Probability – exam questions

Question 1: Jan 2006 – Q2

Xavier, Yuri and Zara attend a sports centre for their judo club's practice sessions. The probabilities of them arriving late are, independently, 0.3, 0.4 and 0.2 respectively.

(a) Calculate the probability that for a particular practice session:

(i) all three arrive late; *(1 mark)*

(ii) none of the three arrives late; *(2 marks)*

(iii) only Zara arrives late. *(2 marks)*

(b) Zara's friend, Wei, also attends the club's practice sessions. The probability that Wei arrives late is 0.9 when Zara arrives late, and is 0.25 when Zara does not arrive late.

Calculate the probability that for a particular practice session:

(i) both Zara and Wei arrive late; *(2 marks)*

(ii) either Zara or Wei, but not both, arrives late. *(3 marks)*

Question 2: Jun 2006 – Q6

A housing estate consists of 320 houses: 120 detached and 200 semi-detached. The numbers of children living in these houses are shown in the table.

	Number of children				
	None	**One**	**Two**	**At least three**	**Total**
Detached house	24	32	41	23	120
Semi-detached house	40	37	88	35	200
Total	64	69	129	58	320

A house on the estate is selected at random.

D denotes the event 'the house is detached'.
R denotes the event 'no children live in the house'.
S denotes the event 'one child lives in the house'.
T denotes the event 'two children live in the house'.
(D' denotes the event 'not D'.)

(a) Find:

(i) $P(D)$; *(1 mark)*

(ii) $P(D \cap R)$; *(1 mark)*

(iii) $P(D \cup T)$; *(2 marks)*

(iv) $P(D \mid R)$; *(2 marks)*

(v) $P(R \mid D')$. *(3 marks)*

(b) (i) Name two of the events D, R, S and T that are mutually exclusive. *(1 mark)*

(ii) Determine whether the events D and R are independent. Justify your answer. *(2 marks)*

(c) Define, in the context of this question, the event:

(i) $D' \cup T$; *(2 marks)*

(ii) $D \cap (R \cup S)$. *(2 marks)*

Question 3: Jan 2007 – Q5

Dafydd, Eli and Fabio are members of an amateur cycling club that holds a time trial each Sunday during the summer. The independent probabilities that Dafydd, Eli and Fabio take part in any one of these trials are 0.6, 0.7 and 0.8 respectively.

Find the probability that, on a particular Sunday during the summer:

(a) none of the three cyclists takes part; *(2 marks)*

(b) Fabio is the only one of the three cyclists to take part; *(2 marks)*

(c) exactly one of the three cyclists takes part; *(3 marks)*

(d) either one or two of the three cyclists take part. *(3 marks)*

Question 4: Jun 2007 – Q2

The British and Irish Lions 2005 rugby squad contained 50 players. The nationalities and playing positions of these players are shown in the table.

		Nationality			
		English	**Welsh**	**Scottish**	**Irish**
Playing position	**Forward**	14	5	2	6
	Back	8	7	2	6

(a) A player was selected at random from the squad for a radio interview. Calculate the probability that the player was:

(i) a Welsh back; *(1 mark)*

(ii) English; *(2 marks)*

(iii) not English; *(1 mark)*

(iv) Irish, given that the player was a back; *(2 marks)*

(v) a forward, given that the player was not Scottish. *(2 marks)*

(b) Four players were selected at random from the squad to visit a school. Calculate the probability that all four players were English. *(3 marks)*

Question 5: Jan 2008 – Q5

A health club has a number of facilities which include a gym and a sauna. Andrew and his wife, Heidi, visit the health club together on Tuesday evenings.

On any visit, Andrew uses either the gym or the sauna or both, but no other facilities. The probability that he uses the gym, $P(G)$, is 0.70. The probability that he uses the sauna, $P(S)$, is 0.55. The probability that he uses both the gym and the sauna is 0.25.

(a) Calculate the probability that, on a particular visit:

(i) he does not use the gym; *(1 mark)*

(ii) he uses the gym but not the sauna; *(2 marks)*

(iii) he uses either the gym or the sauna but not both. *(2 marks)*

(b) Assuming that Andrew's decision on what facility to use is independent from visit to visit, calculate the probability that, during a month in which there are exactly four Tuesdays, he does not use the gym. *(2 marks)*

(c) The probability that Heidi uses the gym when Andrew uses the gym is 0.6, but is only 0.1 when he does not use the gym.

