Trigonometry

Specifications

Trigonometry

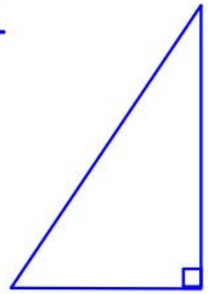
General solutions of trigonometric equations including use of exact values for the sine, cosine and tangent of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$.

E.g. $\sin 2x = \frac{\sqrt{3}}{2}$, $\cos\left(x + \frac{\pi}{6}\right) = -\frac{1}{\sqrt{2}}$, $\tan\left(\frac{\pi}{3} - 2x\right) = 1$

$\sin 2x = 0.3$, $\cos(3x - 1) = -0.2$

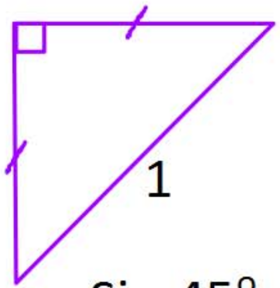
Sine, Cosine and Tangent of "usual" angles

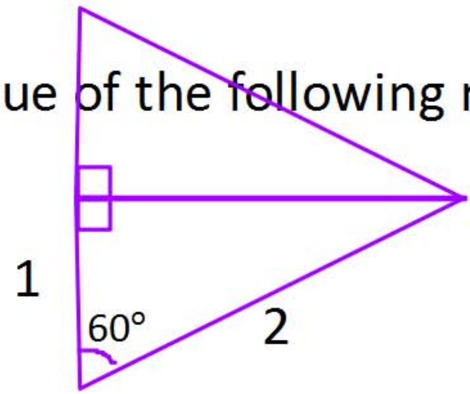
$S^{o}_{h}\ C^{a}_{h}\ T^{o}_{a}$



$$Sin\theta = \frac{opp}{hyp} \qquad Cos\theta = \frac{adj}{hyp} \qquad Tan\theta = \frac{opp}{adj}$$

Application: Work out the exact value of the following ratios

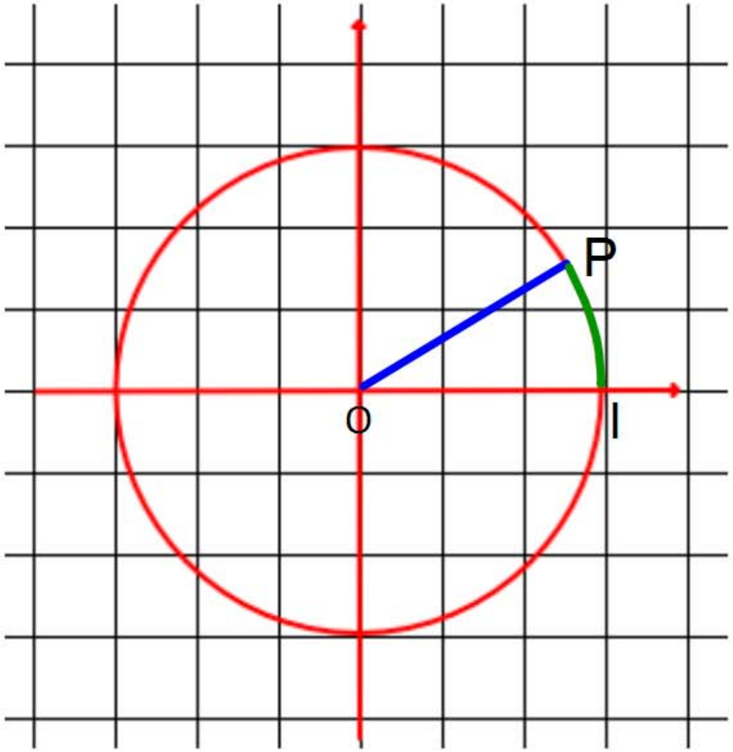




Sin 45° =	Sin 60° =	Sin 30° =
Cos 45° =	Cos 60° =	Cos 30° =
Tan 45° =	Tan 60° =	Tan 30° =

Measuring angles : degrees and radians

Consider a circle in a set of axis, centre O radius 1



Consider a point P on this circle.

A measure of the angle IOP, in radians, is the length of the arc IP

Equivalence table:

degrees	0°	180°	30°	45°	60°	90°	360°
radians							

$\times \frac{\pi}{180}$

Note: an angle in radian does not need to be express in term of π

EXERCISE

1 Change the following to degrees:

(a) $\frac{\pi}{2}$ rads, **(b)** 4π rads, **(c)** $\frac{\pi}{3}$ rads, **(d)** $\frac{\pi}{4}$ rads,

(e) $\frac{5\pi}{6}$ rads, **(f)** $\frac{3\pi}{4}$ rads, **(g)** $\frac{2\pi}{3}$ rads, **(h)** $\frac{11\pi}{6}$ rads,

(i) $\frac{7\pi}{4}$ rads, **(j)** $\frac{5\pi}{2}$ rads.

