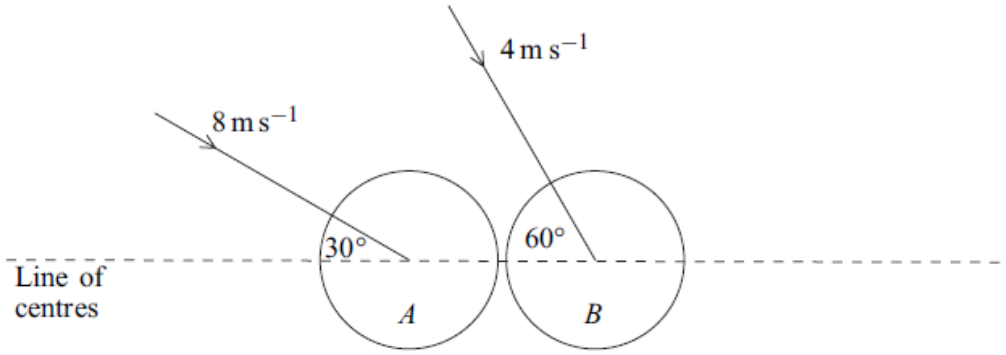
Collisions in 2-D – Exam questions

Question 1: June 2006 – Q6

Two smooth billiard balls A and B, of identical size and equal mass, move towards each other on a horizontal surface and collide. Just before the collision, A has velocity $8\,\text{m s}^{-1}$ in a direction inclined at $30°$ to the line of centres of the balls, and B has velocity $4\,\text{m s}^{-1}$ in a direction inclined at $60°$ to the line of centres, as shown in the diagram.



The coefficient of restitution between the balls is $\frac{1}{2}$.

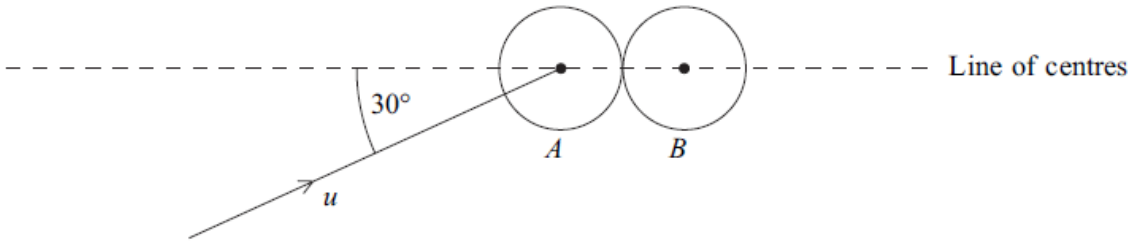
(a) Find the speed of B immediately after the collision. *(9 marks)*

(b) Find the angle between the velocity of B and the line of centres of the balls immediately after the collision. *(2 marks)*

Question 2: June 2007 – Q6

A smooth spherical ball, A, is moving with speed u in a straight line on a smooth horizontal table when it hits an identical ball, B, which is at rest on the table.

Just before the collision, the direction of motion of A makes an angle of $30°$ with the line of the centres of the two balls, as shown in the diagram.



The coefficient of restitution between A and B is e.

(a) Given that $\cos 30° = \frac{\sqrt{3}}{2}$, show that the speed of B immediately after the collision is

$$\frac{\sqrt{3}}{4}u(1+e)$$ *(5 marks)*

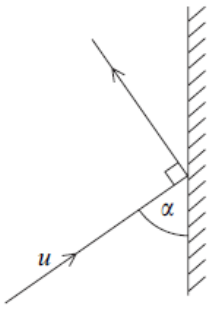
(b) Find, in terms of u and e, the components of the velocity of A, parallel and perpendicular to the line of centres, immediately after the collision. *(3 marks)*

(c) Given that $e = \frac{2}{3}$, find the angle that the velocity of A makes with the line of centres immediately after the collision. Give your answer to the nearest degree. *(3 marks)*

Question 3: June 2008 – Q6

A small smooth ball of mass m, moving on a smooth horizontal surface, hits a smooth vertical wall and rebounds. The coefficient of restitution between the wall and the ball is $\frac{3}{4}$.

Immediately before the collision, the ball has velocity u and the angle between the ball's direction of motion and the wall is α. The ball's direction of motion immediately after the collision is at right angles to its direction of motion before the collision, as shown in the diagram.