Calculate the probability that, on a particular visit, Heidi uses the gym. *(3 marks)*

(d) On any visit, Heidi uses **exactly one** of the club's facilities.

The probability that she uses the sauna is 0.35.

Calculate the probability that, on a particular visit, she uses a facility other than the gym or the sauna. *(2 marks)*

Question 6: Jun 2008 – Q2

A basket in a stationery store contains a total of 400 marker and highlighter pens. Of the marker pens, some are permanent and the rest are non-permanent. The colours and types of pen are shown in the table.

	Colour			
Type	**Black**	**Blue**	**Red**	**Green**
Permanent marker	44	66	32	18
Non-permanent marker	36	53	21	10
Highlighter	0	41	37	42

A pen is selected at random from the basket. Calculate the probability that it is:

(a) a blue pen; *(1 mark)*

(b) a marker pen; *(2 marks)*

(c) a blue pen or a marker pen; *(2 marks)*

(d) a green pen, given that it is a highlighter pen; *(2 marks)*

(e) a non-permanent marker pen, given that it is a red pen. *(2 marks)*

Question 7: Jan 2009 – Q4

Gary and his neighbour Larry work at the same place.

On any day when Gary travels to work, he uses one of three options: his car only, a bus only or both his car and a bus. The probability that he uses his car, either on its own or with a bus, is 0.6. The probability that he uses both his car and a bus is 0.25.

(a) Calculate the probability that, on any particular day when Gary travels to work, he:

(i) does not use his car; *(1 mark)*

(ii) uses his car only; *(2 marks)*

(iii) uses a bus. *(3 marks)*

(b) On any day, the probability that Larry travels to work with Gary is 0.9 when Gary uses his car only, is 0.7 when Gary uses both his car and a bus, and is 0.3 when Gary uses a bus only.

(i) Calculate the probability that, on any particular day when Gary travels to work, Larry travels with him. *(4 marks)*

(ii) Assuming that option choices are independent from day to day, calculate, to three decimal places, the probability that, during any particular week (5 days) when Gary travels to work every day, Larry never travels with him. *(2 marks)*

Question 8: Jun 2009 – Q1

A large bookcase contains two types of book: hardback and paperback. The number of books of each type in each of four subject categories is shown in the table.

		Subject category				
		Crime	**Romance**	**Science fiction**	**Thriller**	**Total**
Type	**Hardback**	8	16	18	18	60
	Paperback	16	40	14	30	100
	Total	24	56	32	48	160

(a) A book is selected at random from the bookcase. Calculate the probability that the book is:

(i) a paperback; *(1 mark)*

(ii) not science fiction; *(2 marks)*

(iii) science fiction or a hardback; *(2 marks)*

(iv) a thriller, given that it is a paperback. *(2 marks)*

(b) Three books are selected at random, without replacement, from the bookcase.

Calculate, to three decimal places, the probability that one is crime, one is romance and one is science fiction. *(4 marks)*

Question 9: Jan 2010 – Q4

Each school-day morning, three students, Rita, Said and Ting, travel independently from their homes to the same school by one of three methods: walk, cycle or bus. The table shows the probabilities of their independent daily choices.

	Walk	**Cycle**	**Bus**
Rita	0.65	0.10	0.25
Said	0.40	0.45	0.15
Ting	0.25	0.55	0.20

(a) Calculate the probability that, on any given school-day morning:

(i) all 3 students walk to school; *(2 marks)*

(ii) only Rita travels by bus to school; *(2 marks)*

(iii) at least 2 of the 3 students cycle to school. *(4 marks)*

(b) Ursula, a friend of Rita, never travels to school by bus. The probability that:

Ursula walks to school when Rita walks to school is 0.9;
Ursula cycles to school when Rita cycles to school is 0.7.