2 Convert the following to radians. Give your answers in terms of π.

(a) 180°, **(b)** 120°, **(c)** 36°, **(d)** 24°,

(e) 108°, **(f)** 540°, **(g)** 80°, **(h)** 225°,

(i) 405°, **(j)** 15°.

3 Convert the following to degrees, giving your answers to 3 sf.

(a) 2 rads, **(b)** 0.5 rads, **(c)** 1.8 rads, **(d)** 3 rads,

(e) 0.3 rads, **(f)** 2.3 rads, **(g)** 1.28 rads, **(h)** 1.6 rads,

(i) $\frac{7\pi}{11}$ rads, **(j)** $\frac{3\pi}{7}$ rads.

4 Convert the following to radians, giving your answers to 3 sf.

(a) 60°, **(b)** 150°, **(c)** 25°, **(d)** 305°,

(e) 96°, **(f)** 78°, **(g)** 14°, **(h)** 82°,

(i) 38°, **(j)** 500°.

EXERCISE

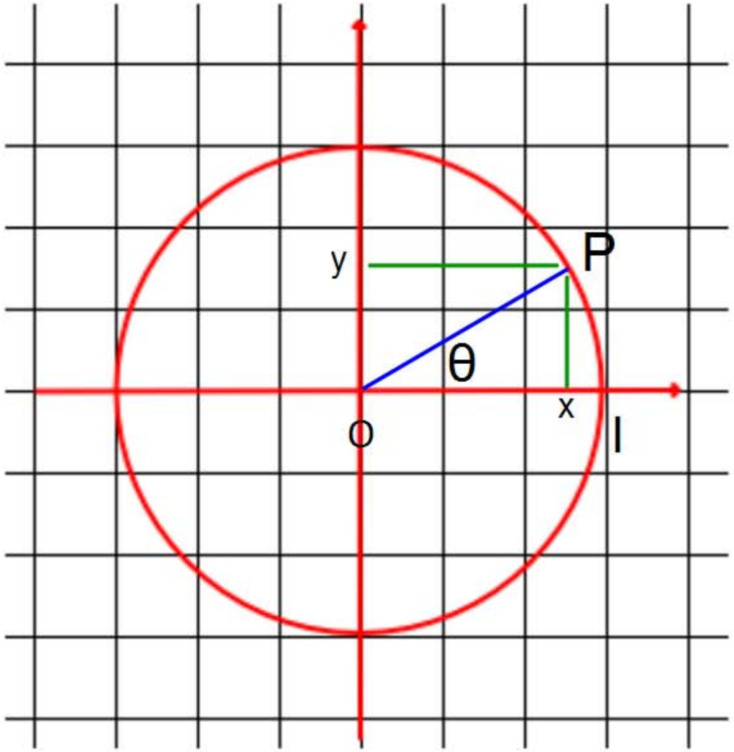
1 (a) 90°, (b) 720°, (c) 60°, (d) 45°, (e) 150°, (f) 135°, (g) 120°, (h) 330°, (i) 315°, (j) 450°.

2 (a) π, (b) $\frac{2\pi}{3}$, (c) $\frac{\pi}{5}$, (d) $\frac{2\pi}{15}$, (e) $\frac{3\pi}{5}$, (f) 3π, (g) $\frac{4\pi}{9}$, (h) $\frac{5\pi}{4}$, (i) $\frac{9\pi}{4}$, (j) $\frac{\pi}{12}$.

3 (a) 115°, (b) 28.6°, (c) 103°, (d) 172°, (e) 17.2°, (f) 132°, (g) 73.3°, (h) 91.7°, (i) 115°, (j) 77.1°.

4 (a) 1.05, (b) 2.62, (c) 0.436, (d) 5.32, (e) 1.68, (f) 1.36, (g) 0.244, (h) 1.43, (i) 0.663, (j) 8.73.

