ u_{1y} \end{pmatrix}$

This gives: $v_{2y} = u_{2y}$ *and* $v_{1y} = u_{1y}$

- The components of the velocities perpendicular to the line of centres are UNCHANGED in the impact

It also gives: $I = m_2 v_{2x} - m_2 u_{2x}$ *and* $-I = m_1 v_{1x} - m_1 u_{1x}$

$$m_2 v_{2x} - m_2 u_{2x} = -\left(m_1 v_{1x} - m_1 u_{1x}\right)$$

$$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$$

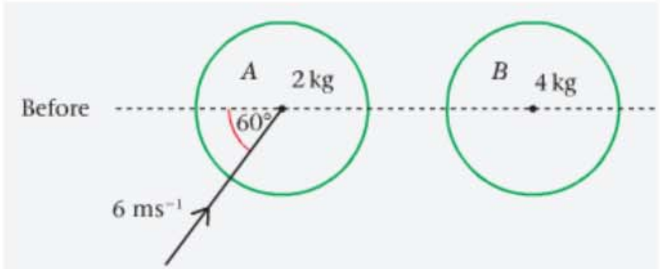
- The law of conservation of momentum applies PARALLEL to the line of centres.

- Newton's law of restitution applies to the components of the velocities parallel to the line of centres.

ie: $(v_{2x} - v_{1x}) = -e(u_{2x} - u_{1x})$

Examples:

A smooth sphere A, of mass 2 kg and moving with speed $6\,\text{m s}^{-1}$ collides obliquely with a smooth sphere B of mass 4 kg. Just before the impact B is stationary and the velocity of A makes an angle of 60° with the lines of centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{4}$. Find the magnitude and direction of the velocities of A and B immediately after the impact.



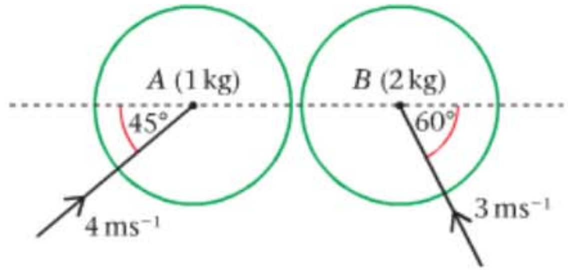
Speed of B: 1.25ms^{-1} along the line of centres.

Speed of A: $\sqrt{27.25}\text{ms}^{-1} = 5.22\text{ms}^{-1}$ at 84.5° to the line of centres.

A small smooth sphere A of mass 1 kg collides with a small smooth sphere B of mass 2 kg. Just before the impact A is moving with a speed of $4\,\text{m s}^{-1}$ in a direction at 45° to the line of centres and B is moving with speed $3\,\text{m s}^{-1}$ at 60° to the line of centres, as shown in the diagram. The coefficient of restitution between the spheres is $\frac{3}{4}$. Find

Work out the magnitude of speed of A and B after the impact

Work out the direction of the speed vectors (angle with the line of centres)

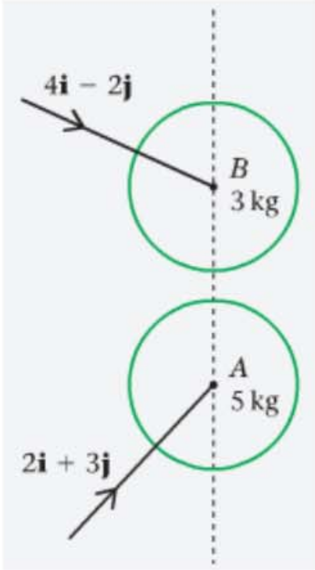


$v_A = 3.596\ \text{ms}^{-1}$ at an angle of 51.9°

$v_B = 2.793\ \text{ms}^{-1}$ at an angle of 68.5°

Initial speeds given as vectors

A smooth sphere A of mass 5 kg is moving on a smooth horizontal surface with velocity $(2\mathbf{i} + 3\mathbf{j})\ \text{m s}^{-1}$. Another smooth sphere B of mass 3 kg and the same radius as A is moving on the same surface with velocity $(4\mathbf{i} - 2\mathbf{j})\ \text{m s}^{-1}$. The spheres collide when their line of centres is parallel to $\mathbf{j}$. The coefficient of restitution between the spheres is $\frac{3}{5}$. Find the velocities of both spheres after the impact.



Velocity of A is $2\mathbf{i}\ \text{m s}^{-1}$, and velocity of B is $(4\mathbf{i} + 3\mathbf{j})\ \text{m s}^{-1}$.

Exercises:

- A smooth sphere A, of mass 2 kg and moving with speed $6\,\text{m}\,\text{s}^{-1}$ collides obliquely with a smooth sphere B of mass 4 kg. Just before the impact B is stationary and the velocity of A makes an angle of $10°$ with the lines of centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{2}$. Find the magnitudes and directions of the velocities of A and B immediately after the impact.

- A smooth sphere A, of mass 4 kg and moving with speed $4\,\text{m}\,\text{s}^{-1}$ collides obliquely with a smooth sphere B of mass 2 kg. Just before the impact B is stationary and the velocity of A makes an angle of $30°$ with the lines of centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{3}$. Find the magnitudes and directions of the velocities of A and B immediately after the impact.

- A smooth sphere A, of mass 3 kg and moving with speed $5\,\text{m}\,\text{s}^{-1}$ collides obliquely with a smooth sphere B of mass 4 kg. Just before the impact B is stationary and the velocity of A makes an angle of $45°$ with the lines of centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{2}$. Find the magnitudes and directions of the velocities of A and B immediately after the impact.

- A small smooth sphere A of mass m and a small smooth sphere B of the same radius but mass $2m$ collide. At the instant of impact, B is stationary and the velocity of A makes an angle θ with the line of centres. The direction of motion of A is turned through $90°$ by the impact. The coefficient of restitution between the spheres is e. Show that

$$\tan^2\theta = \frac{2e-1}{3}.$$

$1.04\,\text{m}\,\text{s}^{-1}$ perpendicular to the line of centres
$2.94\,\text{m}\,\text{s}^{-1}$ parallel to the line of centres
$\frac{4\sqrt{39}}{9}\,\text{m}\,\text{s}^{-1}$ at $46.1°$ to the line of centres
$\frac{16\sqrt{3}}{9}\,\text{m}\,\text{s}^{-1}$ along the line of centres
$\frac{25}{7}\,\text{m}\,\text{s}^{-1}$ at $81.9°$ to the line of centres
$\frac{45\sqrt{2}}{28}\,\text{m}\,\text{s}^{-1}$ along the line of centres