Calculate the probability that, on any given school-day morning, Rita and Ursula travel to school by:

(i) the same method; *(3 marks)*

(ii) different methods. *(1 mark)*

Probability – exam questions

Question 1: Jan 2006 – Q2

(a)	$P(X) = 0.3 \quad P(Y) = 0.4 \quad P(Z) = 0.2$		
(i)	$P(X \cap Y \cap Z) = 0.3 \times 0.4 \times 0.2 = 0.024$	M1	1
(ii)	$P(X' \cap Y' \cap Z') = 0.7 \times 0.6 \times 0.8$	M1	
	$= 0.336$	A1	2
(iii)	$P(X' \cap Y' \cap Z) = 0.7 \times 0.6 \times 0.2$	M1	
	$= 0.084$	A1	
(b)	$P(W \mid Z) = 0.9 \quad P(W \mid Z') = 0.25$		
(i)	$P(Z \cap W) = 0.2 \times 0.9$	M1	
	$= 0.18$	A1	2
(ii)	$P((Z \cap W') \cup (Z' \cap W))$ or $1 - [P((Z \cap W) \cup (Z' \cap W'))]$		
	$= 0.2 \times (1 - 0.9)$	M1	
	$+$ $(1 - 0.2) \times 0.25$	M1	
	$= 0.02 + 0.20$ $= 0.22$	A1	3
	Total		**11**

Question 2: Jun 2006 – Q6

6

	0 (*R*)	1 (*S*)	2 (*T*)	≥3	T
D (*D*)	24	32	41	23	120
SD (*D'*)	40	37	88	35	200
T	64	69	129	58	320

(a)(i)	$P(D) = \frac{120}{320}$ or $\frac{3}{8}$ or 0.375	B1	1
(ii)	$P(D \cap R) = \frac{24}{320}$ or $\frac{3}{40}$ or 0.075	B1	1
(iii)	$P(D \cup T) = \frac{120+88}{320} = \frac{129+24+32+23}{320}$	M1	
	$= \frac{208}{320}$ or $\frac{13}{20}$ or 0.65	A1	2
(iv)	$P(D \mid R) = \frac{P(D \cap R)}{P(R)} = \frac{(ii)}{P(R)} = \frac{24/(320)}{64/(320)}$	M1	
	$= \frac{24}{64}$ or $\frac{3}{8}$ or 0.375	A1	2
(v)	$P(R \mid D') = \frac{P(R \cap D')}{P(D')} = \frac{40/(320)}{200/(320)}$	M1 M1	
	$= \frac{40}{200}$ or $\frac{1}{5}$ or 0.2	A1	3
(b)(i)	*R* and *S* or *R* and *T* or *S* and *T*	B1	1
(ii)	$P(D) = 0.375 = P(D \mid R)$ or (i) = (iv)	M1	
	so YES	A1	2
(c)(i)	A semi-detached house or two children (or both)	B1 B1	2
(ii)	A detached house and/with less than two children	B1 B1	2
	Total		**16**

Question 3: Jan 2007 – Q5

(a)	$P(D' \cap E' \cap F') = 0.4 \times 0.3 \times 0.2$	M1	
	$= 0.024$	A1	2
(b)	$P(D' \cap E' \cap F) = 0.4 \times 0.3 \times 0.8$	M1	
	$= 0.096$	A1	2
(c)	P(One) = $(b) + P(D \cap E' \cap F') + P(D' \cap E \cap F')$	M1	
	$=(b) + (0.6 \times 0.3 \times 0.2) + (0.4 \times 0.7 \times 0.2)$	M1	
	$= 0.096 + 0.036 + 0.056 = 0.188$	A1	3
(d)	P(One or two) = (c) + (3 terms each of 3 probabilities) or = 1 – (a) – (1 term of 3 probabilities)	M1	
	$= 0.188 + (0.6 \times 0.7 \times 0.2) +$ $(0.6 \times 0.3 \times 0.8) + (0.4 \times 0.7 \times 0.8)$ $= 0.188 + 0.084 + 0.144 + 0.224$ or $= 1 - 0.024 - (0.6 \times 0.7 \times 0.8)$ $= 1 - 0.024 - 0.336$	M1	
	$= 0.64$	A1	3
	Total		**10**