Extending the definition of Sine, Cosine and Tangent



A point P(x,y) is on the circumference of the circle, creating an angle with OI,

$$\text{Cos}(\theta) = x$$

$$\text{Sin}(\theta) = y$$

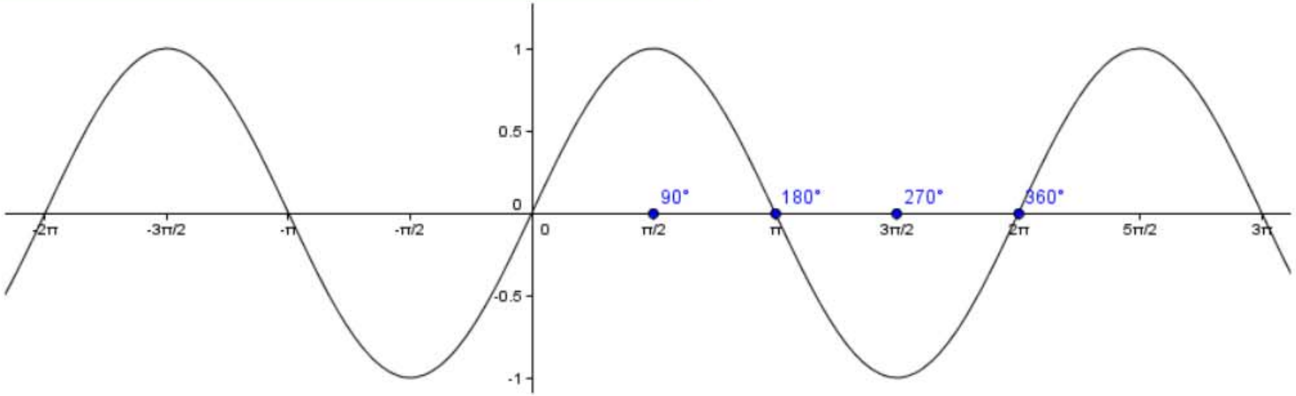
and

$$\text{Tan}(\theta) = \frac{Sin(\theta)}{Cos(\theta)} = \frac{y}{x}$$

The graphs of Sin, Cos and Tan

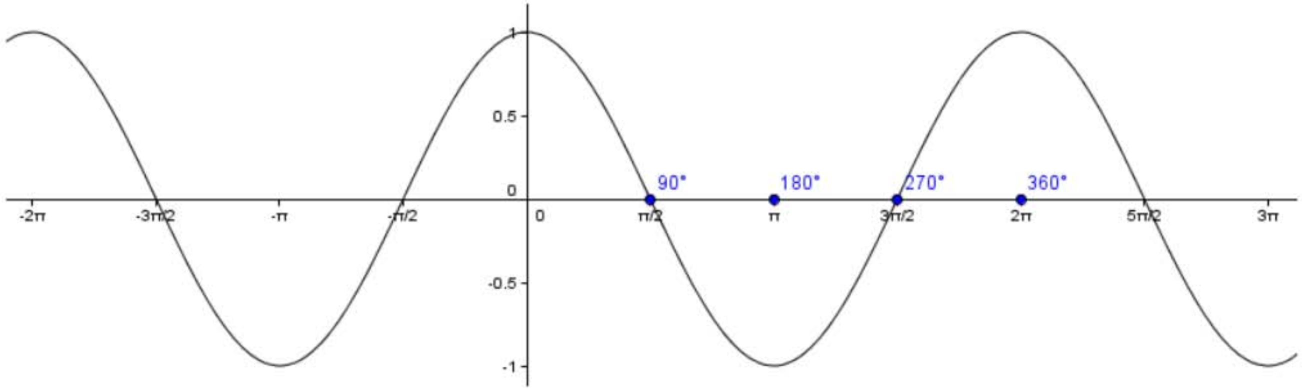
y=Sin(θ)

period: 2π



y=Cos(θ)

period: 2π



y=Tan(θ)

