OCR ADVANCED SUBSIDIARY GCE IN

MATHEMATICS (MEI)	**(3895)**
FURTHER MATHEMATICS (MEI)	**(3896/3897)**
PURE MATHEMATICS (MEI)	**(3898)**

OCR ADVANCED GCE IN

MATHEMATICS (MEI)	**(7895)**
FURTHER MATHEMATICS (MEI)	**(7896/7897)**
PURE MATHEMATICS (MEI)	**(7898)**

Specimen Question Papers and Mark Schemes

These specimen question papers and mark schemes are intended to accompany the OCR Advanced Subsidiary GCE and Advanced GCE specifications in Mathematics (MEI) for teaching from September 2004.

The specimen assessment material accompanying the new specifications is provided to give centres a reasonable idea of the general shape and character of the planned question papers in advance of the first operational examination.

CONTENTS

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
INTRODUCTION TO ADVANCED MATHEMATICS, C1

4751

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Solve the equations:

(i) $x^{\frac{1}{2}} = 9$ **[1]**

(ii) $x^{-3} = \frac{1}{8}$ **[1]**

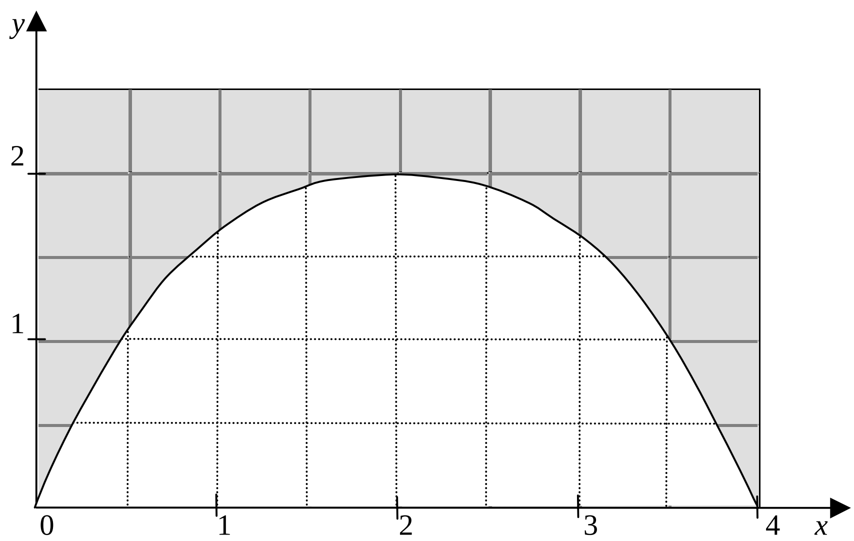
(iii) $(x^{10})^{\frac{1}{2}} = 32$ **[1]**

2 Make x the subject of the equation $ax^2 + b = -x^2 + d$. **[3]**

3 Solve the equation $2x^2 - 5x = 3$. **[3]**

4 Find the term in x^3 in the binomial expansion of $(1-2x)^5$. **[3]**

5 The diagram shows a bridge.
The units are metres.



It is suggested that the curved underside of the bridge can be modelled by the curve $y = \frac{1}{2}x(4-x)$ for $0 \le x \le 4$.

(i) Give two different reasons why this is a good model. **[2]**

(ii) Give also one reason why it is not a perfect model. **[1]**

6 A line l passes through the point (-1, 2) and has gradient 3.
Determine whether the point (-100, -294) lies above the line l, on it or below it. **[4]**

7 The coordinates of points A, B, C and D are (-2, -1), (2, 1), (5, 4) and (1, 2) respectively.
Prove that ABCD is a parallelogram but not a rhombus. **[4]**

8 The quadratic equation $x^2 + 6x + p = 0$ has equal roots.
State the value of p and hence find x. **[4]**

9 **(i)** Simplify $(\sqrt{2}+1)(\sqrt{2}-1)$. **[1]**

(ii) Express $\dfrac{\sqrt{2}}{\sqrt{2}+1}$ in the form $a + b\sqrt{2}$, where a and b are integers to be determined. **[3]**

10 Find the coordinates of the points of intersection of the line $y = 2x + 2$ and the curve $y = x^2 - 4x + 1$, giving your answers as surds. **[5]**

Section B (36 marks)

11

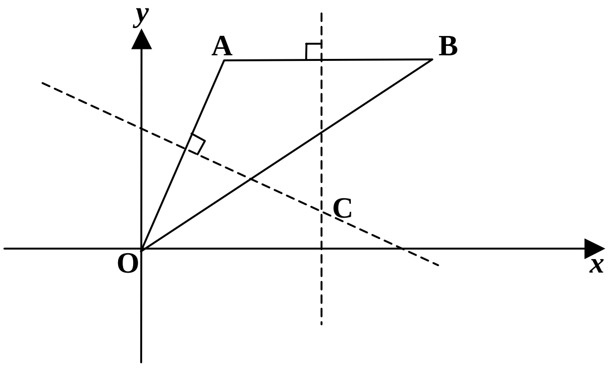


Fig. 1

Fig. 1 shows a triangle with vertices O (0, 0), A (2, 6) and B (12, 6). The perpendicular bisectors of OA and AB meet at C.

(i) Write down the equation of the perpendicular bisector of AB.
Find the equation of the perpendicular bisector of OA.
Hence show that the coordinates of C are (7, 1). **[6]**

(ii) Show that the point C is the centre of the circle which passes through O, A and B.
Find the equation of this circle.
Find the *y*-coordinate of the point other than O where the circle cuts the *y*-axis. **[6]**

12 In this question, $\mathrm{f}(x) = x^3 - 3x^2 - 6x + 8$.

(i) Show that $x - 1$ is a factor of $\mathrm{f}(x)$. **[1]**

(ii) Factorise $\mathrm{f}(x)$ completely and hence sketch the graph of $y = \mathrm{f}(x)$. **[7]**

(iii) On the same axes sketch the graph of $y = -x^3 + 3x^2 + 6x - 8$. **[2]**

(iv) Sketch the graph of $y = \mathrm{f}(x+2)$, marking the *x*-coordinates of the points where it crosses the *x*-axis. You need not calculate the *y*-intercept. **[2]**

13 **(i)** Express $x^2 - 6x + 10$ in the form $(x + a)^2 + b$ where a and b are constants to be determined.

Hence show that the value of $x^2 - 6x + 10$ is positive for all values of x. **[4]**

(ii) Sketch the graph of $y = x^2 - 6x + 10$.

Mark the axis of symmetry and give its equation.

State the co-ordinates of the lowest point of the curve. **[3]**

(iii) On the same axes sketch the graph of $y = x - 3$.

State, with reasons, what your graph tells you about the solution of the equation $x^2 - 6x + 10 = x - 3$. **[3]**

(iv) Solve the inequality $x^2 - 6x + 10 < 2$. **[2]**

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
INTRODUCTION TO ADVANCED MATHEMATICS, C1

4751

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1(i)	$x = 81$	B1 **[1]**	
1(ii)	$x = 2$	B1 **[1]**	
1(iii)	$x = 2$	B1 **[1]**	
2	$ax^2 + x^2 = d - b$	M1	
	$x^2 = \dfrac{d-b}{a+1}$	A1	
	$x = \pm\sqrt{\dfrac{d-b}{a+1}}$	A1 **[3]**	cao including ±
3	$2x^2 - 5x - 3 = 0$	B1	May be implied
	$(2x+1)(x-3) = 0$	M1	
	$\Rightarrow x = -0.5$ or 3	A1 **[3]**	cao
4	${}^5C_3 \times (-2)^3$	M1 B1	Binomial coefficient cao
	$= -80$ Or use of Pascal's triangle	A1 **[3]**	
5(i)	Good reasons: The model curve passes through (0, 0) (or (4, 0)) The model curve passes through (2, 2) The model curve is flat in the middle The model curve is symmetrical	B1,B1	Any two good reasons
5(ii)	Reasons why not: The point (1, 1.5) is on the model curve but below the bridge	B1	
6	Find equation of l using $y - y_1 = m(x - x_1)$	M1	
	$y = 3x + 5$	A1	
	Substituting $x = -100$ in line l gives (-100, -295)	M1	
	(-100, -294) is above l	A1 **[4]**	

Qu	Answer	Mark	Comment
Section A (continued)			
7	Gradient of AB = gradient of DC = ½ Gradient of BC = gradient of AD = 1 $\therefore$ ABCD is a parallelogram AB = √20, BC = √18 so AB ≠ BC $\therefore$ ADCD is not a rhombus	M1 E1 M1 E1 **[4]**	
8	$(x+3)^2=0$ $p=9$ $x=-3$	M1,A1 B1 B1 **[4]**	Or use of discriminant
9(i)	1	B1 **[1]**	
9(ii)	$\dfrac{\sqrt{2}}{\sqrt{2}+1}\times\dfrac{\sqrt{2}-1}{\sqrt{2}-1}=2-\sqrt{2}$ a = 2, b = –1	M1,A1 A1 **[3]**	 cao
10	$x^2-4x+1=2x+2$ $x^2-6x-1=0$ $x=\dfrac{6\pm\sqrt{36+4}}{2}$ $x=3+\sqrt{10}$ or $3-\sqrt{10}$ Substitute in $y=2x+2$ $y=8+2\sqrt{10}$ or $y=8-2\sqrt{10}$ respectively	M1 M1 A1 M1 A1 **[5]**	
			Section A Total: 36
Section B			
11(i)	Mid point of AB is (7, 6) Perpendicular bisector: x = 7 Mid point of OA is (1, 3) Gradient of OA is 3 Gradient of perpendicular is $-\frac{1}{3}$ $\Rightarrow y=-\dfrac{1}{3}x+\dfrac{10}{3}$ Intersects x = 7 at (7, 1)	B1 B1 M1 M1 A1 E1 **[6]**	

Qu	Answer	Mark	Comment
Section B (continued)			
11(ii)	Show that CO = CA = CB All are $\sqrt{50}$ $(x-7)^2+(y-1)^2=50$ Cuts y-axis at (0, 2)	M1 A1 B1,B1 M1,A1 **[6]**	 Radius, centre
12(i)	Show $\mathrm{f}(1)=0$	B1 **[1]**	
12(ii)	$\mathrm{f}(x)=(x-1)(x-4)(x+2)$ Shape of sketch. Points of intersection with x-axis. Point of intersection with y-axis.	M1 M1 A1 B1,B1 B1 B1 **[7]**	Take out $(x-1)$ Factorise quotient
12(iii)	Recognition that this is $y=-\mathrm{f}(x)$ Curve consistent with answer to **12(ii)**	M1 A1 **[2]**	May be implied
12(iv)	Their curve moved 2 to left Points of intersection with x-axis	B1 B1 **[2]**	
13(i)	$(x-3)^2+1$ $a=-3$ and $b=1$ $(x-3)^2\geq 0$ for all x and $+1>0$	B1,B1 M1,E1 **[4]**	
13(ii)	U-shaped curve Line of symmetry $x=3$ Lowest point (3, 1)	B1 B1 B1 **[3]**	
13(iii)	Correct straight line No solution/no real roots The line and the curve do not intersect	B1 B1 B1 **[3]**	
13(iv)	$2<x<4$	M1 A1 **[2]**	Solving $x^2-6x+8=0$ or verifying roots read from graph
			Section B Total: 36
			Total: 72

AO	Range	Total	Question Number												
			1	2	3	4	5	6	7	8	9	10	11	12	13
1	28-36	34	3	1	-	2	-	2	-	1	3	3	6	7	6
2	28-36	33	-	2	3	1	-	2	3	3	1	2	5	5	6
3	0-8	3	-	-	-	-	3	-	-	-	-	-	-	-	-
4	0-8	2	-	-	-	-	-	-	1	-	-	-	1	-	-
5	0-4	0	-	-	-	-	-	-	-	-	-	-	-	-	-
	Totals	**72**	3	3	3	3	3	4	4	4	4	5	12	12	12

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
CONCEPTS FOR ADVANCED MATHEMATICS, C2

4752

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Find the values of x for which $\sin x = 2\cos x$ given that $0° < x < 360°$. **[3]**

2 A sector of a circle has radius 15 cm and angle 0.6 radians.
Find the perimeter and area of the sector. **[4]**

3 Given that $y = 6x^2 + \sqrt{x} - 17$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **[4]**

4 The first two terms of a geometric sequence are 6144, 1536.

(i) Find the exact value of the 10th term. **[2]**

(ii) Find the sum of the first ten terms, giving your answer to 4 decimal places. **[2]**