(a) Show that $\tan\alpha = \frac{2}{\sqrt{3}}$. *(5 marks)*

(b) Find, in terms of u, the speed of the ball immediately after the collision. *(2 marks)*

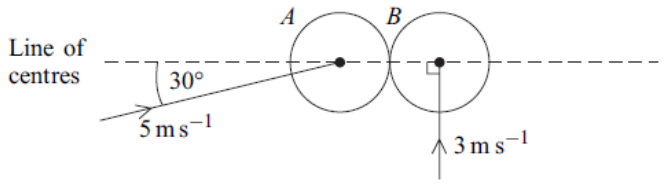
(c) The force exerted on the ball by the wall acts for 0.1 seconds.

Given that $m = 0.2\,\text{kg}$ and $u = 4\,\text{m s}^{-1}$, find the average force exerted by the wall on the ball. *(6 marks)*

Question 4: June 2009 – Q5

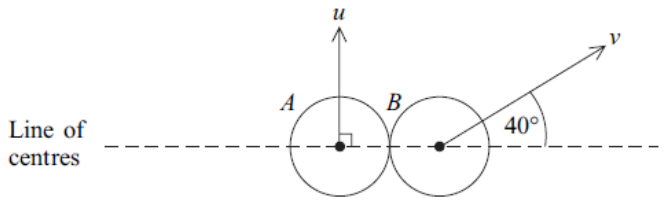
Two smooth spheres, A and B, of equal radii and different masses are moving on a smooth horizontal surface when they collide.

Just before the collision, A is moving with speed $5\,\text{m s}^{-1}$ at an angle of 30° to the line of centres of the spheres, and B is moving with speed $3\,\text{m s}^{-1}$ perpendicular to the line of centres, as shown in the diagram below.



Before collision

Immediately after the collision, A and B move with speeds u and v in directions which make angles of 90° and 40° respectively with the line of centres, as shown in the diagram below.



After collision

(a) Show that $v = 4.67\,\text{m s}^{-1}$, correct to three significant figures. *(3 marks)*

(b) Find the coefficient of restitution between the spheres. *(3 marks)*

(c) Given that the mass of A is 0.5 kg, show that the magnitude of the impulse exerted on A during the collision is 2.17 N s, correct to three significant figures. *(3 marks)*

(d) Find the mass of B. *(3 marks)*

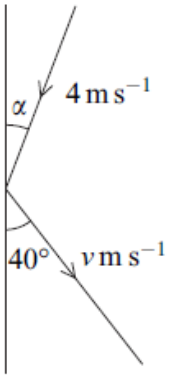
Question 5: June 2010 – Q5

A smooth sphere is moving on a smooth horizontal surface when it strikes a smooth vertical wall and rebounds.

Immediately before the impact, the sphere is moving with speed $4\,\text{m s}^{-1}$ and the angle between the sphere's direction of motion and the wall is α.

Immediately after the impact, the sphere is moving with speed $v\,\text{m s}^{-1}$ and the angle between the sphere's direction of motion and the wall is $40°$.

The coefficient of restitution between the sphere and the wall is $\frac{2}{3}$.



(a) Show that $\tan\alpha = \frac{3}{2}\tan 40°$. *(3 marks)*

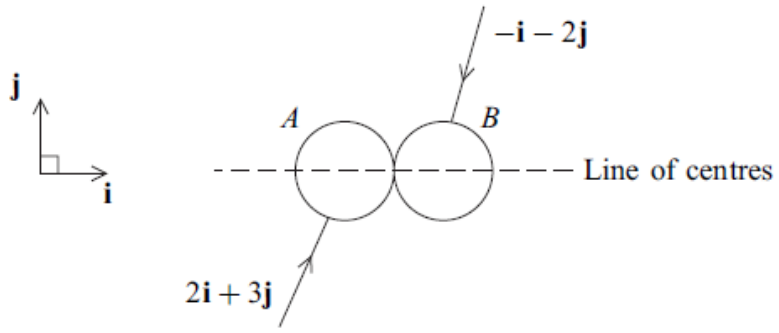
(b) Find the value of v. *(3 marks)*

Question 6: June 2010 – Q6

Two smooth spheres, A and B, have equal radii and masses 1 kg and 2 kg respectively.

The sphere A is moving with velocity $(2\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and the sphere B is moving with velocity $(-\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$ on the same smooth horizontal surface.

The spheres collide when their line of centres is parallel to the unit vector $\mathbf{i}$, as shown in the diagram.