period: π

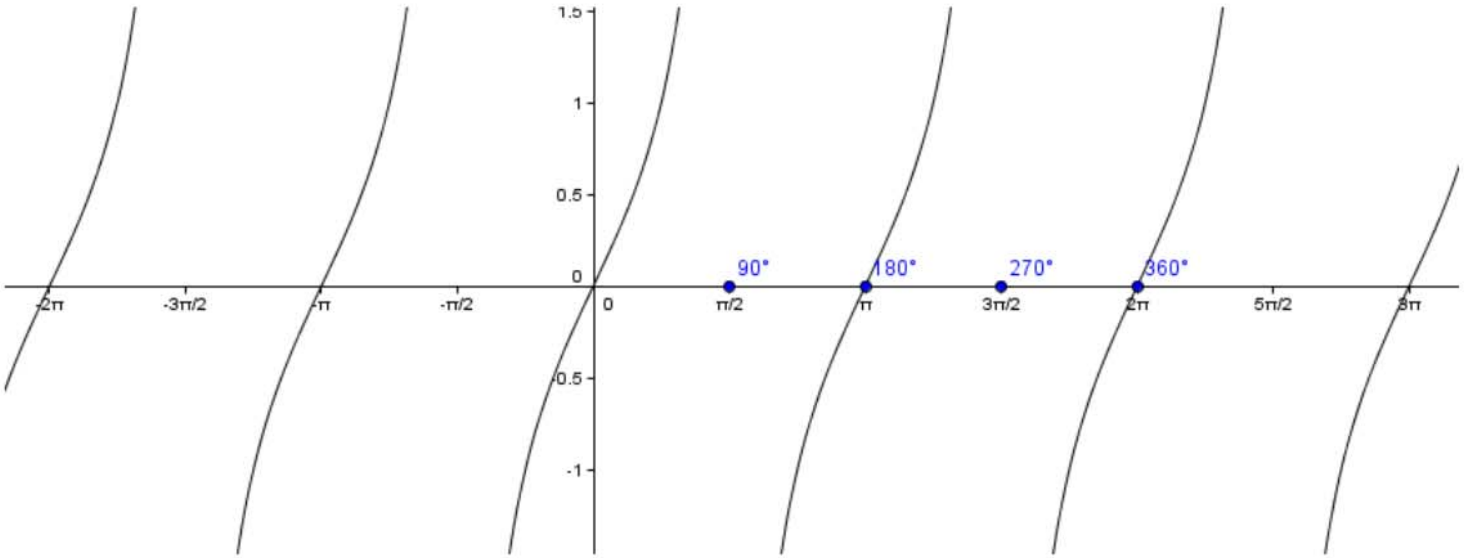
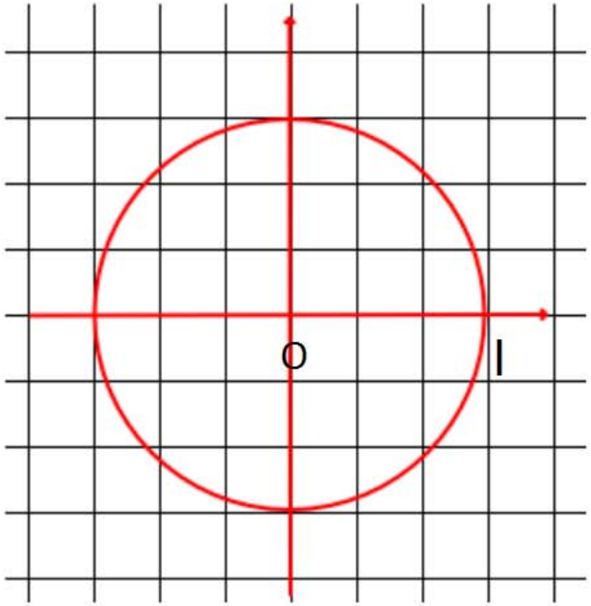
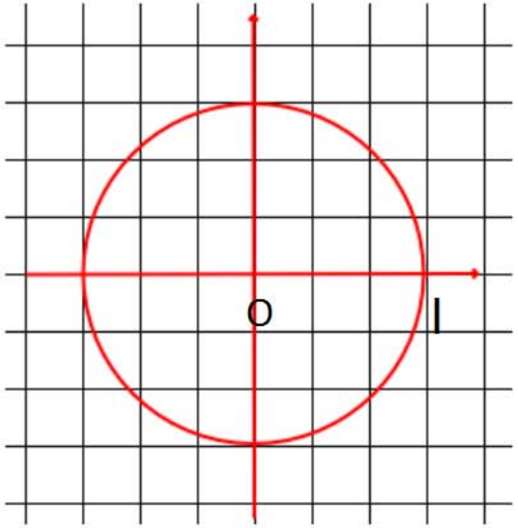


Table of values, sin cos and tan of obtuse, reflex and negative angles



θ in degrees	θ in radians	$\sin\ \theta$	$\cos\ \theta$	$\tan\ \theta$
0				
30				
45				
60				
90				
180				
360				



Express, in terms of Cos(θ)

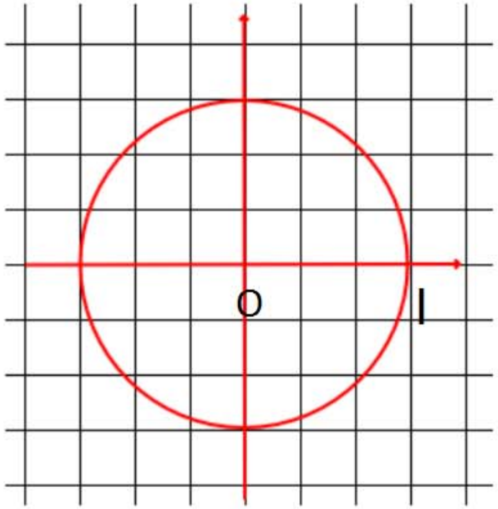
$$Cos(-\theta) =$$

$$Cos(180-\theta) =$$

$$Cos(180+\theta) =$$

$$Cos(360-\theta) =$$

$$Cos(360+\theta) =$$



Express, in terms of Sin(θ)

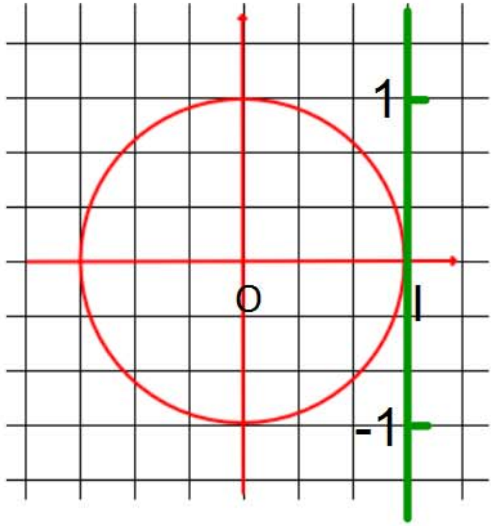
$$Sin(-\theta) =$$

$$Sin(180-\theta) =$$

$$Sin(180+\theta) =$$

$$Sin(360-\theta) =$$

$$Sin(360+\theta) =$$



Express, in terms of Tan(θ)

$$Tan(-\theta) =$$

$$Tan(180-\theta) =$$

$$Tan(180+\theta) =$$

$$Tan(360-\theta) =$$

$$Tan(360+\theta) =$$

This identity are equivalent when the angles are expressed in radians.