(iii) Find the sum to infinity of the sequence. **[1]**

5 Some values of the function $\mathrm{f}(x) = \frac{1}{1+x^2}$ are given in the table below.

The figures are rounded to 5 decimal places.

x	0.0	0.2	0.4	0.6	0.8	1.0
f(x)		0.96154	0.86207		0.60976	

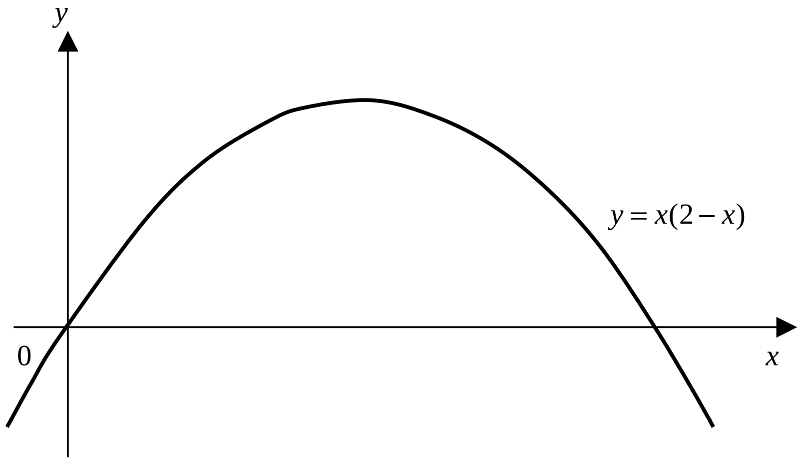
(i) Find the values of $\mathrm{f}(x)$ missing from the table. **[1]**

(ii) Use the trapezium rule with 5 strips to estimate the value of: $\int_0^1 \frac{1}{1+x^2}\,\mathrm{d}x$. **[4]**

6 The gradient of a curve is given by: $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - \frac{5}{x^2}$.

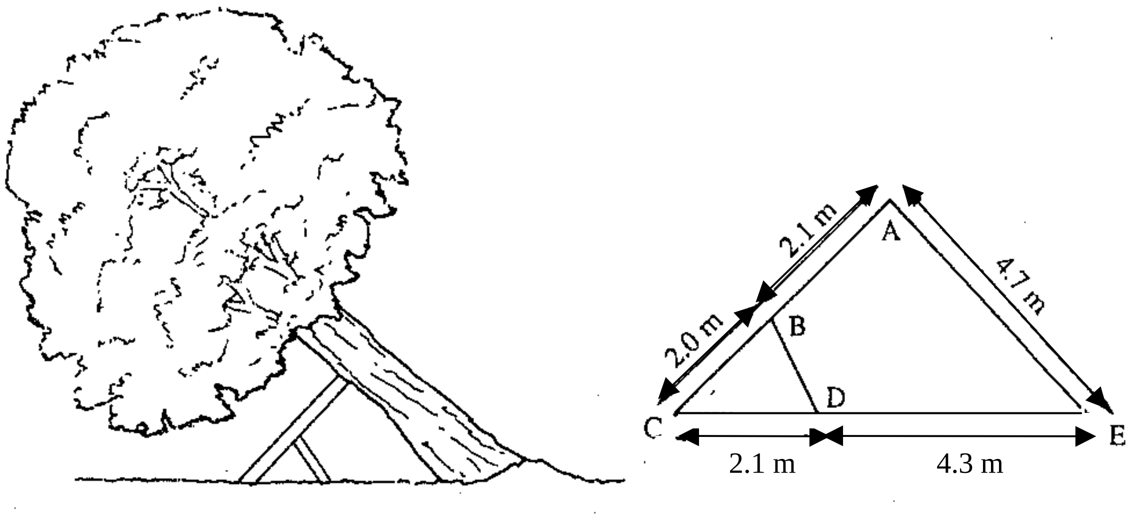
The curve passes through the point (-1, 3).
Find the equation of the curve. **[5]**

7



The graph shows the curve with equation $y = x(2-x)$.
Find the area of the region enclosed between the curve and the x-axis. **[5]**

8

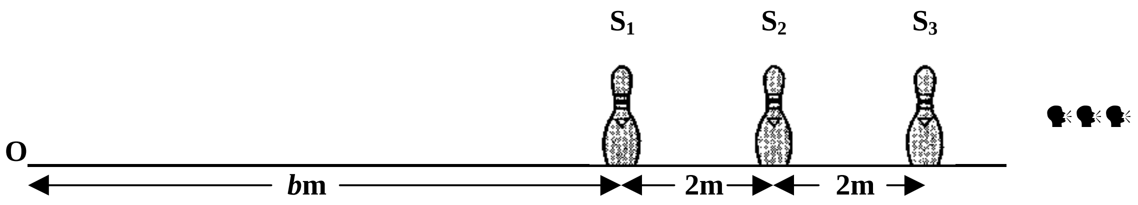


In the gales last year, a tree started to lean and needed to be supported by struts that were wedged as shown above. There is also a simplified diagram giving dimensions.
Calculate the angle the tree makes with the vertical, giving your answer to the nearest degree. **[5]**

Section B (36 marks)

9 In a race, skittles S_1, S_2, S_3, …are placed in a line, spaced 2 metres apart.

Contestants run from the starting point O, *b* metres from the first skittle. They pick up the skittles, one at a time and in order, returning them to O each time.



(i) Show that the total distance of a race with 3 skittles is $6(b+2)$ metres. **[1]**

(ii) Show that the total distance of a race with *n* skittles is $2n(b+n-1)$ metres. **[4]**

(iii) With $b=5$, the total distance is 570 metres. Find the number of skittles in this race. **[3]**

A football coach uses this race for training the team. The total distance for each contestant is exactly 1000 metres. The skittles are still 2 metres apart and the value of *b* is a whole number less than 20.

(iv) How many skittles are there in this form of the race? **[3]**

10 A virus is spreading through a population and so a vaccination programme is introduced.

Thereafter, the numbers of new cases are as follows:

Week number, x	1	2	3	4	5
Number of new cases, y	240	150	95	58	38

The number of new cases, y, in week x is to be modelled by an equation of the form $y = pq^x$, where p and q are constants.

(i) Copy and complete this table of values.

x	1	2	3	4	5
$\log_{10} y$					

[1]

(ii) Plot a graph of $\log_{10} y$ against x, taking values of x from 0 to 8. **[2]**

(iii) Explain why the graph confirms that the model is appropriate. **[2]**

(iv) Use the graph to predict the week in which the number of new cases will fall below 20.
Explain why you should treat your answer with caution. **[3]**

(v) Estimate the values of p and q.
Use your values of p and q, and the equation $y = pq^x$, to calculate the value of y when $x = 3$.
Comment on your answer. **[5]**

11 The equation of a curve is given by $y = x^4 - 8x^2 + 7$.

(i) Use calculus to show that the function has a turning point at (2, -9) and find the coordinates of the other turning points. **[7]**

(ii) Sketch the curve. **[2]**

(iii) Show that the line $y = -12x + 12$ is a tangent to the curve at one of the points where it crosses the x-axis. **[3]**

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
CONCEPTS FOR ADVANCED MATHEMATICS, C2

4752

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	$\tan x = 2$ $\arctan 2 = 63.4°$ $x = 63.4°$ or $243.4°$	M1 A1 B1 **[3]**	Use of tan 180°+ previous answer if acute
2	Arc length = $15 \times 0.6 = 9$ Perimeter = $2 \times 15 + 9 = 39$ cm Area $= 0.5 \times 15^2 \times 0.6 = 67.5\,\text{cm}^2$	M1 A1 M1,A1 **[4]**	Correct use of formula + radians Correct use of formula + radians
3	$12x + \frac{1}{2\sqrt{x}}$	M1 M1,A1 A1 **[4]**	Differentiating Handling the $\sqrt{\ }$ No extra terms
4(i) **4(ii)**	$6144 \times (0.25)^9$ 0.0234 $\frac{a(1-r^n)}{(1-r)} = \frac{6144(1-0.25^{10})}{(1-0.25)}$ 8191.9922 $\frac{a}{1-r} = \frac{6144}{1-0.25} = 8192$	M1 A1 M1 A1 **[4]** B1 **[1]**	Attempt to use correct formula for M1 Use of correct formula
5(i) **5(ii)**	<u>x</u> <u>f(x)</u> 0 **1** 0.2 0.96154 0.4 0.86207 0.6 **0.73529** 0.8 0.60976 1.0 **0.5** $\frac{1}{2} \times 0.2 \times [(1+0.5) + 2 \times (0.96154 + ...)]$ 0.78373	 B1 **[1]** M1 A1 A1 A1 **[4]**	 All 3 missing values Interval and end values 2 x Sum of middle values cao

Qu	Answer	Mark	Comment
Section A (continued)			
6	$y = 2x^3 + \frac{5}{x} + c$ $c = 10$	B1 B1 B1 M1 A1 **[5]**	$2x^3$ $\frac{5}{x}$ c Substitution ft
7	$\int_0^2 (2x - x^2)\mathrm{d}x = \left[x^2 - \frac{x^3}{3}\right]_0^2$ $= 1\frac{1}{3}$ sq. units	M1 A1 A1 M1 A1 **[5]**	Use of integral for area Correct integration Correct limits Use of limits
8	$\cos E = \frac{4.7^2 + 6.4^2 - 4.1^2}{2 \times 4.7 \times 6.4}$ $E = 39.8°$ so $40°$ Angle with vertical is $90° - 40° = 50°$	M1 A1,A1 A1 A1 **[5]**	Cosine rule Top line, bottom line cao ft
			Section A Total: 36
Section B			
9(i)	$2b + 2(b+2) + 2(b+4) = 6b + 12 = 6(b+2)$	B1 **[1]**	
9(ii)	AP with first term $2b$, common difference 4 Sum to n terms is: $\frac{1}{2}n(2a + (n-1)d) = 2n(b + n - 1)$	M1 A1 M1,A1 **[4]**	Recognition of AP First term and common difference Use of appropriate formula
9(iii)	$2n(5 + n - 1) = 570$ $n^2 + 4n - 285 = 0$ $(n-15)(n+19) = 0$ 15 skittles	M1 A1 A1 **[3]**	Forming an equation
9(iv)	$2n(b + n - 1) = 1000$ $n(b + n - 1) = 500$ n is a factor of 500 and only 25 works, giving $b = 16$	M1 M1 A1 **[3]**	Equation involving n and b Correct reasoning cao

Qu	Answer	Mark	Comment
Section B (continued)			
10(i)	**x** 1 2 3 4 5 **$\log_{10}y$** **2.38** **2.18** **1.98** **1.76** **1.58**	B1 **[1]**	All correct
10(ii)	Straight line graph	B1,B1 **[2]**	
10(iii)	$y = pq^x \Rightarrow \log y = \log p + x \log q$ Plotting $\log y$ against x should give a straight line and it does	M1 E1 **[2]**	Taking logarithms
10(iv)	log 20 = 1.30 From graph this will be in week 7 It involves extrapolation	M1 A1 B1 **[3]**	Using logarithms
10(v)	Gradient of graph is: $\log q \approx \frac{1.58 - 2.38}{5 - 1} = -0.2$ $q = 10^{-0.2} = 0.63$ Intercept is $\log p = 2.58$ $p = 380$ $y = 380 \times 0.63^3 = 95.0$ Agrees with data	 M1 A1 B1 M1 A1 **[5]**	 Use of gradient ft
11(i)	$\frac{dy}{dx} = 4x^3 - 16x$ $\frac{dy}{dx} = 0 \Rightarrow x = -2, 2 \text{ or } 0$ $x = 2 \Rightarrow y = 2^4 - 8 \times 2^2 + 7 = -9$ (-2, -9) and (0, 7)	M1,A1 M1 A1 E1 B1,B1 **[7]**	Differentiation Setting = 0 For verification for $x = z$
11(ii)	Sketch with coordinates of all 3 turning points.	B1 B1 **[2]**	
11(iii)	$y = -12x + 12$ cuts x-axis at x =1 (1, 0) lies on the curve When $x = 1, \frac{dy}{dx} = -12$	B1 B1 B1 **[3]**	
			Section B Total: 36
			Total: 72

AO	Range	Total	Question Number										
			1	2	3	4	5	6	7	8	9	10	11
1	21-29	28	1	2	2	2	3	2	2	-	3	5	6
2	21-29	28	1	2	2	-	-	3	3	2	6	3	6
3	0-8	3	-	-	-	-	-	-	-	1	1	1	-
4	0-8	3	-	-	-	-	-	-	-	1	1	1	-
5	7-15	10	1	-	-	3	2	-	-	1	-	3	-
	Totals	**72**	3	4	4	5	5	5	5	5	11	13	12

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
METHODS FOR ADVANCED MATHEMATICS, C3

4753

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 It is suggested that the function $f(x) = (x+1)^2$ is even.
Prove this is false. **[2]**

2 Find $\int x \sin 2x \, dx$. **[4]**

3 Make t the subject in $P = P_0 e^{0.1(t-3)}$. **[5]**

4 Sketch the graph of $y = |2x+3|$.
Hence, or otherwise, solve the equation $|2x+3| = 2 - x$. **[5]**

5 Using the substitution $u = 2x - 1$, or otherwise, calculate the exact value of $\int_0^{0.5} 4x(2x-1)^7 \, dx$. **[5]**

6 Differentiate $\sqrt{2x+1}$ with respect to x and show that $\dfrac{d}{dx}(x^2\sqrt{2x+1}) = \dfrac{5x^2+2x}{\sqrt{2x+1}}$. **[7]**

7 The function f(x) is defined as $f(x) = \dfrac{\cos x}{e^x}$ for $-\pi \le x \le \pi$.

Show that $f(x) \ge 0$ for $\dfrac{-\pi}{2} \le x \le \dfrac{\pi}{2}$.