(a) Briefly state why the components of the velocities of A and B parallel to the unit vector $\mathbf{j}$ are not changed by the collision. *(1 mark)*

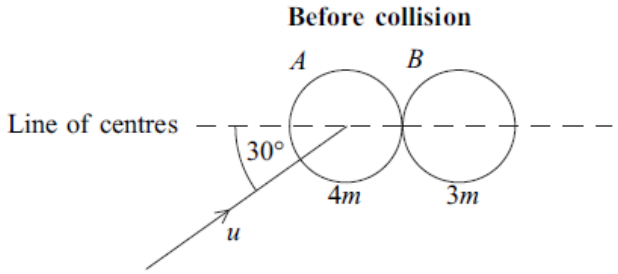
(b) The coefficient of restitution between the spheres is 0.5.

Find the velocities of A and B immediately after the collision. *(6 marks)*

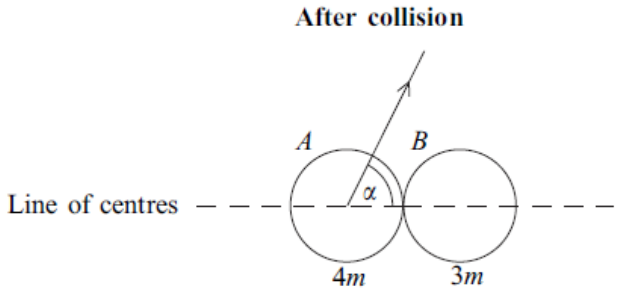
Question 7: June 2011 – Q7

Two smooth spheres, A and B, have equal radii and masses $4m$ and $3m$ respectively. The sphere A is moving on a smooth horizontal surface and collides with the sphere B, which is stationary on the same surface.

Just before the collision, A is moving with speed u at an angle of 30° to the line of centres, as shown in the diagram below.



Immediately after the collision, the direction of motion of A makes an angle α with the line of centres, as shown in the diagram below.



The coefficient of restitution between the spheres is $\frac{5}{9}$.

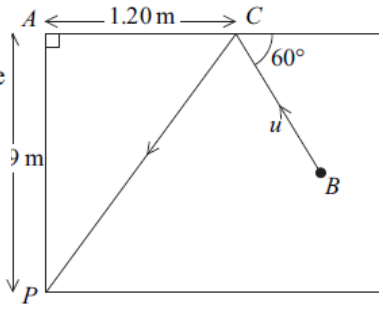
(a) Find the value of α. *(10 marks)*

(b) Find, in terms of m and u, the magnitude of the impulse exerted on B during the collision. *(3 marks)*

Question 8: June 2012 – Q4

The diagram shows part of a horizontal snooker table of width 1.69 m.

A player strikes the ball B directly, and it moves in a straight line. The ball hits the cushion of the table at C before rebounding and moving to the pocket at P at the corner of the table, as shown in the diagram. The point C is 1.20 m from the corner A of the table. The ball has mass 0.15 kg and, immediately before the collision with the cushion, it has velocity u in a direction inclined at 60° to the cushion. The **table** and the **cushion** are modelled as smooth.



(a) Find the coefficient of restitution between the ball and the cushion. *(5 marks)*

(b) Show that the magnitude of the impulse on the cushion at C is approximately $0.236u$. *(4 marks)*

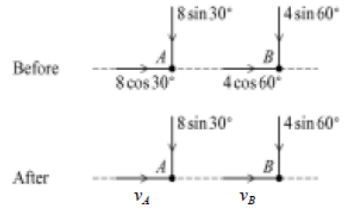
(c) Find, in terms of u, the time taken between the ball hitting the cushion at C and entering the pocket at P. *(3 marks)*

(d) Explain how you have used the assumption that the cushion is smooth in your answers. *(1 mark)*

Collisions in 2-D – Exam questions - MS

Question 1: June 2006 – Q6

6(a) Components of velocities:



conservation of linear momentum along the line of centres:

$m\times 8\cos 30 + m\times 4\cos 60 = mv_A + mv_B$ — M1A1

$v_A + v_B = 8.93$

Law of restitution along the line of centre:

$\dfrac{v_B - v_A}{8\cos 30 - 4\cos 60} = \dfrac{1}{2}$ — M1A1

$v_B - v_A = 2.46$

$v_B = 5.70$ — m1

A1F

momentum of B perpendicular to the line of centres is unchanged — B1

Speed of $B = \sqrt{u_B^{\,2} + v_B^{\,2}}$

$= \sqrt{(4\sin 60)^2 + (5.70)^2}$ — m1

$= 6.67$ — A1F — 9

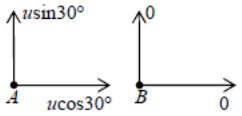
(b) direction of $B = \tan^{-1}\dfrac{4\sin 60}{5.70} = 31.3°$ — m1 A1F — 2