Examples:

Cos(300) =

Cos(120) =

Sin(240)=

Sin(-120)=

EXERCISE

1 Find the exact values of the following:

(a) $\sin 300°$, **(b)** $\tan 120°$, **(c)** $\cos 225°$,
(d) $\sin 150°$, **(e)** $\tan 510°$, **(f)** $\cos 150°$,
(g) $\sin 270° + \tan 210° + \cos 420°$,
(h) $\sin(-315°) + \tan(-405°) - \cos(-120°)$.

2 Find the exact values of the following:

(a) $\sin \frac{2\pi}{3}$, **(b)** $\tan \frac{3\pi}{4}$, **(c)** $\cos \frac{7\pi}{6}$,
(d) $\sin\left(-\frac{\pi}{6}\right)$, **(e)** $\tan\left(-\frac{2\pi}{3}\right)$, **(f)** $\cos\left(-\frac{5\pi}{4}\right)$,
(g) $\sin \frac{11\pi}{6}$, **(h)** $\tan \frac{14\pi}{3}$, **(i)** $\cos \frac{8\pi}{3}$,
(j) $\sin(-2\pi)$, **(k)** $\tan\left(\frac{-7\pi}{6}\right)$, **(l)** $\cos(-7\pi)$.

3 Without using a calculator, verify that $\frac{\pi}{8}$ is a solution of the equation $\sin 3x = \cos x$.

4 Find the exact value of $\dfrac{3\tan\frac{5\pi}{4}}{\sin\frac{\pi}{3}} + \cos\left(-\frac{7\pi}{6}\right)$.

5 Given that $x = \frac{\pi}{4}$ is a solution of the equation

$3\sin^2 x - k\tan x = \frac{1}{3}$, find the value of the constant k.

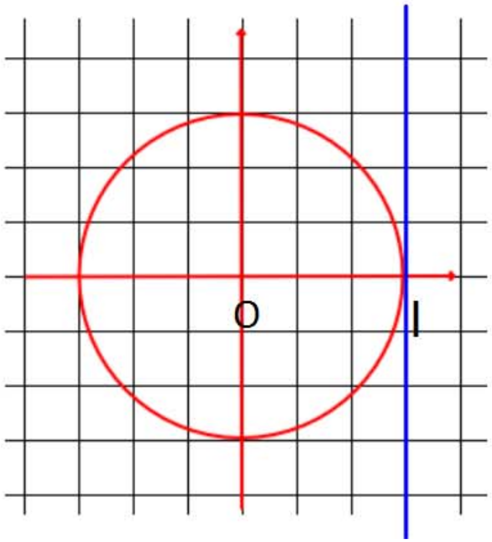
6 Given that $\sin\left(-\frac{5\pi}{6}\right) + k\tan\frac{7\pi}{6} + \cos\frac{5\pi}{6} = 0$, find the value of the constant k.

EXERCISE

1 (a) $-\frac{\sqrt{3}}{2}$; (b) $-\sqrt{3}$; (c) $-\frac{1}{\sqrt{2}}$; (d) $\frac{1}{2}$;
(e) $-\frac{1}{\sqrt{3}}$; (f) $\frac{-\sqrt{3}}{2}$; (g) $-\frac{1}{2}+\frac{1}{\sqrt{3}}$; (h) $-\frac{1}{2}+\frac{1}{\sqrt{2}}$.

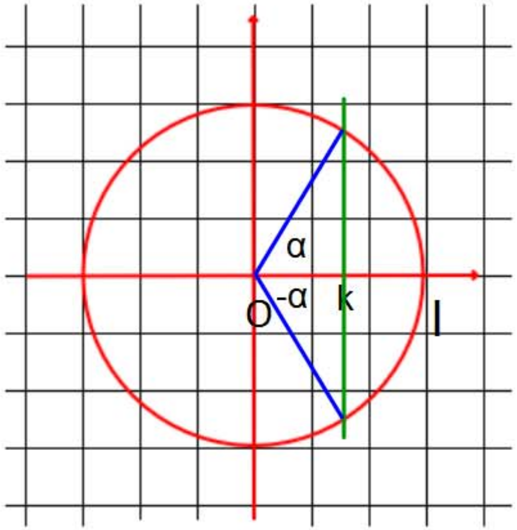
2 (a) $\frac{\sqrt{3}}{2}$; (b) -1 (c) $-\frac{\sqrt{3}}{2}$; (d) $-\frac{1}{2}$;
(e) $\sqrt{3}$; (f) $-\frac{1}{\sqrt{2}}$; (g) $-\frac{1}{2}$; (h) $-\sqrt{3}$;
(i) $-\frac{1}{2}$; (i) 0; (k) $-\frac{1}{\sqrt{3}}$; (l) -1.