State the values of x for which f(x) = 0.

Show, using calculus, that the maximum value of f(x) is 1.55, correct to 2 decimal places. **[8]**

Section B (36 marks)

8 Fig. 8.1 shows a sketch of the graph $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \sqrt{4-x}$ for $0 \le x \le 4$.

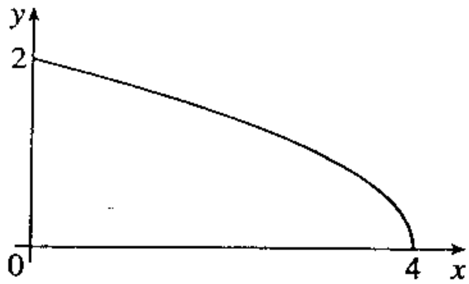


Fig. 8.1

(i) Write down the domain and range of $\mathrm{f}(x)$. **[2]**

(ii) (***A***) Find the inverse function $\mathrm{f}^{-1}(x)$. **[3]**

(***B***) Copy Fig 8.1 and draw the graph of $y = \mathrm{f}^{-1}(x)$ on the same diagram.
What is the connection between the graph of $y = \mathrm{f}(x)$ and the graph of $y = \mathrm{f}^{-1}(x)$? **[2]**

(iii) Figs. 8.2, 8.3 and 8.4 below show the graph of $y = \mathrm{f}(x)$, together with the graphs of $y = \mathrm{f}_1(x)$, $y = \mathrm{f}_2(x)$ and $y = \mathrm{f}_3(x)$ respectively, each of which is a simple transformation of the graph $y = \mathrm{f}(x)$.
Find expressions in terms of x for each of the functions $\mathrm{f}_1(x)$, $\mathrm{f}_2(x)$ and $\mathrm{f}_3(x)$.

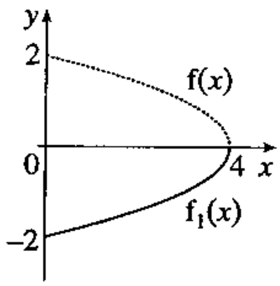


Fig. 8.2

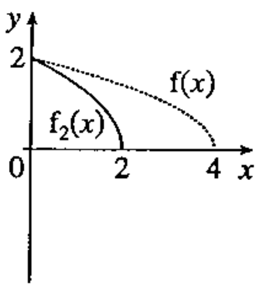


Fig. 8.3

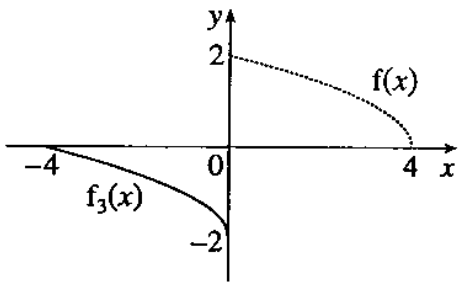


Fig. 8.4 **[4]**

(iv) The function $\mathrm{g}(x)$ is defined in such a way that the composite function $\mathrm{gf}(x)$ is given by $\mathrm{gf}(x) = x - 4$.
Find the functions $\mathrm{g}(x)$ and $\mathrm{g}^2(x)$. **[3]**

(v) State the range of the function $\mathrm{f}^2(x)$.
Hence show that the equation $\mathrm{f}^2(x) = x$ must have a solution.
[You are **not** required to solve the equation.] **[4]**

9 Fig. 9 shows a sketch of the graph $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{\ln x}{x}$ $(x > 0)$.

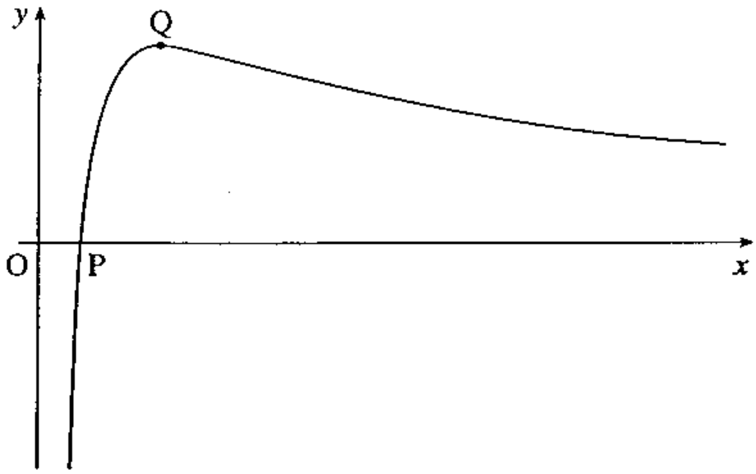


Fig. 9

The graph crosses the x-axis at the point P and has a turning point at Q.

(i) Write down the x-coordinate of P. **[2]**

(ii) Find the first and second derivatives $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$, simplifying your answers as far as possible. **[5]**

(iii) **(*A*)** Hence show that the x-coordinate of Q is e. **[2]**

(*B*) Find the y-coordinate of Q in terms of e. **[1]**

(*C*) Find $\mathrm{f}''(\mathrm{e})$ and use this result to verify that Q is a maximum point. **[2]**

(iv) Find the exact area of the finite region between the graph $y = \mathrm{f}(x)$, the x-axis, and the line $x = 2$. **[6]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

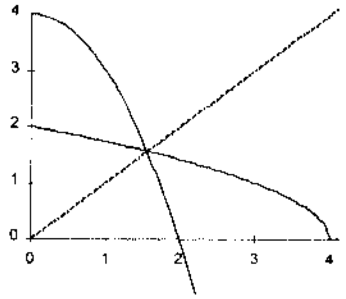
METHODS FOR ADVANCED MATHEMATICS, C3

4753

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	Take a counter-example, e.g. $x=1 \Rightarrow \mathrm{f}(x)=4, \mathrm{f}(-x)=0$ $\therefore \mathrm{f}(x) \neq \mathrm{f}(-x)$	M1 E1 **[2]**	Use must be shown
2	Integration by parts with: $u=x$ and $\dfrac{\mathrm{d}v}{\mathrm{d}x}=\sin 2x$ $v=-\dfrac{1}{2}\cos 2x$ $\left[x\times(-\dfrac{1}{2}\cos 2x)\right]-\int(-\dfrac{1}{2}\cos 2x)\mathrm{d}x$ $-\dfrac{1}{2}x\cos 2x+\dfrac{1}{4}\sin 2x+c$	 M1 A1 A1 A1 **[4]**	 Use of parts
3	$\dfrac{P}{P_0}=\mathrm{e}^{0.1(t-3)}$ $\ln P-\ln P_0=0.1(t-3)$ $t-3=10(\ln P-\ln P_0)$ $t=3+10\ln\dfrac{P}{P_0}$	M1 M1,A1 A1 A1 **[5]**	Separation of e M for use of ln
4	Graph: Segment to right of (-1.5, 0) Segment to left of (-1.5, 0) $2x+3=2-x \Rightarrow x=-\dfrac{1}{3}$ $-(2x+3)=2-x \Rightarrow x=-5$	B1 B1 B1 M1 A1 **[5]**	 Use of $-(2x+3)$
5	Let $u=2x-1$ $\Rightarrow x=\dfrac{1}{2}(u+1),\ \mathrm{d}x=\dfrac{1}{2}\mathrm{d}u$ Limits become –1 and 0 $\int_{-1}^{0}(u+1)u^7\mathrm{d}u=\left\|\dfrac{u^9}{9}+\dfrac{u^8}{8}\right\|_{-1}^{0}$ $-\dfrac{1}{72}$	 M1 M1 M1,A1 A1 **[5]**	 Change of variable Change of limits Integration

Qu	Answer	Mark	Comment
Section A (continued)			
6	**Either** by inspection $2\times\frac{1}{2}\times(2x+1)^{-\frac{1}{2}}$ $=\frac{1}{\sqrt{2x+1}}$	 M1 A1 A1	 Dealing with $\sqrt{\ }$ Use of 2 and ½
	(**Or** Chain rule: Let $t=2x+1,\ \frac{\mathrm{d}t}{\mathrm{d}x}=2$ $y=t^{\frac{1}{2}},\ \frac{\mathrm{d}y}{\mathrm{d}t}=\frac{1}{2}t^{-\frac{1}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{1}{\sqrt{2x+1}}$)	 (M1 A1 A1)	 Chain rule
	Product rule: $x^2\times\frac{1}{\sqrt{2x+1}}+2x\times\sqrt{2x+1}=\frac{5x^2+2x}{\sqrt{2x+1}}$	M1,A1 A1,E1 **[7]**	Product rule
7	$\cos x\geq 0$ for $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$ $\mathrm{e}^x>0$ for all x $\therefore \mathrm{f}(x)\geq 0$ for $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$. $\therefore \mathrm{f}(x)=0$ for $x=-\frac{\pi}{2}$ and $x=\frac{\pi}{2}$. $\mathrm{f}'(x)=\frac{\mathrm{e}^x(-\sin x)-\cos x\,\mathrm{e}^x}{(\mathrm{e}^x)^2}$ $=-\frac{(\sin x+\cos x)}{\mathrm{e}^x}$ For maximum: $\mathrm{f}'(x)=0$ $\Rightarrow \tan x=-1$ $x=-\frac{\pi}{4}$ $\mathrm{f}(x)=1.5508...\to 1.55$	 B1 B1 M1,A1 A1 M1 A1 E1 **[8]**	
			Section A Total: 36

Qu	Answer	Mark	Comment
Section B			
8(i)	Domain $0 \le x \le 4$ Range $0 \le \mathrm{f}(x) \le 2$	B1 B1 **[2]**	
8(ii)(*A*)	$y^2 = 4 - x$ $x = 4 - y^2$ $\mathrm{f}^{-1}(x) = 4 - x^2$	M1 A1 A1 **[3]**	Solving for *y* or reversing flowchart method
8(ii)(*B*)		B1 **[1]**	Correct shape through (0,4) and (2,0)
8(ii)(*C*)	Reflection in $y = x$	B1 **[1]**	
8(iii)	$\mathrm{f}_1(x) = -\sqrt{4-x}$ $\mathrm{f}_2(x) = \sqrt{4-2x}$ $\mathrm{f}_3(x) = \sqrt{(-x)} - 2$	B1 B1 B1,B1 **[4]**	cao cao
8(iv)	$\mathrm{g}(x) = -x^2$ $\mathrm{g}^2(x) = -x^4$	B1,B1 B1 **[3]**	1 for – sign cao
8(v)	$\mathrm{f}^2(x) = \sqrt{4 - \sqrt{4-x}}$ Range is $\sqrt{2} \le \mathrm{f}^2(x) \le 2$ since $y = x$ goes from (0, 0) to (4, 4) and $y = \mathrm{f}^2(x)$ from $(0, \sqrt{2})$ to (4, 2), the two intersect. $\therefore \mathrm{f}^2(x) = x$ has a solution.	M1 A1 E1 E1 **[4]**	