Question 4: Jun 2007 – Q2

2	Ratios: Penalise first occurrence only of a correct answer		
(a)(i)	P(Welsh back) = $\frac{7}{50}$ or 0.14	B1	1
(ii)	P(English) = $\frac{14+8}{50}$ =	B1	
	$\frac{22}{50}$ or $\frac{11}{25}$ or 0.44	B1	2
(iii)	P(not English) = 1 – (ii) =		
	$\frac{28}{50}$ or $\frac{14}{25}$ or 0.56	B1√	1
(iv)	P(Irish | back) = $\frac{P(\text{Irish} \cap \text{back})}{P(\text{back})} = \frac{6}{\sum(\text{back})} =$	M1	
	$\frac{6}{23}$ or 0.26 to 0.261	A1	2
(v)	P(forward | not Scottish) = $\frac{P(\text{forward} \cap \text{not Scottish})}{P(\text{not Scottish})} =$ $\frac{14+5+6}{50-4} = \frac{27-2}{50-4} =$	M1	
	$\frac{25}{46}$ or 0.54 to 0.544	A1	2
(b)	P(4 × English) = $\left(\frac{22}{50}\right) \times \left(\frac{21}{49}\right) \times \left(\frac{20}{48}\right) \times \left(\frac{19}{47}\right) =$	M1 M1	
	$\frac{175560}{5527200}$ or $\frac{209}{6580}$ or 0.0317 to 0.032	A1	3
	Total		**11**

Question 5: Jan 2008 – Q5

5(a)(i)	$P(G') = 1 - 0.70 = 0.3(0)$	B1	1
(ii)	$P(G \cap S') =$ $0.70 - (0.25 \text{ or } 0.55 \text{ or } 0.45)$ or $1 - 0.55$	M1	
	$= 0.45$	A1	2
(iii)	P(1 only) $=$ $0.70 + 0.55 - (2 \times 0.25)$ or $1 - 0.25$ or $0.45 + 0.30$	M1	
	$= 0.75$	A1	2
(b)	$P(G' \cap G' \cap G' \cap G') = [(a)(i)]^4$	M1	
	$= 0.0081$	A1	2
(c)	$P(H_G) = P(A_G \cap H_G) + P(A_{G'} \cap H_G) =$		
	(0.70×0.60) or 0.42	M1	
	(0.30×0.10) or 0.03	M1	
	$= 0.42 + 0.03 = 0.45$	A1	3
(d)	$P(H_O) = 1 - [0.35 + (c)]$	M1	
	$= 0.2(0)$	A1	2
	Total		12

Question 6: Jun 2008 – Q2

2(a)	$P(Blue) = \frac{160}{400} = 0.4$ or $\frac{2}{5}$ or $\frac{160}{400}$	B1	1
	In (b) to (e), method marks are for single fractions, or equivalents, only		
(b)	$P(Marker) = \frac{280}{400}$	M1	
	$= 0.7$ or $\frac{7}{10}$ or $\frac{280}{400}$	A1	2
(c)	$P(B \text{ or } M) = P(B \cup M) =$		
	$\frac{160+280-119}{400} = \frac{280+41}{400} = \frac{321}{400}$	M1	
	$= 0.802$ to 0.803 or $\frac{321}{400}$	A1	2
(d)	$P(Green \mid Highlighter) = P(G \mid H) = \frac{42}{120}$	M1	
	$= 0.35$ or $\frac{7}{20}$ or $\frac{42}{120}$	A1	2
(e)	$P(Non\text{-}Permanent \mid Red) = P(P' \mid R) = \frac{21}{90}$	M1	
	$= 0.233$ to 0.234 or $\frac{7}{30}$ or $\frac{21}{90}$	A1	2
	Total		9