4 $\frac{3\sqrt{3}}{2}$. **5** $\frac{7}{6}$. **6** $\frac{3+\sqrt{3}}{2}$.

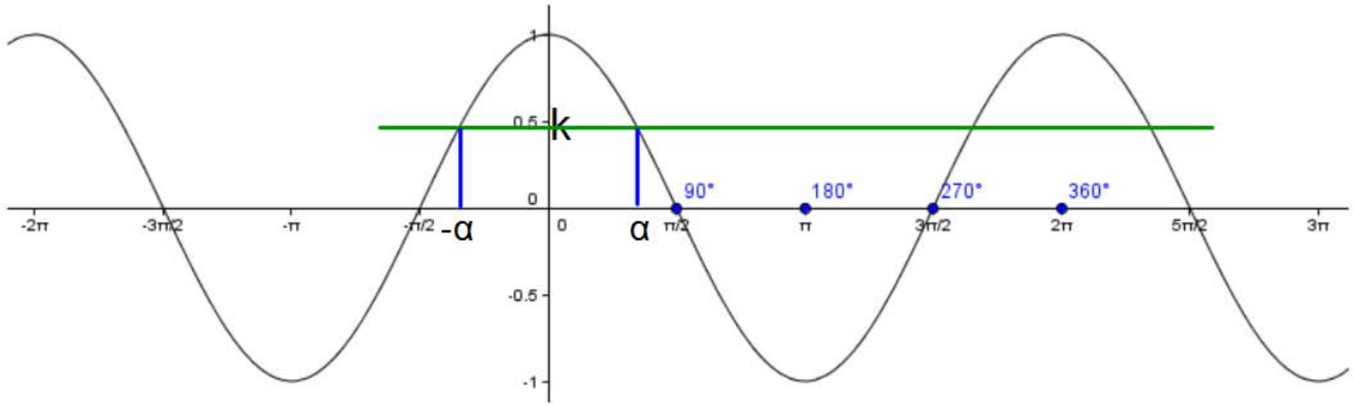


Solving trigonometric equations

Cos(x) = k   with $-1 \leq k \leq 1$



$\alpha = Cos^{-1}(k)$



$$Cos(x) = k$$

$$x = Cos^{-1}(k) + n \times 360$$

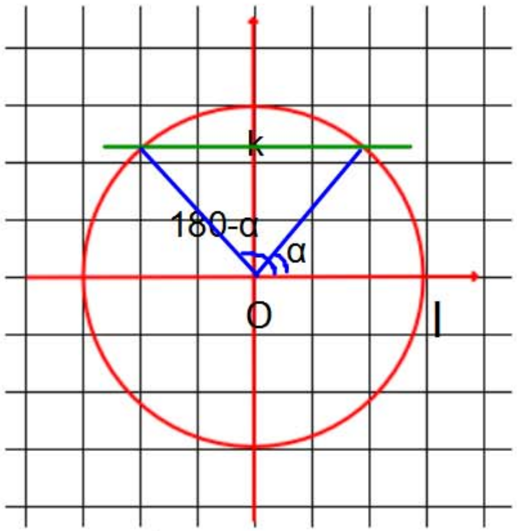
$$or \qquad x = -Cos^{-1}(k) + n \times 360$$

Examples:

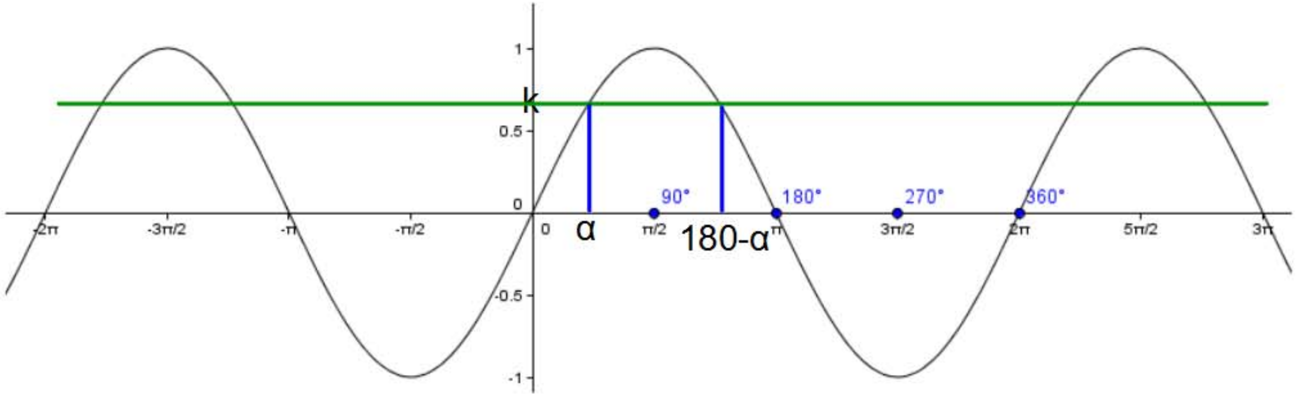
Find the GENERAL SOLUTIONS of the equation i) Cos(x)= 0.3

ii) Cos(2x) = -0.1

Sin(x) = k with $-1 \le k \le 1$



$\alpha = Sin^{-1}(k)$



$$Sin(x) = k$$

$$x = Sin^{-1}(k) + n \times 360$$

$$or \qquad x = 180 - Sin^{-1}(k) + n \times 360$$

Examples:

find the GENERAL SOLUTIONS of the equation i) Sin(x)=0.9

ii) Sin(3x) = Sin(π/6)