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\ln x = 0$ $\Rightarrow x = 1$ coordinates are (1,0)	M1 A1 **[2]**	 $x = 1$
9(ii)	**Either:** $f'(x) = \dfrac{x.\frac{1}{x} - \ln x.1}{x^2} = \dfrac{1-\ln x}{x^2}$	M1 A1 A1	Quotient rule (consistent with their derivatives) Correct numerator $\dfrac{1-\ln x}{x^2}$ cao
	(**Or:** $x^{-1}.\dfrac{1}{x} - x^{-2}\ln x = x^{-2} - x^{-2}\ln x$ **)**	(M1 A1 A1)	(Product rule Correct expression Simplified correctly (allow –ve indices))
	$f''(x) = \dfrac{x^2.(-\frac{1}{x}) - (1-\ln x).2x}{x^4}$	M1	Any expression for $f''(x)$ consistent with their $f'(x)$ (condone missing brackets)
	$= \dfrac{-x - 2x + 2x\ln x}{x^4}$ $= \dfrac{2\ln x - 3}{x^3}$	A1 **[5]**	$\dfrac{2\ln x - 3}{x^3}$ or $\dfrac{2x\ln x - 3x}{x^4}$ or $2x^{-3}\ln x - 3x^{-3}$
9(iii)(*A*)	**Either:** $f'(x) = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow x = \mathrm{e}$	M1 E1 **[2]**	Their $f'(x) = 0$ soi or calculates $f'(\mathrm{e}) \Rightarrow x = \mathrm{e}$ or $1 - \ln \mathrm{e} = 0$ www
	(Or: $f'(\mathrm{e}) = \dfrac{1-\ln \mathrm{e}}{\mathrm{e}^2} = 0$ **)**		
9(iii)(*B*)	when $x = \mathrm{e}$, $y = \dfrac{\ln \mathrm{e}}{\mathrm{e}} = \dfrac{1}{\mathrm{e}}$	B1 **[1]**	$y = \dfrac{1}{\mathrm{e}}$
9(iii)(*C*)	$f''(\mathrm{e}) = \dfrac{2\ln \mathrm{e} - 3}{\mathrm{e}^3} = -\dfrac{1}{\mathrm{e}^3} < 0$ $f''(\mathrm{e}) < 0 \Rightarrow$ Q is a maximum point	M1 A1 **[5]**	Substituting e into their $f''(x)$ cao $f''(x) = -\dfrac{1}{\mathrm{e}^3}$ or –0.04989…<0 $\Rightarrow$ Q is a maximum point [must evaluate $f''(\mathrm{e})$]

Qu	Answer	Mark	Comment
Section B (continued)			
9(iv)	$A = \int_1^2 \frac{\ln x}{x} \mathrm{d}x$	M1	Correct integral and limits
	let $u = \ln x$	M1	Using substitution $u = \ln x$
	$\Rightarrow \mathrm{d}u = \frac{1}{x} \mathrm{d}x$		
	$A = \int_{\ln 1}^{\ln 2} u \mathrm{d}u$	A1	Integral in terms of u with limits
	$= \left[\frac{1}{2} u^2 \right]_0^{\ln 2}$	A1	
	$= \frac{1}{2} (\ln 2)^2 = \ln 2$	M1 A1 **[6]**	Use of limits cao
			Section B Total: 36
			Total: 72

AO	Range	Total	Question Number									CWk
			1	2	3	4	5	6	7	8	9	
1	36-41	38	-	1	2	3	3	3	2	10	10	4
2	36-41	37	1	3	3	2	2	4	5	6	7	4
3	0-9	0	-	-	-	-	-	-	-	-	-	-
4	0-9	4	1	-	-	-	-	-	-	2	-	1
5	9-18	11	-	-	-	-	-	-	1	-	1	9
	Totals	**90**	2	5	4	5	5	7	8	18	18	18

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
APPLICATIONS OF ADVANCED MATHEMATICS, C4
PAPER A

4754

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Find the binomial expansion of $\sqrt{1+2x}$ up to and including the term in x^3, simplifying the coefficients.
State the values of x for which this expansion is valid. **[5]**

2 PQR is a straight line, with the points in that order.
The coordinates of P and Q are (2, 1) and (7, 1) respectively. The point S has coordinates (7,13).
The length of SR is 20 units.

Find $\tan \mathrm{P\hat{S}Q}$ and $\tan \mathrm{Q\hat{S}R}$ and hence show that $\tan \mathrm{P\hat{S}R} = \frac{63}{16}$. **[5]**

3 Write $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta+\alpha)$ where R and α are to be determined.
Solve $3\sin\theta + 4\cos\theta = 1$ for $0° \le \theta \le 360°$. **[6]**

4 **(i)** A curve, C, has parametric equations

$$x = \sin\theta - \cos\theta + 1$$

$$y = \sin 2\theta$$

Show that the cartesian equation of the curve is $y = -x^2 + 2x$. **[4]**

(ii) Sketch the curve $y = -x^2 + 2x$ and indicate which part of it corresponds to the curve C. **[2]**

5 Show that $\frac{x}{x+1} = 1 - \frac{1}{x+1}$.

The curve $y = \frac{x}{x+1}$, from $x = 0$ to 2, is rotated through $360°$ about the x-axis.

Show that the volume of revolution is $(\frac{8}{3} - 2\ln 3)\pi$. **[7]**

6 A curve has parametric equations $x = 3t,\ y = \frac{4}{t}$.

Show that the straight line joining (0, 4) to (12, 0) is a tangent to the curve and state the value of t at the point where the line touches the curve. **[7]**

Section B (36 marks)

7 The population of a city is P millions at time t years. When $t=0$, $P=1$.

(i) A simple model is given by the differential equation: $\frac{\mathrm{d}P}{\mathrm{d}t}=kP$ where k is a constant.

(*A*) Verify that $P=A\mathrm{e}^{kt}$ satisfies this differential equation, and show that $A=1$.
Given that $P=1.24$ when $t=1$, find k. **[5]**

(*B*) Why is this model unsatisfactory in the long term? **[1]**

(ii) An alternative model is given by the differential equation: $4\frac{\mathrm{d}P}{\mathrm{d}t}=P(2-P)$.

(*A*) Express $\frac{4}{P(2-P)}$ in partial fractions. **[3]**

(*B*) Hence, by integration, show that: $\frac{P}{2-P}=\mathrm{e}^{\frac{1}{2}t}$. **[5]**

(*C*) Express P in terms of t.
Verify that, when $t=1$, P is approximately 1.24. **[3]**

(*D*) According to this model, what happens to the population of the city in the long term? **[1]**

8 Fig. 8 illustrates the flight path of a helicopter H taking off from an airport.
Coordinate axes O*xyz* are set up with the origin O at the base of the airport control tower. The *x*-axis is due east, the *y*-axis due north, and the *z*-axis vertical. The units of distance are kilometres throughout.
The helicopter takes off from the point G. The position vector *r* of the helicopter *t* minutes after take-off is given by: $\mathbf{r} = (1+t)\mathbf{i} + (0.5+2t)\mathbf{j} + 2t\mathbf{k}$.

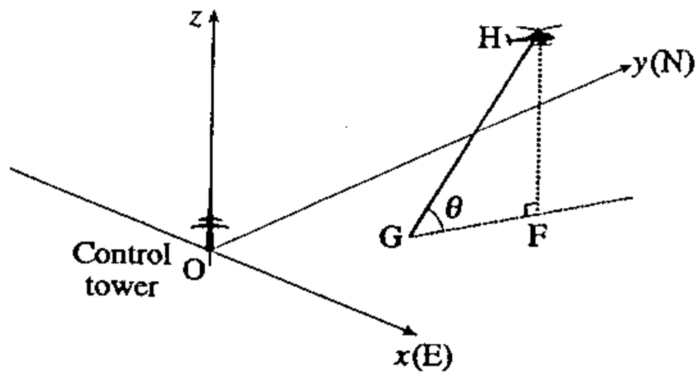


Fig. 8

(i) Write down the coordinates of G. **[1]**

(ii) Find the angle the flight path makes with the horizontal. (This angle is shown as θ in Fig. 8). **[3]**

(iii) Find the bearing of the flight path. (This is the bearing of the line GF shown in Fig. 8). **[2]**

(iv) The helicopter enters a cloud at a height of 2 km.
Find the coordinates of the point where the helicopter enters the cloud. **[3]**

(v) A mountain top is situated at M (5, 4.5, 3).
Find the value of *t* when HM is perpendicular to the flight path GH.
Find the distance from the helicopter to the mountain top at this time. **[5]**

(vi) Find, in vector form, the equation of the line GM.
Find also the angle between the line from G to the mountain top and the helicopter's flight path. **[4]**

Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4754**

APPLICATIONS OF ADVANCED MATHEMATICS, C4
PAPER A

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	$(1+2x)^{\frac{1}{2}}$	M1	Handling $\sqrt{\ }$
	$1+\frac{1}{2}(2x)+\frac{1}{2}(\frac{1}{2}-1)\frac{(2x)^2}{2!}+\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)\frac{(2x)^3}{3!}+\ ...$	M1,A1	Expansion of right form
	$1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+...$	A1	
	$-\frac{1}{2}<x<\frac{1}{2}$	B1 **[5]**	
2	$\tan\mathrm{P\hat{S}Q}=\frac{5}{12}$ and $\tan\mathrm{Q\hat{S}R}=\frac{16}{12}$	B1,B1	
	Let $\mathrm{P\hat{S}Q}=A,\ \mathrm{Q\hat{S}R}=B$		
	$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	M1	Use of formula
	$\tan(A+B)=\frac{\frac{5}{12}+\frac{16}{12}}{1-\frac{5}{12}\times\frac{16}{12}}$	A1	
	$\sin\mathrm{P\hat{S}R}=\frac{\frac{21}{12}}{\frac{144-80}{144}}=\frac{63}{16}$	E1 **[5]**	
3	$5(\sin\theta\times\frac{3}{5}+\cos\theta\times\frac{4}{5})$	M1	Correct form
	$5\sin(\theta+53.1°)$, $R=5,\ \alpha=53.13...°$	A1,A1	
	$5\sin(\theta+53.1°)=1$		
	$\sin(\theta+53.1°)=0.2$, $\arcsin(0.2)=11.536...°$		
	$\theta+53.1°=...\ 11.5°,\ 168.5°,\ 371.5°,\ 528.5°,\ ...$	M1	Search for many roots
	In range, $\theta=115.3°,\quad 318.4°$	A1,A1 **[6]**	
4(i)	$x^2=\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta+2\sin\theta-2\cos\theta+1$	M1,A1	
	$x^2=-\sin 2\theta+2\sin\theta-2\cos\theta+2$	B1	
	$x^2=-y+2x$		
	$y=-x^2+2x$	E1 **[4]**	
4(ii)	Sketch graph of $y=-x^2+2x$	B1	
	Part between approx. (-0.4, -1) and (2.4, -1) highlighted.	B1 **[2]**	

Qu	Answer	Mark	Comment
Section A (continued)			
5	$1-\dfrac{1}{x+1}=\dfrac{x+1-1}{x+1}=\dfrac{x}{x+1}$	B1	
	Volume = $\pi\displaystyle\int_0^2\left(\frac{x}{x+1}\right)^2\mathrm{d}x$	M1	Volume of revolution procedure
	$=\pi\displaystyle\int_0^2\left(1-\frac{2}{x+1}+\frac{1}{(x+1)^2}\right)\mathrm{d}x$	A1	
	$=\pi\left\|x-2\ln\|x+1\|-\dfrac{1}{x+1}\right\|_0^2$	A1	logarithm
		A1	$-\dfrac{1}{x+1}$
	$=[2-2\ln 3-\frac{1}{3}]\pi-[-1]\pi$	M1	Use of limits
	$=\left(\dfrac{8}{3}-2\ln 3\right)\pi$	A1 **[7]**	
6	Gradient of line is $-\dfrac{1}{3}$	M1	
	For curve $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\dfrac{-\frac{4}{t^2}}{3}=-\dfrac{4}{3t^2}$	M1,A1	Procedure for finding gradient
	$-\dfrac{4}{3t^2}=-\dfrac{1}{3}$ when $t=2$ or -2	M1 A1	Equating gradient to $-\dfrac{1}{3}$
	When $t=2$, the curve is at (6, 2) and (6, 2) lies on the line $x+3y=12$	M1 E1 **[7]**	
			Section A Total: 36
Section B			
7(i)(*A*)	$P=Ae^{kt}$ $\Rightarrow\dfrac{\mathrm{d}P}{\mathrm{d}t}=kAe^{kt}$	M1	Differentiating
	$=kP$	E1	Replacing by P
	when $t=0$, $P=1$, $\Rightarrow 1=Ae^0=A$	B1	Verifying $A=1$ (may come first)
	when $t=1$, $P=1.24=1.e^k$ $\Rightarrow k=\ln 1.24=0.215$	M1 A1 **[5]**	Substituting $t=1$ 0.215… accept ln1.24 or 0.22 or better
7(i)(*B*)	As $t\to\infty, P\to\infty$, so population grows without limit	B1 **[1]**	Unlimited growth