Total — 11

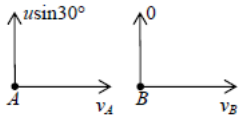
Question 2: June 2007 – Q6

6(a)

Before:



After:



Con. of Mom. along the line of centres:

$mu\cos 30° = mv_A + mv_B$ — M1

$v_A + v_B = \dfrac{\sqrt{3}}{2}u$ ----------(1) — A1

Newton's experimental law :

$e = \dfrac{v_B - v_A}{u\cos 30° - 0}$ — M1

$v_B - v_A = \dfrac{\sqrt{3}}{2}ue$ ----------(2) — A1

Solving (1) and (2) :

$v_B = \dfrac{\sqrt{3}}{4}u(1+e)$ — A1 — 5

(b) $\perp$ $u\sin 30° = \dfrac{1}{2}u$ — B1

$\parallel$ $v_A = \dfrac{\sqrt{3}}{2}u - \dfrac{\sqrt{3}}{4}u(1+e)$ — M1 A1F — 3

$v_A = \dfrac{\sqrt{3}}{4}u(1-e)$

(c) $\alpha = \tan^{-1}\dfrac{\frac{1}{2}u}{\frac{\sqrt{3}}{4}u\left(1-\frac{2}{3}\right)}$ — M1 A1F

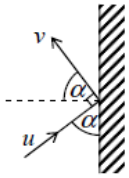
$\alpha = \tan^{-1}\dfrac{6}{\sqrt{3}}$

$\alpha = 74°$ — A1F — 3

Total — 11

Question 3: June 2008 – Q6

(a)



Parallel to the wall : velocity is unchanged

$u\cos\alpha = v\sin\alpha$ — M1

Perpendicular to the wall : Law of Restitution

$\dfrac{v\cos\alpha}{u\sin\alpha} = \dfrac{3}{4}$ — M1

$\dfrac{v\cos\alpha}{v\tan\alpha\sin\alpha} = \dfrac{3}{4}$ — m1

$\dfrac{\cos^2\alpha}{\sin^2\alpha} = \dfrac{3}{4}$ — m1

$\tan^2\alpha = \dfrac{4}{3}$

$\tan\alpha = \dfrac{2}{\sqrt{3}}$ — A1 — 5

(b)

$v = \dfrac{u}{\tan\alpha}$ — M1

$v = \dfrac{\sqrt{3}}{2}u$ or $0.866u$ — A1 — 2

(c) Magnitude of Impulse = Change in momentum perpendicular to the wall — M1

$= 0.2\times v\cos\alpha - (-0.2\times 4\sin\alpha)$ — A1 A1

$= 0.2\times\dfrac{\sqrt{3}}{2}\times 4\cos\alpha + 0.2\times 4\sin\alpha$ — m1

$= 1.06$ Ns — A1F

Average Force $= \dfrac{1.06}{0.1} = 10.6$ N — A1F — 6

Total — 13

Question 4: June 2009 – Q5

5(a) Momentum of B perpendicular to the line of centres is unchanged

$m_B v\sin 40° = 3m_B$ — M1A1

$v = 4.667$ ms^{-1} $= 4.67$ ms^{-1} (3sf) — A1 — 3

(b) $e = \dfrac{4.67\cos 40°}{5\cos 30°}$ — M1A1

$e = 0.826$ — A1F — 3

(c) Impulse on A = change in momentum of A along the line of centres

$= 0.5\times 5\cos 30° = 2.165$ — M1A1

$= 2.17$ Ns — A1 — 3

(d) $2.165 = m_B(4.667)\cos 40°$ — M1A1

$m_B = 0.6056 = 0.606$ kg (3sf) — A1F — 3

Total — 12

Question 5: June 2010 – Q5

5(a)	Parallel to the wall		
	$4\cos\alpha = v\cos 40^\circ$	M1	
	Perpendicular to the wall		
	$v\sin 40^\circ = \frac{2}{3}\times 4\sin\alpha$	M1	
	$\tan\alpha = \frac{3}{2}\tan 40^\circ$	A1	3
(b)	$\alpha = 51.5^\circ$	M1	
	$v = \frac{4\cos 51.5^\circ}{\cos 40^\circ}$	M1	
	$v = 3.25 \quad \text{ms}^{-1}$	A1	3
	Total		**6**