EXERCISE

1 Find the general solutions, in degrees, of these equations:

(a) $\sin x = \frac{1}{\sqrt{2}}$, **(b)** $\cos x = 0$,

(c) $\sin 2x = -\frac{\sqrt{3}}{2}$, **(d)** $\cos 4x = \frac{1}{2}$,

(e) $\sin 2x = 0.6$, **(f)** $1 + \cos 3x = 0.3$,

(g) $\sin (x + 40°) = \sin 50°$ **(h)** $\cos(50° - 2x) = \cos 20°$.

2 Find the general solutions, in radians, of these equations:

(a) $\cos x = \frac{1}{\sqrt{2}}$, **(b)** $\sin x = 0$,

(c) $\cos 4x = \frac{\sqrt{3}}{2}$, **(d)** $\sin 2x = -\frac{1}{2}$,

(e) $\cos 2x = 0.6$, **(f)** $1 + \sin(3x + 1) = 0.3$,

(g) $\cos\left(x - \frac{\pi}{2}\right) = \cos \frac{\pi}{3}$, **(h)** $\sin\left(\frac{\pi}{4} - 2x\right) = 1$.

3 Find the general solution, in radians, of the equation $2 \sin x \cos x - \cos x = 0$.

4 Find the general solution, in degrees, of the equation $2 \cos^2 x = \cos x$.

5 Find the general solution, in radians, of the equation $2 \cos^2 x = 1$.

6 Find the general solution, in degrees, of the equation $4 \sin^2 2x = 3$.

EXERCISE

Correct answers to general solutions may be given in a different form to those below; changing n to $n + 1$ or n to $n - 1$ will usually allow you to compare correct alternatives, for example in **1(c)**, a correct alternative to $(180n - 60)°$ is $(180n + 120)°$.

1 **(a)** $x = (360n + 45)°, (360n + 135)°$; **(b)** $x = (360n \pm 90)°$;

(c) $x = (180n - 30)°, (180n - 60)°$; **(d)** $x = (90n \pm 15)°$;

(e) $x = (180n + 18.4)°, (180n + 71.6)°$; **(f)** $x = (120n \pm 44.8)°$;

(g) $x = (360n + 10)°, (360n + 90)°$;

(h) $x = (15 - 180n)°, (35 - 180n)°$.

2 **(a)** $x = 2n\pi \pm \frac{\pi}{4}$;

(b) $x = 2n\pi, 2n\pi + \pi$ (Note: these can be combined to give $x = n\pi$);

(c) $x = \frac{n\pi}{2} \pm \frac{\pi}{24}$;

(d) $x = n\pi - \frac{\pi}{12}, n\pi + \frac{5\pi}{12}$;

(e) $x = n\pi \pm 0.4636^c$;

(f) $x = \frac{2n\pi}{3} - 0.5918^c, x = \frac{2n\pi}{3} + 0.9723^c$;

(g) $x = 2n\pi + \frac{\pi}{6}, x = 2n\pi + \frac{5\pi}{6}$;

(h) $x = -n\pi - \frac{\pi}{8}$ (This can be written as $x = n\pi - \frac{\pi}{8}$).

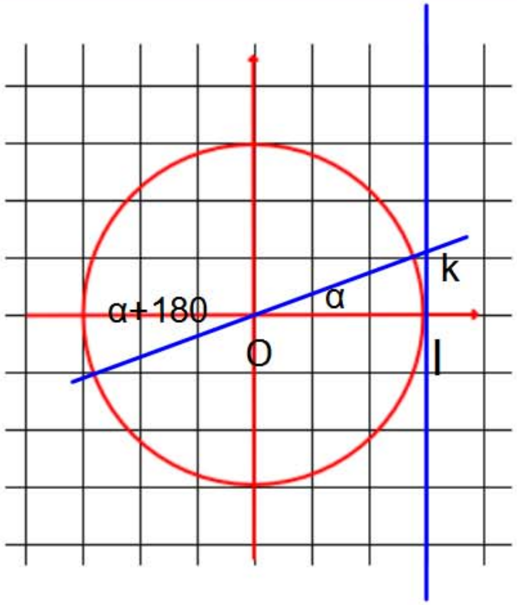
3 $x = 2n\pi \pm \frac{\pi}{2}, 2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}$.