Qu	Answer	Mark	Comment
Section B (continued)			
7(ii)(*A*)	$\frac{4}{P(2-P)} \equiv \frac{A}{P} + \frac{B}{2-P}$	M1	$\frac{A}{P} + \frac{B}{2-P}$
	$\Rightarrow 4 \equiv A(2-P) + BP$		
	$P = 0 \ \Rightarrow 4 = 2A$		
	$\Rightarrow A = 2$	A1	$A = 2$
	$P = 2 \ \Rightarrow 4 = 2B$		
	$\Rightarrow B = 2$	A1	$B = 2$
	so $\frac{4}{P(2-P)} \equiv \frac{2}{P} + \frac{2}{2-P}$	**[3]**	
7(ii)(*B*)	$\int \frac{4}{P(2-P)} \mathrm{d}P = \int \mathrm{d}t$	M1	$\int \frac{4}{P(2-P)} \mathrm{d}P = \int \mathrm{d}t$
	$\Rightarrow 2\int (\frac{1}{P} + \frac{1}{2-P}) \mathrm{d}P = \int \mathrm{d}t$		
	$\Rightarrow 2[\ln P - \ln(2-P)] = t + c$	B1,B1	LHS $= 2[\ln P - \ln(2-P)]$
	$\Rightarrow \ln \frac{P}{2-P} = \frac{1}{2}t + c$		
	when $t = 0$, $P = 1$, $\Rightarrow \ln 1 = c = 0$	DM1	Evaluating c at any stage
	$\Rightarrow \frac{P}{2-P} = \mathrm{e}^{\frac{1}{2}t}$ *	E1 **[5]**	Deriving*
7(ii)(*C*)	$\frac{P}{2-P} = \mathrm{e}^{\frac{1}{2}t}$	M1	Multiplying through by $2-P$ and expanding
	$\Rightarrow P = (2-P)\mathrm{e}^{\frac{1}{2}t} = 2\mathrm{e}^{\frac{1}{2}t} - P\mathrm{e}^{\frac{1}{2}t}$		Collecting Ps
	$\Rightarrow P(1 + \mathrm{e}^{\frac{1}{2}t}) = 2\mathrm{e}^{\frac{1}{2}t}$	A1	cao $P = \frac{2\mathrm{e}^{\frac{1}{2}t}}{1+\mathrm{e}^{\frac{1}{2}t}}$ or $P = \frac{2}{\mathrm{e}^{-\frac{1}{2}t} + 1}$
	$\Rightarrow P = \frac{2\mathrm{e}^{\frac{1}{2}t}}{1+\mathrm{e}^{\frac{1}{2}t}}$	E1	P = 1.2449 or 1.245 accept 1.24 or better
	when t = 1.24, P = 1.2449 $\approx$ 1.24	**[3]**	SC putting t = 1 and verifying P = 1.24 (B1)
7(ii)(*D*)	As $t \to \infty, P \to 2$	B1 **[1]**	$P \to 2$

Qu	Answer	Mark	Comment
Section B (continued)			
8(i)	G is (1, 0.5, 0)	B1 **[1]**	(1, 0.5, 0) accept: $\begin{pmatrix}1\\0.5\\0\end{pmatrix}$, $\mathbf{i}+\frac{1}{2}\mathbf{j}$, (1, 0.5)
8(ii)	Direction of GH is $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$	M1	Direction of GH
	$\tan\theta=\frac{2}{\sqrt{5}}\Rightarrow\theta=42°$	A1	$\tan\theta=\frac{2}{\sqrt{5}}$ or equivalent
		A1 **[3]**	42°
8(iii)	Direction of GF is $\mathbf{i}+2\mathbf{j}$	M1	$\mathbf{i}+2\mathbf{j}$ or $\arctan\frac{1}{2}$ seen anywhere
	Angle with north is $\arctan\frac{1}{2}=27°$	A1	27° or 027°
	Bearing is 027°	**[2]**	
8(iv)	$z=2$ when $t=1$, $r=2\mathbf{i}+2.5\mathbf{j}+2\mathbf{k}$ coordinates are (2, 2.5, 2)	M1 A1 A1 **[3]**	$z=2$ $\Rightarrow t=1$ (2, 2.5, 2)
8(v)	$\overrightarrow{HM}=5\mathbf{i}+4.5\mathbf{j}+3\mathbf{k}-[(1+t)\mathbf{i}+(0.5+2t)\mathbf{j}+2t\mathbf{k}]$ $=(4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)\mathbf{k}$	M1	$\overrightarrow{HM}=(4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)\mathbf{k}$
	perpendicular when $\overrightarrow{HM}.\overrightarrow{GH}=0$	M1	$\overrightarrow{HM}.\overrightarrow{GH}=0$ allow this (M1) for $\overrightarrow{HM}$.(their $\overrightarrow{GH}$)
	$\Rightarrow[(4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)\mathbf{k}].[\mathbf{i}+2\mathbf{j}+2\mathbf{k}]=0$ $\Rightarrow 4-t+8-4t+6-4t=0$	A1	
	$\Rightarrow 18-9t=0\Rightarrow t=2$	A1	$t=2$ f.t. their equation
	at this time $\overrightarrow{HM}=2\mathbf{i}+\mathbf{k}$, $\lvert\text{HM}\rvert=\sqrt{5}$ km	A1 **[5]**	cao $\sqrt{5}=2.24$ km
8(vi)	$\overrightarrow{GM}=(5\mathbf{i}+4.5\mathbf{j}+3\mathbf{k})-(\mathbf{i}+0.5\mathbf{j})$ $=4\mathbf{i}+4\mathbf{j}+3\mathbf{k}$	M1	
	So line GM is $\mathbf{r}=(\mathbf{i}+0.5\mathbf{j})+\lambda(4\mathbf{i}+4\mathbf{j}+3\mathbf{k})$	A1	
	Angle MGH is between vectors $(4\mathbf{i}+4\mathbf{j}+3\mathbf{k})$ and $(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$	M1	
	$(4\mathbf{i}+4\mathbf{j}+3\mathbf{k})\bullet(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$ $=\sqrt{4^2+4^2+3^2}\sqrt{1^2+2^2+2^2}\cos\theta$		
	$\Rightarrow\theta=20.4°$	A1 **[4]**	
			Section B Total: 36
			Total: 72

AO	Range	Total	Paper A Question Number								Paper B Comprehension
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
1	27-32	27	2	2	2	2	4	3	6	6	-
2	27-32	29	2	2	2	4	3	4	8	4	-
3	9-18	10	-	-	-	-	-	-	4	6	-
4	13-23	19	1	-	-	-	-	-	-	-	18
5	4-14	5	-	1	2	-	-	-	-	2	-
	Totals	**72**	5	5	6	6	7	7	18	18	18

Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4
PAPER B: COMPREHENSION

4754

Specimen Insert

TIME Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- This insert contains the text for use with the questions in the related Specimen Paper.

Acknowledgement

The research referred to in this article was carried out by Martyn Gorman and John Speakman of the University of Aberdeen, and Michael Mills and Jacobus Raath from the Kruger National Park in South Africa. OCR would particularly like to thank Martyn Gorman for his help in the production of this article.

Saving the African Wild Dog

Introduction

The African Wild Dog (*Lycaon pictus*) was once plentiful south of the Sahara. However, in recent years its numbers have declined sharply and it is believed that as few as 5000 individuals now remain.

This article outlines some recent work on a mathematical model for one possible cause of its decline, and considers the implications for conservation measures.

The African Wild Dog

The African Wild Dog is a completely different species from the domestic dog and it is illustrated in Fig. 1. The large rounded ears are a characteristic feature.

African Wild Dogs live in packs of up to about 40 individuals and survive by hunting. They usually prey on larger animals such as wildebeest, impala and gazelle. Their method is to approach a herd, select an individual, and then chase it until it is exhausted. At any time two dogs take the lead with the others following on behind; when those two get tired another two take over.

A pack of dogs hunts twice a day, in the morning and the evening, and spends the rest of its time eating and resting.

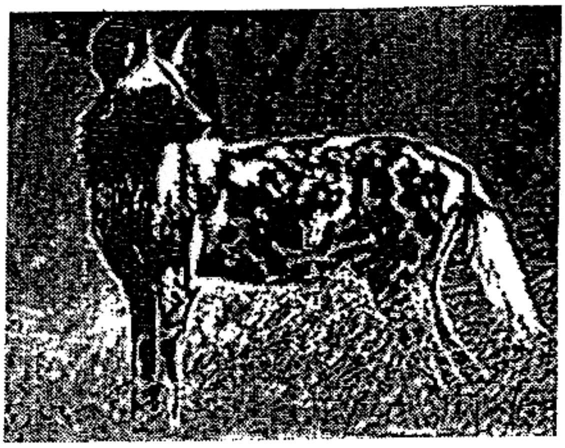


Fig. 1: An African Wild Dog

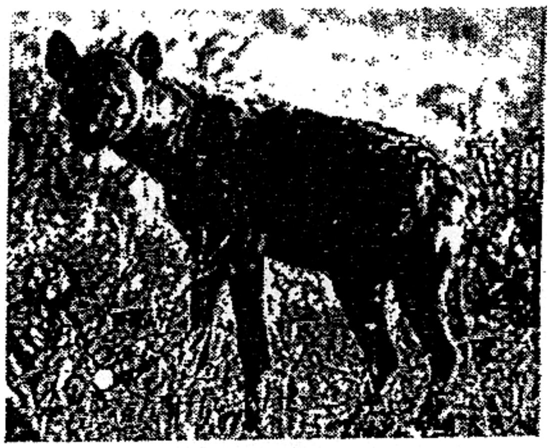


Fig. 2: A Spotted Hyena

Various reasons have been suggested for the decline in the numbers of African Wild Dogs. One of these is their relationship with the Spotted Hyena (*Crocuta crocuta*) (Fig. 2).

Data from a number of places in Africa suggest that where the density of hyenas is high, the density of wild dogs is low, and vice versa.

Hyenas are much feared by other animals and consequently are able to steal food which others, such as cheetahs and wild dogs, have just hunted and killed. (This habit is called *kleptoparasitism*.) It is believed that this may be a cause of the diminishing number of wild dogs.

Balancing Energy

Before considering the effect of food being stolen it is helpful to model the simpler situation in which the dogs eat all the meat they capture. This is done in terms of energy.

For any dog the energy output over a reasonable period of time must be the same as the energy input over the same period. A common unit for energy is the megajoule (MJ) and this is used throughout this article.

Taking a period of 24 hours gives the equation

$$E = ht + r(24 - t) \qquad \textbf{(1)}$$

where

E is the energy, in MJ, expended in a 24-hour day, the *daily energy expenditure*,
t is the number of hours hunting per day,
h is the rate of energy output when hunting, in MJ per hour,
r is the rate of energy output when not hunting, in MJ per hour.

Notice that these variables represent average values. They will vary from day to day and from one dog to another. This article is looking at a typical dog on a typical day.

The dog takes in energy by eating meat that has been captured. (One kilogram of meat coverts into about 4.4 MJ of energy.) The rate of capturing meat can thus be thought of as a rate of energy capture as a result of hunting. A further variable, c, is thus needed, where

c is the rate of energy capture while hunting, in MJ per hour.

The energy captured in a day's hunting, in MJ, is therefore ct.

Thus, assuming the dogs eat all the meat they capture, the energy balance is expressed by the equation

$$ct = ht + r(24 - t)\,. \qquad \textbf{(2)}$$

Equation **(2)** can be rearranged to make t the subject, giving

$$t = \frac{24r}{c + r - h}\,. \qquad \textbf{(3)}$$

Equation **(3)** gives the number of hours that a dog needs to hunt in a day. The sketch graph in Fig. 3 shows t plotted against c.