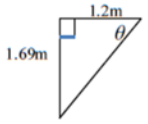
Question 6: June 2010 – Q6

6(a)	The spheres are smooth, no force acting in **j** direction	E1	1
(b)	$\mathbf{v}_A = a\mathbf{i} + b\mathbf{j}$		
	$\mathbf{v}_B = c\mathbf{i} + d\mathbf{j}$		
	C.L.M. along **i**: $1(2) + 2(-1) = 1(a) + 2(c)$	M1A1	
	$a + 2c = 0$		
	Restitution along **i**: $c - a = 0.5(2 - (-1))$	M1A1	
	$c - a = 1.5$		
	$c = 0.5$		
	$a = -1$		
	$\mathbf{v}_A = -\mathbf{i} + 3\mathbf{j}$	A1F	
	$\mathbf{v}_B = 0.5\mathbf{i} - 2\mathbf{j}$	A1F	6
	Total		**7**

Question 7: June 2011 – Q7

(a)	Momentum of A is unchanged $\perp$ to the line of centres		
	$4mu\sin 30 = 4mv_A\sin\alpha$	M1	
	$v_A = \frac{u}{2\sin\alpha}$ .....(1)	A1	
	C.L.M.:	M1A1	
	$4mu\cos 30 = 4mv_A\cos\alpha + 3mv_B$		
	$2\sqrt{3}u = 4v_A\cos\alpha + 3v_B$ .....(2)	A1F	
	Restitution along the line of centres:		
	$\frac{v_B - v_A\cos\alpha}{u\cos 30} = \frac{5}{9}$	M1A1	
	$v_B = v_A\cos\alpha + \frac{5\sqrt{3}u}{18}$ .....(3)	B1	
	$2\sqrt{3}u = 4\frac{u}{2\sin\alpha}\cos\alpha + 3\frac{u}{2\sin\alpha}\cos\alpha + \frac{15\sqrt{3}u}{18}$	m1	
	$\frac{7\sqrt{3}}{6} = \frac{7}{2\tan\alpha}$		
	$\tan\alpha = \sqrt{3}$		
	$\alpha = 60^\circ$ or $\frac{\pi}{3}$	A1F	10
b)	Impulse on B = Change in momentum of B along the line of centres		
	$v_B = \frac{u}{2\sin 60}\cos 60 + \frac{5\sqrt{3}u}{18}$		
	$v_B = \frac{u}{2\sqrt{3}} + \frac{5\sqrt{3}u}{18} \quad (= \frac{4\sqrt{3}}{9})$	M1	
	$I = 3m(\frac{u}{2\sqrt{3}} + \frac{5\sqrt{3}u}{18}) - 3m(0)$	M1	
	$I = \frac{4mu}{\sqrt{3}}$ or $2.31mu$	A1F	3
			13

Question 8: June 2012 – Q4

(a)			
	$\theta = \tan^{-1}\frac{1.69}{1.2} = 54.623^\circ$	B1	
	$u\cos 60^\circ = v\cos 54.623^\circ$	M1	
	$eu\sin 60^\circ = v\sin 54.623^\circ$	M1	
	$e = \frac{v\sin 54.623^\circ}{\frac{v\cos 54.623^\circ}{\cos 60^\circ}\times\sin 60^\circ}$	m1	
	$e = 0.813$ or 0.812	A1	5
(b)	$I = 0.15u\sin 60^\circ + 0.15v\sin 54.623^\circ$	M1A1	
	$= 0.15u\sin 60^\circ + 0.15\times\frac{u\cos 60^\circ}{\cos 54.623^\circ}\times\sin 54.623^\circ$	m1	
	$= 0.236u$	A1	4
(c)	Attempt at considering motion parallel or perpendicular to AC	M1	
	$t = \frac{1.2}{u\cos 60^\circ}$	M1	
	$t = \frac{12}{5u}$ or $\frac{2.4}{u}$	A1	3
	Alternative:		
	$CP = \frac{1.2}{\cos 54.623^\circ} \quad (= 2.072703844 \text{ m})$	(M1)	
	$t = \frac{\frac{1.2}{\cos 54.623^\circ}}{\frac{u\cos 60^\circ}{\cos 54.623^\circ}}$	(M1)	
	$= \frac{12}{5u}$ or $\frac{2.4}{u}$	(A1)	(3)
(d)	Velocity (momentum) parallel to the cushion is unchanged, or, Restitution only affects motion perpendicular to the cushion	E1	1
	Total		**13**