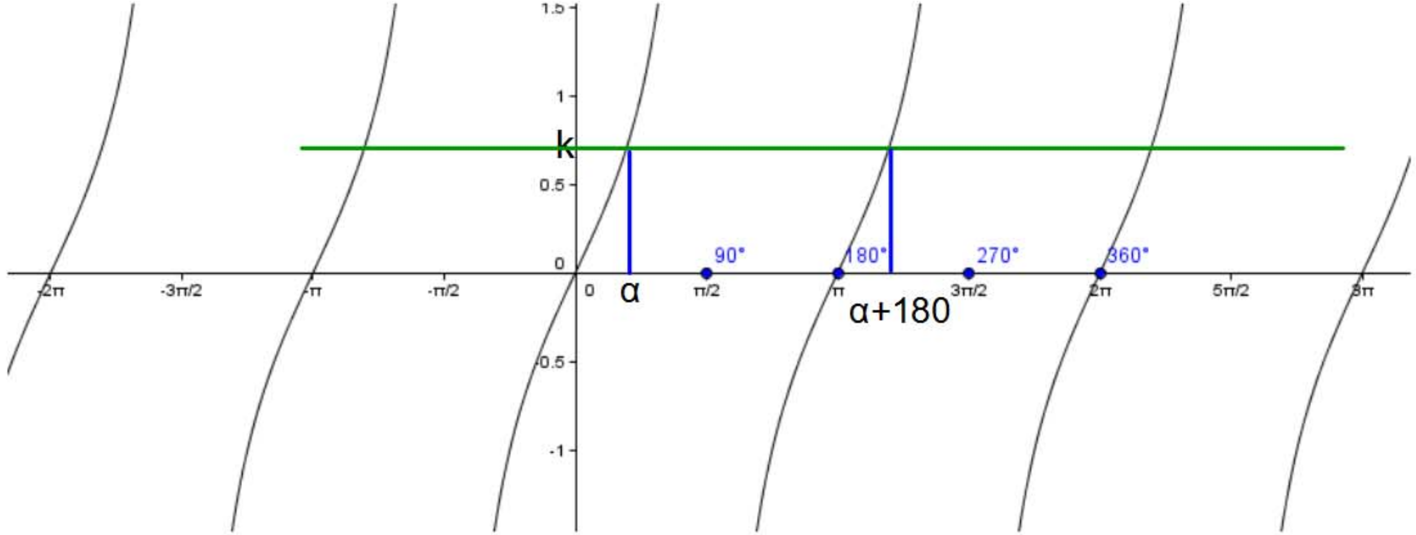
4 $x = (360n \pm 60)°, (360n \pm 90)°$.

5 $x = 2n\pi \pm \frac{\pi}{4}, 2n\pi \pm \frac{3\pi}{4}$.

6 $x = (180n \pm 30)°, (180n \pm 60)°$.

Tan(x) = k   k is any real number





$$Tan(x) = k$$

$$x = Tan^{-1}(k) + n \times 180$$

Examples:

Find the GENERAL SOLUTIONS of the equation i) Tan(x)=3

ii) $Tan(2x) = Tan(\pi/4)$

EXERCISE

1 Find the general solutions, in degrees, of these equations:

(a) $\tan x = \tan 20°$, (b) $\tan x = -1$,

(c) $\tan 2x = \tan 70°$, (d) $\tan 4x = \tan(-40°)$,

(e) $\tan(x - 75°) = -\tan 50°$, (f) $2\tan(50° - 2x) = 5$.

2 Find the general solutions, in radians, of these equations:

(a) $\tan x = 1$, (b) $\tan x = -\sqrt{3}$,

(c) $\tan 4x + 1 = 0$, (d) $\tan 2x = \frac{1}{\sqrt{3}}$,

(e) $\tan(2x - \frac{\pi}{2}) = \tan\frac{\pi}{3}$, (f) $\tan\left(\frac{\pi}{4} - 2x\right) = -\tan\frac{\pi}{3}$.

3 Find the general solution, in radians, of the equation $3\tan^2 x = 1$.

4 Find the general solution, in degrees, of the equation $(\tan x + 1)(\tan x - 3) = 0$.

5 Find the general solution, in radians, of the equation $\tan 2x = \tan x$.

6 Find the general solution, in degrees, of the equation $\tan 4x = \tan(x + 42°)$.

EXERCISE

1 (a) $x = (180n + 20)°$; (b) $x = (180n - 45)°$;

(c) $x = (90n + 35)°$; (d) $x = (45n - 10)°$;

(e) $x = (180n + 25)°$; (f) $x = (-90n - 9.1)°$.

2 (a) $x = n\pi + \frac{\pi}{4}$; (b) $x = n\pi - \frac{\pi}{3}$;

(c) $x = \frac{n\pi}{4} - \frac{\pi}{16}$; (d) $x = \frac{n\pi}{2} + \frac{\pi}{12}$;

(e) $x = \frac{n\pi}{2} + \frac{5\pi}{12}$; (f) $x = \frac{7\pi}{24} - \frac{n\pi}{2}$.

3 $x = n\pi \pm \frac{\pi}{6}$.

4 $x = (180n - 45)°, (180n + 71.57)°$.

5 $x = n\pi$.

6 $x = (60n + 14)°$.