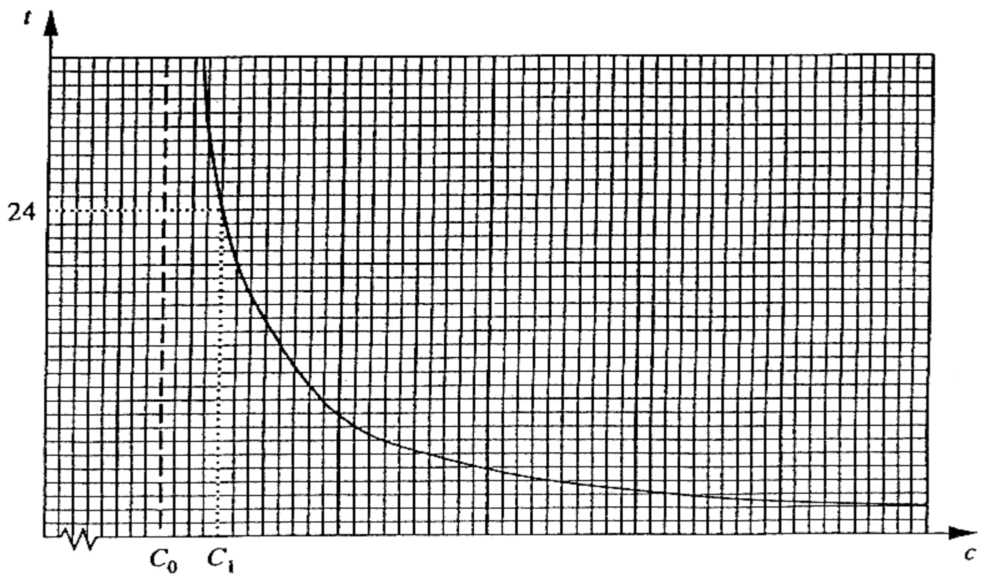


Fig. 3

There are a number of features of this graph to notice.

- The larger the value of c, the less time a dog needs to hunt.
- There is an asymptote for a certain value of c, marked C_0. The value of c must exceed C_0.
- There is however another value of c, marked as C_1 ($C_1 > C_0$), which corresponds to $t = 24$. Unless c exceeds C_1, there are not enough hours in a day for a dog to catch sufficient meat to fulfil its energy requirements.

The value C_1 represents a theoretical rather than a practical limit. No dog can hunt for anything like 24 hours a day; the value of c must be sufficiently greater than C_1 for t to have a realistic value, much less than 24.

Finding Values for the Variables

Recent research on a pack of wild dogs in the Kruger National Park has meant that, for the first time, it is possible for estimates to be made of the values of all the variables used in this article.

- The dogs' hunting times were recorded for days when their meat was not stolen and an average value calculated: $t \approx 3.45$ in hours (i.e. 3 hours 27 minutes).
- Measurements on **six** of the dogs in the pack were used to estimate the daily energy expenditure of a wild dog: $E \approx 15.3$ in MJ.
- The quantity r was estimated using an established experimental formula for domestic dogs relating the mass of a dog to its rate of energy expenditure when resting: $r \approx 0.22$ in MJ per hour.
- Substituting these figures into equation **(1)** allows an estimate to be made of the rate of energy expenditure when hunting: $h \approx 3.12$ in MJ per hour.
- Substitution also gives an estimate of the rate of energy capture when no meat is stolen: $c \approx 4.43$ in MJ per hour.

These values of h and r would suggest that the value, C_0, of c for which the asymptote occurs in Fig. 3 is 2.90. The value of c obtained above, 4.43, is quite well above this.

Food Loss

The model used so far has assumed that the dogs eat all the meat they capture. This is not the case; it is observed that hyenas often steal meat from wild dogs.

At first sight it would seem that the loss of, say, 10% of a pack's food would be made up by spending about 10% extra time hunting. Since wild dogs only hunt for a few hours a day this would represent a minor loss of their leisure time. However a suitable refinement of the model shows that this is not the case.

A further variable p is introduced to represent the proportion of the food (or energy) that is stolen: $0 \le p < 1$.

So, although the energy a dog captures in a 24-hour period is ct, its energy intake over that period is $(1-p)ct$.

Replacing ct by $(1-p)ct$ in equation **(2)**, and rearranging it to make t the subject, gives

$$t = \frac{24r}{(1-p)c + r - h} \qquad \textbf{(4)}$$

The graph in Fig. 4, in which t is plotted against p, illustrates this relationship. The values taken for r, h and c are those calculated on page 4.

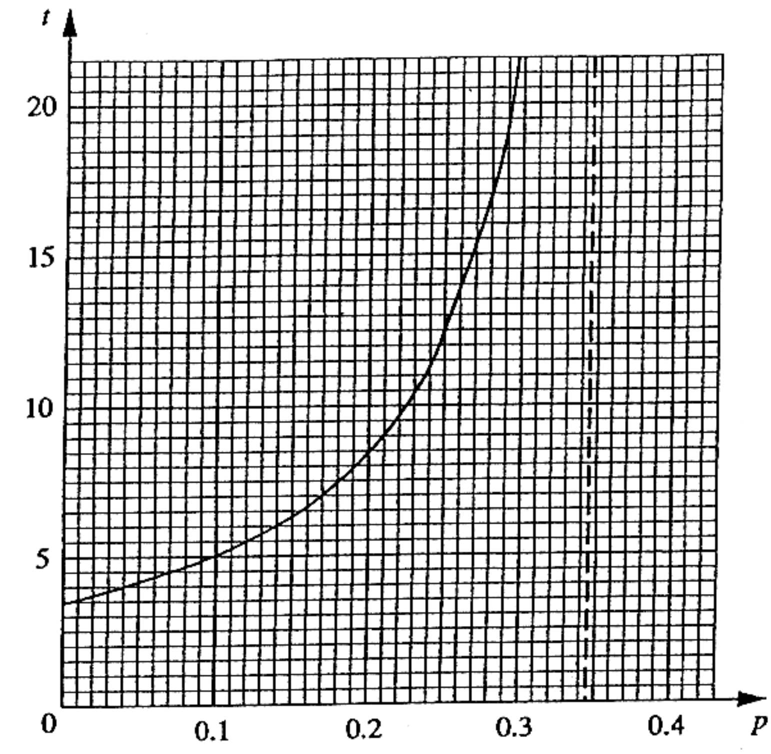


Fig. 4

Conclusions

Fig. 4 shows just how close to the limit the dogs are living. For example, if just 25% of the meat they capture is stolen, they must increase their hunting from about 3½ hours to over 12 hours a day.

Even without having any meat stolen, wild dogs work extremely hard. A wild dog is comparable in size to a collie sheep dog. A working collie sheep dog has an energy output of about 8 MJ per day, compared with the estimated 15.3 MJ for a wild dog. This is believed to be close to the limit of what the wild dogs' bodies can take. The extra energy requirements produced by having quite small quantities of food stolen may well prove fatal.

As a result of the study described in this article, some conservationists have concluded that it is pointless to try to protect the African Wild Dog in open country where there are many hyenas, and where the hyenas find it easy to detect that a kill has just taken place. The situation is different in areas of thick vegetation, both because few hyenas live there and because they are less likely to detect a kill.

Consequently efforts to save the African Wild Dog from extinction are now likely to be concentrated on those populations living in areas of thick vegetation.

Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4754**
APPLICATIONS OF ADVANCED MATHEMATICS, C4
PAPER B: COMPREHENSION

Specimen Paper

Additional materials: Answer booklet
MEI Examination Formulae and Tables (MF 2)

TIME Up to 1 hour

Candidate Name	Centre Number	Candidate Number

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces above.
- Write your answers, in blue or black ink.
- Answer **all** the questions.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.

1 State the meaning of the terms ht and $r(24-t)$ in equation **(1)**.

[2]

2 In line 62, the value of h is stated to be 3.12.
Explain how this figure was obtained from the information given in lines 54 to 60.

[2]

3 Show how equation **(3)** is obtained from equation **(2)**.

[2]

4 In Fig. 3, there is an asymptote at $c = C_0$.
Find an expression, in terms of r and h, for C_0.
Justify the value given for C_0 in line 66.

[3]

5 Using figures given in the article calculate the hunting time if 20% of the meat is stolen.

____

____

____ **[2]**

6 Use equation **(4)** to calculate the value of *p* at the asymptote in Fig. 4.

____

____

____

____ **[2]**

7 The article gives estimates that were made of the values of the variables involved. While these estimates were the best that could be obtained under the circumstances, it is possible that they are not particularly accurate.

(i) State one likely source of error.

____

____ **[1]**

(ii) Explain briefly how you could assess the effect of any such errors on the value of *p* for which the asymptote in Fig. 4 occurs.

____

____

____ **[1]**

8 The article contains information that allows you to calculate the average number of kilograms of meat that a wild dog eats in a day.
Find this information and carry out the calculation.

____

____

____ **[3]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4754**

APPLICATIONS OF ADVANCED MATHEMATICS, C4

PAPER B: COMPREHENSION

MARK SCHEME

Qu	Answer	Mark
1	ht is the energy expended when hunting. $r(24-t)$ is the energy expended when not hunting.	B1 B1 **[2]**
2	Equation **(1)** is $E = ht + r(24-t)$ $t = 3.45, E = 15.3, r = 0.22$ $\Rightarrow$ $15.3 = 3.45h + 0.22(24-3.45)$ $\Rightarrow$ $h = (15.3 - 4.521)/3.45$ $= 3.1243$	 M1 A1 **[2]**
3	Equation **(2)** is $ct = ht + r(24-t)$ $\Rightarrow$ $ct + rt - ht = 24r$ $\Rightarrow$ $(c + r - h)t = 24r$ $\Rightarrow$ Equation **(3)**	 M1 E1 **[2]**
4	Equation **(3)** is $t = \dfrac{24r}{c + r - h}$ This has an asymptote when $c + r - h = 0$ $\Rightarrow$ $c = h - r$ $\Rightarrow$ $C_0 = h - r$ Hence $C_0 = 3.12 - 0.22 = 2.90$	 M1 A1 E1 **[3]**
5	If 20% of the meat is stolen, $p = 0.2$ Equation **(4)** is $t = \dfrac{24r}{(1-p)c + r - h}$ $\Rightarrow$ $t = 24x0.22/((1-0.2)4.43\ldots + 0.22 - 3.12\ldots$ $= 8.2$	 M1 A1 **[2]**
6	From equation **(4)** the asymptote occurs when $(1-p)c + r - h = 0$ $(1 - p = (h - r)/c$ $= (3.12 - 0.22)/4.43$ $= 0.655(0.6546\ldots)$ $p = 0.345$	 M1 A1 **[2]**

Qu	Answer	Mark
7(i)	One sensible comment such as: The sample size of six dogs was very small; The formula for calculating r applies to domestic dogs and so may not be accurate for wild dogs.	B1 **[1]**
7(ii)	Any sensible answer, such as: Give the various inputs somewhat different values, say by 10%, and repeat the calculation to find the corresponding error in the value of p.	B1 **[1]**
8	Energy output 15.3 Mj per day Energy from 1 kg of meat = 4.4 Mj Meat consumed $\frac{15.3}{4.4} \simeq 3.5$ kg	B1 B1 B1
		Total: 18

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1 **4755**

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Find the values of A, B and C in the identity $x^2 = A(x-1)^2 + B(x-2) + C$. **[3]**

2 Solve the inequality $x^2 \geq \frac{1}{x}$. **[4]**

3 Matrices **A** and **B** are given by:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & k \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 6 & 0 & -2k \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{pmatrix}.$$

Find the matrix product **AB**.
Hence write down the inverse of matrix **A** in the case when $k = 3$. **[4]**

4 A complex number α is given by $\alpha = 1 + 5\mathrm{j}$.

(i) Find the modulus of α. **[1]**

(ii) Write down the complex conjugate α^*. **[1]**

(iii) Write down the value of $\alpha\alpha^*$. **[1]**

(iv) Express $\dfrac{\alpha + \alpha^*}{\alpha^*}$ in the form $a + b\mathrm{j}$. **[2]**

5 The matrix $\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ defines a transformation in the (x, y)-plane.

(i) Find $\mathbf{A}^2$ and $\mathbf{A}^3$. **[3]**

(ii) Describe fully the transformation represented by **A**. **[3]**

6 Find $\sum_{r=1}^{n} r(6r+1)$, giving your answer in a fully factorised form. **[6]**

7 The quadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$ has roots α, $-\alpha$, β and β.

(i) Express p, q, r and s in terms of α and β, simplifying your answers. **[6]**

(ii) Hence show that $pr - 4s = 0$. **[2]**

Section B (36 marks)

8 A curve has equation $y = \dfrac{x(x-1)}{(x+2)(x-3)}$.

(i) Write down the values of x for which $y = 0$. **[1]**

(ii) Write down the equations of the 3 asymptotes. **[3]**

(iii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your description. **[3]**

(iv) Sketch the curve. **[3]**

(v) The equation $\dfrac{x(x-1)}{(x+2)(x-3)} = k$ has no real roots.
What can you say about the value of k? **[4]**