Question 7: Jan 2009 – Q4

4	$P(C) = 0.6 \quad P(C \cap B) = 0.25$ $\{P(C \text{ only}) = 0.35 \quad P(B \text{ only}) = 0.4\}$		
(a) (i)	$P(C') = 1 - P(C) = 1 - 0.6 = 0.4$	B1	1
(ii)	$P(C \cap B') = 0.6 - 0.25$ $= 1 - (0.4 + 0.25)$ $= 0.35$	M1 A1	2
(iii)	$P(B) = (i) + p$ with $p < 0.6$ $= (i) + 0.25$ $= 0.65$ **OR** $P(B) = 1 - (ii)$ $= 0.65$ **OR** $1 = P(C) + P(B) - P(C \cap B)$ Thus $P(B) = 1 - (0.6 - 0.25)$ $= 0.65$	M1 A1 A1 (M2) (A1) (M1) (A1) (A1)	3
(b)	$P(L \mid G_C) = 0.9 \quad P(L \mid G_{CB}) = 0.7$ $P(L \mid G_B) = 0.3$		
(i)	$P(G \cap L) \Rightarrow (a)(ii) \times 0.9 \quad (0.315)$	M1	
	$0.25 \times 0.7 \quad (0.175)$	M1	
	$[(a)(iii) - 0.25] \times 0.3 \quad (0.12)$	M1	
	Note: Each pair of multiplied probabilities must be > 0 to score the corresponding method mark		
	$\Rightarrow 0.315 + 0.175 + 0.12 = 0.61$	A1	4
(ii)	Probability $= \{1 - (b)(i)\}^5$	M1	
	$= 0.39^5 = 0.009$	A1	2
		Total	12

Question 8: Jun 2009 – Q1

(i)	$P(P) = 100/160 = 50/80 = 25/40 = 10/16$ $= 5/8 = 0.625$	B1	1
(ii)	$P(S') = 1 - \frac{32}{160}$ or $P(S) = \frac{32}{160}$	M1	
	$= 128/160 = 64/80 = 32/40 = 16/20 = 8/10$ $= 4/5 = 0.8$	A1	2
(iii)	$P(S \text{ or } H) = P(S \cup H) =$ $\frac{60+32-18}{160}$ or $\frac{60+14}{160}$ or $\frac{32+8+16+18}{160}$	M1	
	$= 74/160 = 37/80 = 0.462$ to 0.463	A1	2
(iv)	$P(T \mid P) = \frac{30/160}{(i)}$	M1	
	$= 3/100 = 3/10 = 0.3$	A1	2
(b)	$P(1C \ \& \ 1R \ \& \ 1S) =$		
	$\frac{24}{160} \times \frac{56}{159} \times \frac{32}{158}$	M1 M1	
	$(0.15 \times 0.35220 \times 0.20253)$ $\times 6$	M1	
	$= 0.064$ to 0.0644	A1	
	Special Case: (Any given subject total) $\div$ 160 seen anywhere in (b)	(M1)	4
		Total	11

Question 9: Jan 2010 – Q4

4(a)(i)	P(all 3 walk) = $0.65 \times 0.40 \times 0.25$	M1	
	= 65/1000 = 13/200 = 0.065	A1	2
(ii)	P(Rita by bus) = $0.25 \times (1-0.15) \times (1-0.20)$	M1	
	= 17/100 = 0.17	A1	2
(iii)	P(2 cycle) = $0.10 \times 0.45 \times (0.25 + 0.20)$ = 0.02025 + $0.10 \times (0.40 + 0.15) \times 0.55$ = 0.03025 + $(0.65 + 0.25) \times 0.45 \times 0.55$ = 0.22275 (0.27325)	B1	
	P(3 cycle) = $0.10 \times 0.45 \times 0.55$ = 0.02475	B1	
	P(≥ 2 cycle) = P(2 cycle) + P(3 cycle)	M1	
	= 0.298	A1	4
	or P(0 cycle) = $0.90 \times 0.55 \times 0.45$ = 0.22275	(B1)	
	P(1 cycles) = $0.10 \times 0.55 \times 0.45$ = 0.02475 + $0.90 \times 0.45 \times 0.45$ = 0.18225 (0.47925) + $0.90 \times 0.55 \times 0.55$ = 0.27225	(B1)	
	P(≥ 2 cycle) = 1 – [P(0 cycle) + P(1 cycles)]	(M1)	
	1 – 0.702 = 0.298	(A1)	
(b)(i)	P(WW) = (0.65×0.90) = 0.585 P(CC) = (0.10×0.70) = 0.070	B1	
	P(WW or CC) = 0.585 + 0.070	M1	
	= 0.655	A1	3
(ii)	P(different) = 1 – (b)(i) = 0.345	B1F	1
		Total	**12**