9 **(i)** Given that $\alpha = -1 + 2\text{j}$, express α^2 and α^3 in the form $a + b\text{j}$.
Hence show that α is a root of the cubic equation: $z^3 + 7z^2 + 15z + 25 = 0$. **[5]**

(ii) Find the other two roots of this cubic equation. **[4]**

(iii) Illustrate the three roots of the cubic equation on an Argand diagram. **[2]**

10 Prove by induction that $\sum_{r=1}^{n}(3r^2 - r) = n^2(n+1)$ for all positive integers, n. **[11]**

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1 **4755**

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	$A = 1$ $B = 2$ $C = 3$	B1 B1 B1 **[3]**	
2	Sketch graph showing $y = x^2$ and $y = \frac{1}{x}$ Point of intersection, (1, 1) $x \geq 1$ or $x < 0$	M1 B1 B1,B1 **[4]**	 Accept correct answer for all four marks
3	$\mathbf{AB} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ $\mathbf{A}^{-1} = \frac{1}{6}\mathbf{B}$ When $k = 3$, $\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & \frac{1}{2} & -\frac{1}{6} \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$	B1,B1 M1 A1 **[4]**	 Finding $\mathbf{A}^{-1}$.
4(i)	$\lvert\alpha\rvert = \sqrt{26}$	B1 **[1]**	
4(ii)	$\alpha^* = 1 - 5\text{j}$	B1 **[1]**	
4(iii)	$\alpha\alpha^* = 1^2 - (5\text{j})^2 = 26$	B1 **[1]**	
4(iv)	$\frac{2(1+5\text{j})}{(1-5\text{j})(1+5\text{j})} = \frac{1}{13} + \frac{5}{13}\text{j}$	M1,A1 **[2]**	Use of conjugate.
5(i)	$\mathbf{A}^2 = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ $\mathbf{A}^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B2 B1 **[3]**	B1 for 2 correct numbers
5(ii)	Rotation centre (0, 0) 120° anticlockwise	B1 B1 B1 **[3]**	

Qu	Answer	Mark	Comment
Section A (continued)			
6	$6\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$	M1,A1	Separate sums.
	$6\times\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)$	M1,A1	Use of Σ formulae.
	$\frac{1}{2}n(n+1)[2(2n+1)+1]$	M1	Factorising.
	$\frac{1}{2}n(n+1)(4n+3)$	A1 **[6]**	
7(i)	$p=2\beta$	B1	
	$q=-\alpha^2+\alpha\beta+\alpha\beta-\alpha\beta-\alpha\beta+\beta^2$	M1	
	$=-\alpha^2+\beta^2$	A1	
	$r=-(-\alpha^2\beta-\alpha^2\beta+\alpha\beta^2-\alpha\beta^2)$	M1	
	$=2\alpha^2\beta$	A1	
	$s=-\alpha^2\beta^2$	B1 **[6]**	
7(ii)	$pr=-2\beta\times2\alpha^2\beta=-4\alpha^2\beta^2=4s$	M1	
	$\Rightarrow pr-4s=0$	E1 **[2]**	
			Section A Total: 36
Section B			
8(i)	$x=0$ and 1	B1 **[1]**	Both.
8(ii)	$x=-2,\ x=3$ and $y=1$	B1, B1,B1 **[3]**	
8(iii)	Large positive x, $y\to1^+$	B1	
	(e.g. consider $x=100$)	B1	Evidence needed for this mark.
	Large negative x, $y\to1^+$	B1 **[3]**	
8(iv)	Curve		
	3 branches.	B1	
	Correct approaches to vertical asymptotes.	B1	
	Correct approaches to horizontal asymptotes (from above).	B1 **[3]**	

Qu	Answer	Mark	Comment
Section B (continued)			
8(v)	Graph is symmetrical about $x=\frac{1}{2}$	B1	
	There is a maximum at $x=\frac{1}{2}$	B1	Allow equivalent wording; this mark may be awarded implicitly.
	When $x=\frac{1}{2}$, $y=\frac{1}{25}$	M1	
	so $\frac{1}{25}<k<1$	A1 **[4]**	Use of algebra also acceptable.
9(i)	$\alpha^2=-3-4\mathrm{j}$, $\alpha^3=11-2\mathrm{j}$	B1,B1	
	Substituting: $(11-2\mathrm{j})+7((-3-4\mathrm{j})+15(-1+2\mathrm{j})+25$	M1	Substituting and equating real and imaginary parts.
	$=0+0\mathrm{j}$	A1	
	$\therefore\alpha$ is a root.	E1 **[5]**	
9(ii)	Roots occur in conjugate pairs.	M1	Use of conjugacy.
	$\alpha^*=-1-2\mathrm{j}$ is another root.	A1	
	Sum of roots = -7 $\Rightarrow -5$ is the third root	M1	A correct method to find the third root.
	(or equivalent method).	A1 **[4]**	
9(iii)	Argand diagram.	M1	Use of Argand diagram.
	Real and imaginary axes. Points marked.	A1 **[2]**	

Qu	Answer	Mark	Comment
Section B (continued)			
10	For $k=1$, LHS = 3-1 =2	M1	Initialisation.
	RHS = $1^2 \times (1+1) = 2$		
	Therefore it is true for $k=1$.	E1	
	Assume true for *k*,	M1	Assuming true.
	Next term is $3(k+1)^2 - (k+1)$	M1	Next term.
	Add to both sides	M1	Add to both sides.
	RHS = $k^3 + k^2 + 3(k+1)^2 - (k+1)$		
	$= k^3 + k^2 + 3k^2 + 6k + 3 - k - 1$	A1	Expansion
	$= k^3 + 3k^2 + 3k + 1 + k^2 + 2k + 1$		
	$= (k+1)^3 + (k+1)^2$	A1	
	$= (k+1)^2 (k+2)$	A1	
	But this is the given result with $(k+1)$ replacing *k*.	M1	Explicit statement required.
	Therefore if it is true for *k* it is true for $(k+1)$.	M1	
	Since it is true for $k=1$ it is also true for $k = 2, 3, \ldots$	E1	
		[11]	
			Section B Total: 36
			Total: 72

AO	Range	Total	Question Number 1	2	3	4	5	6	7	8	9	10
1	25-33	32	1	2	2	4	3	3	4	4	6	3
2	25-33	32	2	2	1	1	3	2	4	6	5	6
3	0-8	-	-	-	-	-	-	-	-	-	-	-
4	0-8	7	-	-	1	-	-	-	-	4	-	2
5	0-8	1	-	-	-	-	-	1	-	-	-	-
	Totals	**72**	3	4	4	5	6	6	8	14	11	11

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2 **4756**

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions in Section A and **one** question from Section B.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (54 marks)
Answer all the questions

1 **(a)** **(i)** Given that $f(x) = \arctan(1+x)$, find $f'(x)$ and $f''(x)$. **[4]**

(ii) Find the Maclaurin series for $f(x)$, as far as the term in x^2. **[4]**

(b) A curve Q has polar equation $r = a(1+2\cos\theta)$ for $-\frac{2}{3}\pi \le \theta \le \frac{2}{3}\pi$.

(i) Sketch the curve Q. **[3]**

(ii) Find the area of the region enclosed by the curve Q. **[7]**

2 **(i)** Express $e^{jk\theta}$ and $e^{-jk\theta}$ in the form $a + jb$.
Show that $e^{j\pi} = -1$. **[5]**

(ii) Show that $\dfrac{1}{1-e^{j\theta}} = \dfrac{1}{2}(1 + j\cot\dfrac{1}{2}\theta)$. **[5]**

(iii) Find the sixth roots of 8j in the form $re^{j\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$.
Illustrate these roots on an Argand diagram. **[6]**

(iv) Show that two of these sixth roots have the form $m + jn$, where m and n are integers. **[2]**

3 A matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} -1 & -1 & 1 \\ 6 & 2 & k \\ 0 & -2 & 1 \end{pmatrix}$.

(i) Find, in terms of k,

(*A*) the determinant of $\mathbf{M}$ **[2]**

(*B*) the inverse matrix $\mathbf{M}^{-1}$. **[4]**

One of the eigenvalues of $\mathbf{M}$ is 2.

(ii) Find the value of k, and show that the other two eigenvalues are 1 and -1. **[7]**

(iii) Find the eigenvector corresponding to the eigenvalue of 2. **[3]**

(iv) Find integers p, q and r such that $\mathbf{M}^2 = p\mathbf{M} + q\mathbf{I} + r\mathbf{M}^{-1}$. **[2]**

Section B (18 marks)
Answer one question

Option 1: Hyperbolic functions

4 **(i)** Given that $k \geq 1$ and $\cosh x = k$, prove that $x = \pm\ln\left(k + \sqrt{k^2 - 1}\right)$. **[5]**

In the remainder of this question, $\mathrm{f}(x) = 2\sinh^2 x - 5\cosh x$.

(ii) Solve the equation $\mathrm{f}(x) = 10$, giving your answers in an exact logarithmic form. **[4]**

(iii) Find the coordinates of the stationary points on the curve $y = \mathrm{f}(x)$. **[6]**

(iv) Sketch the curve $y = \mathrm{f}(x)$. **[3]**

Option 2: Geometry

5 **(i)** A curve, C, has parametric equations

$$x = 6\cos T$$
$$y = 6\sin T$$

Prove that this curve is a circle. **[1]**

Before proceeding with the rest of this question, you are advised to enter this curve into your calculator and to set the scales so that it appears as a circle.

(ii) Another curve, H, has parametric equations:

$$x = 5\cos T + \cos 5T$$
$$y = 5\sin T - \sin 5T$$

Enter this curve, also, onto your calculator.
Sketch and describe its main features of the curve, including its greatest and least distances from the origin. **[4]**

The curve H is a particular member of a family of curves.
The general member is defined by the parametric equations:

$$x = k\cos T + \cos kT$$
$$y = k\sin T - \sin kT$$

for positive integer values of k.

(iii) Generalise, in terms of k, the features described in part **(ii)**. **[2]**

(iv) Show that the distance, r, of the point (x, y) from the origin is given by:
$r^2 = k^2 + 2k\cos(k+1)T + 1$.
Show that this result is consistent with your answer to part **(iii)**. **[10]**

The curves in this family are called hypocycloids.
A hypocycloid is the locus of a point on the circumference of a circle as it rolls round the inside of another circle of larger radius.

(v) In this case, the radius of the smaller circle is 1 unit.
Write down the radius of the larger circle. **[1]**

RECOGNISING ACHIEVEMENT

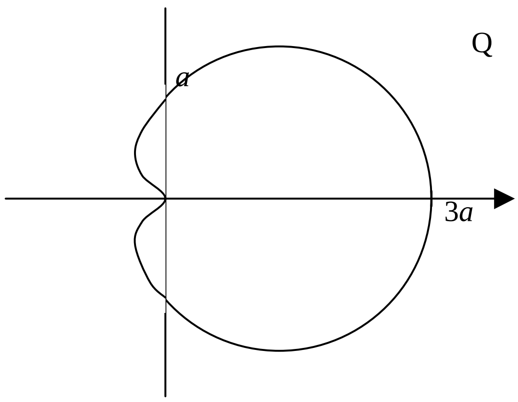
Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

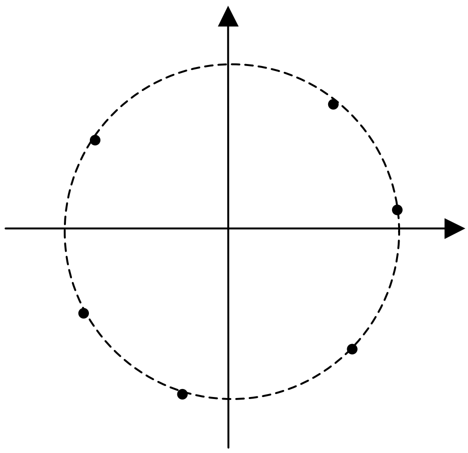
FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2

4756

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1(a)(i)	$f'(x) = \dfrac{1}{1+(1+x)^2}$	M1,A1	
	$f''(x) = \dfrac{-2(1+x)}{\{1+(1+x)^2\}^2}$	M1,A1 **[4]**	Use of chain (or quotient) rule
1(a)(ii)	$f(0) = \frac{1}{4}\pi$, $f'(0) = \frac{1}{2}$, $f''(0) = -\frac{1}{2}$	M1,A1	Evaluating (at least two) when $x = 0$
	Maclaurin series is $\arctan(1+x) = \frac{1}{4}\pi + \frac{1}{2}x - \frac{1}{4}x^2 + \ldots$	A2 **[4]**	F.t. Give A1 ft for two terms correct
1(b)(i)		B1 B1 B1 **[3]**	General shape (0, 0) (3*a*, 0)
1(b)(ii)	Area $= \displaystyle\int_{-\frac{2}{3}\pi}^{\frac{2}{3}\pi} \frac{1}{2}a^2(1+2\cos\theta)^2\,d\theta$	M1,A1	
	$= \dfrac{1}{2}a^2 \displaystyle\int_{-\frac{2}{3}\pi}^{\frac{2}{3}\pi} (1+4\cos\theta+4\cos^2\theta)\,d\theta$		
	$= \dfrac{1}{2}a^2 \displaystyle\int_{-\frac{2}{3}\pi}^{\frac{2}{3}\pi} (3+4\cos\theta+2\cos 2\theta)\,d\theta$	M1,A1	Handling $\cos^2\theta$
	$= \dfrac{1}{2}a^2 \left[3\theta+4\sin\theta+\sin 2\theta\right]_{-\frac{2}{3}\pi}^{\frac{2}{3}\pi}$	B1	
	$= a^2(2\pi + 2\sqrt{3} - \dfrac{\sqrt{3}}{2})$	B1	
	$= (2\pi + \dfrac{3\sqrt{3}}{2})a^2$	A1 **[7]**	

Qu	Answer	Mark	Comment
Section A (continued)			
2(i)	$e^{jk\theta} = \cos k\theta + j\sin k\theta$, $e^{-jk\theta} = \cos k\theta - j\sin k\theta$ For $k\theta = \pi$, $e^{j\pi} = \cos\pi + j\sin\pi$ $e^{j\pi} = -1$	B1,B1 M1,A1 E1 **[5]**	Allow $e^{-jk\theta} = \cos(-k\theta) + j\sin(-k\theta)$
2(ii)	$\dfrac{1}{1-e^{j\theta}} = \dfrac{e^{-\frac{1}{2}j\theta}}{e^{-\frac{1}{2}j\theta} - e^{\frac{1}{2}j\theta}}$ $= \dfrac{\cos\frac{1}{2}\theta - j\sin\frac{1}{2}\theta}{-2j\sin\frac{1}{2}\theta}$ $= \frac{1}{2}j\cot\frac{1}{2}\theta + \frac{1}{2}$	M1,A1 A1,A1 A1	
	(Or: $\dfrac{1}{1-e^{j\theta}} = \dfrac{1}{1-\cos\theta - j\sin\theta}$ $= \dfrac{1-\cos\theta + j\sin\theta}{(1-\cos\theta)^2 + \sin^2\theta}$ $= \dfrac{2\sin^2\frac{1}{2}\theta + 2j\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{4\sin^2\frac{1}{2}\theta}$ $= \frac{1}{2} + \frac{1}{2}j\cot\frac{1}{2}\theta$ **)**	**(Or:** M1,A1 A1,A1 A1**)**	
	(Or: $\dfrac{1}{2\sin^2\frac{1}{2}\theta - 2j\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}$ $= \dfrac{1}{2\sin\frac{1}{2}\theta(\sin\frac{1}{2}\theta - j\cos\frac{1}{2}\theta)}$ $= \dfrac{\sin\frac{1}{2}\theta + j\cos\frac{1}{2}\theta}{2\sin\frac{1}{2}\theta}$ $= \frac{1}{2}j\cot\frac{1}{2}\theta + \frac{1}{2}$ **)**	**(Or:** B1 B1 M1,A1 A1**)**	
		[5]	

Qu	Answer	Mark	Comment
Section A (continued)			
2(iii)	$8\mathrm{j} = 8\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}$, so sixth roots $r\mathrm{e}^{\mathrm{j}\theta}$ have	B1	Accept $\sqrt[6]{8}$ *or* $8^{\frac{1}{6}}$ *or* 1.4
	$r = \sqrt[6]{8} = \sqrt{2}$ $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, -\frac{\pi}{4}, -\frac{7\pi}{12}, -\frac{11\pi}{12}$	B3	Give B1 for one correct, B2 for 3 correct Allow $\theta = \frac{\pi}{12} + \frac{2k\pi}{6}$, $k = 0, \pm 1, \pm 2$ etc. Accept decimals. Deduct 1 for degrees
		B2 **[6]**	Give B1 for three points correct
2(iv)	$\sqrt{2}\mathrm{e}^{\mathrm{j}\frac{3\pi}{4}} = -1 + \mathrm{j}$, $\sqrt{2}\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}} = 1 - \mathrm{j}$	B1,B1 **[2]**	
3(i)(*A*)	Det $\mathbf{M} = -1(2+2k) + 1(6) + 1(-12)$ $= -2k - 8$	M1 A1 **[2]**	
3(i)(*B*)	$\mathbf{M}^{-1} = \frac{-1}{2k+8}\begin{pmatrix} 2k+2 & -1 & -k-2 \\ -6 & -1 & k+6 \\ -12 & -2 & 4 \end{pmatrix}$	B4 **[4]**	F.t. Deduct 1 for missing (or wrong) determinant, failure to transpose, one or two wrong elements
3(ii)	Det $(\mathbf{M} - 2\mathbf{I}) = 0$ $-3(2k) + 1(-6) + 1(-12) = 0$ $k = -3$	M1 M1 A1	
	Det $(\mathbf{M} - \lambda\mathbf{I}) = 0$ $(-1-\lambda)[(2-\lambda)(1-\lambda) - 6] + 1[6(1-\lambda)] - 12 = 0$	M1A1	F.t.
	$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$ λ = 1, -1, 2	A1 A1 **[7]**	cao Or any correct factorised form
3(iii)	$\begin{pmatrix} -1 & -1 & 1 \\ 6 & 2 & -3 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ p \\ q \end{pmatrix} = \begin{pmatrix} 2 \\ 2p \\ 2q \end{pmatrix}$	M1	
	$\Rightarrow p = -1,\ q = 2$	A1,A1 **[3]**	

Qu	Answer	Mark	Comment
Section A (continued)			
3(iv)	$\mathbf{M}^3 - 2\mathbf{M}^2 - \mathbf{M} + 2\mathbf{I} = 0$ $\mathbf{M}^2 = 2\mathbf{M} + \mathbf{I} - 2\mathbf{M}^{-1}$	M1 A1 **[2]**	 cao
			Section A Total: 54
Section B			
4(i)	$\frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x}) = k$ $\mathrm{e}^{2x} - 2k\mathrm{e}^x + 1 = 0$ $\mathrm{e}^x = \frac{2k \pm \sqrt{4k^2 - 4}}{2}$ $(= k \pm \sqrt{k^2 - 1})$	M1 M1 A1	 (± not required)
	(Or: $\sinh x = \sqrt{k^2 - 1}$ (when $x > 0$ $k + \sqrt{k^2 - 1} = \cosh x + \sinh x = \mathrm{e}^x$ **)**	**(Or:** M1 M1,A1**)**	
	(Or: $\frac{\mathrm{d}}{\mathrm{d}k}\ln(k + \sqrt{k^2 - 1}) = \ldots = \frac{1}{\sqrt{k^2 - 1}}$ $\ln(k + \sqrt{k^2 - 1}) = \cosh^{-1} k + C$ When $k = 1$, $0 = 0 + C$, so $C = 0$ **)**	**(Or:** B2 B1**)**	
	$(k - \sqrt{k^2 - 1})(k + \sqrt{k^2 - 1}) = k^2 - (k^2 - 1) = 1$ so $k - \sqrt{k^2 - 1} = \frac{1}{k + \sqrt{k^2 - 1}}$ so $\ln(k - \sqrt{k^2 - 1}) = -\ln(k + \sqrt{k^2 - 1})$ $x = \pm\ln(k + \sqrt{k^2 - 1})$	M1 A1 **[5]**	 ag
4(ii)	$2(\cosh^2 x - 1) - 5\cosh x = 10$ $2\cosh^2 x - 5\cosh x - 12 = 0$ $(\cosh x - 4)(2\cosh x + 3) = 0$ $\cosh x = 4$ $x = \pm\ln(4 + \sqrt{15})$	M1 M1 A1 A1 **[4]**	 Dependent on previous M1 Ignore $\cosh x = -\frac{3}{2}$ if stated Cao Or $x = \ln(4 \pm \sqrt{15})$ Give A0 if any other solutions stated

Qu	Answer	Mark	Comment
Section B (continued)			
4(iii)	$\mathrm{f}'(x) = 4\sinh x \cosh x - 5\sinh x$ $= \sinh x(4\cosh x - 5)$ $\mathrm{f}'(x) = 0$ when $\sinh x = 0$, $\cosh x = \frac{5}{4}$ $x = 0$, $x = \pm\ln 2$ Stationary points $(0, -5)$, $(\ln 2, -\frac{41}{8})$, $(-\ln 2, -\frac{41}{8})$	M1,A1 M1 A1 A2 **[6]**	One term is sufficient for M1 Accept 0.69 or $\cosh^{-1}\frac{5}{4}$ for x but $y = -5.125$ must be exact Give A1 for one correct
4(iv)	Curve shows Max at $(0, -5)$ Minima either side $y \to \infty$ for large x (+ or –)	 B1 B1 B1 **[3]**	

Qu	Answer	Mark	Comment
5(i)	$x^2+y^2=36\cos^2 T+36\sin^2 T=36$ Circle of radius 6	E1 **[1]**	
5(ii)	A continuous closed curve 6 cusps Max distance 6 Min distance 4	B1 B1 B1 B1 **[4]**	Allow comment on minimum values of r
5(iii)	Curve has $(k+1)$ cusps Bounded by circles of radii $(k+1)$, $(k-1)$	B1 B1 **[2]**	All required
5(iv)	$r^2=x^2+y^2$	M1	Eliminating x and y
	$x^2=k^2\cos^2 T+2k\cos T\cos kT+\cos^2 kT$ $y^2=k^2\sin^2 T-2k\sin T\sin kT+\sin^2 kT$	B1	Both
	$r^2=k^2(\cos^2 T+\sin^2 T)+2k(\cos T\cos kT-\sin T\sin kT)$ $+\cos^2 kT+\sin^2 kT$ $=k^2+2k\cos[(k+1)T]+1$	A1,E1	Condition for greatest (or least) value
	Greatest value of r^2 is when $\cos[(k+1)T]=1$ and is $k^2+2k+1=(k+1)^2$ Greatest value of r is $(k+1)$	M1 A1 A1	
	Least value of r^2 is when $\cos[(k+1)T]=-1$ and is $k^2-2k+1=(k-1)^2$		
	Least value of r is $(k-1)$	A1	
	Greatest values occur when $(k+1)T=2n\pi$ i.e. at $T=0,\dfrac{2\pi}{n},\dfrac{4\pi}{n},\ldots,\dfrac{2k\pi}{n}$ so $(k+1)$ times So there are $(k+1)$ extreme points	M1 B1 **[10]**	cao
5(v)	Radius $(k+1)$	B1 **[1]**	
			Section B Total: 18
			Total: 72

AO	Range	Total	Question Number				
			1	**2**	**3**	**4**	**5**
1	31-41	38	10	7	9	7	5
2	31-41	40	8	9	8	10	5
3	0-9	0	-	-	-	-	-
4	0-9	5	-	1	1	-	3
5	0-9	7	-	1	-	1	5
	Totals	**90**	18	18	18	18	18

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

FURTHER APPLICATIONS OF ADVANCED MATHEMATICS, FP3 **4757**

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **three** questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Option 1: Vectors

1 Four points A, B, C and D have co-ordinates (0, 5, 0), (3, 10, -4), (7, 0, 24) and (10, *k*, 20), where *k* is a constant.

(i) When $k \neq 5$, find the following, giving your answers (in terms of *k* where appropriate) as simply as possible:

(***A***) the area of the triangle ABC; **[4]**

(***B***) the volume of the tetrahedron ABCD; **[4]**

(***C***) the shortest distance from D to the plane ABC; **[2]**

(***D***) the shortest distance between the lines AB and CD. **[7]**

(ii) When $k = 5$, find the shortest distance between the lines AB and CD. **[7]**

Option 2: Multi-Variable Calculus

2 A surface *S* has equation $\mathrm{g}(x, y, z) = 0$, where $\mathrm{g}(x, y, z) = (y - x)(x + 2y - z)^2 - 32$.

(i) Show that the point (2, 10, 20) lies on the plane. **[2]**

(ii) Show that $\frac{\partial g}{\partial x} = (x + 2y - z)(z - 3x)$, and find $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. **[6]**

(iii) Verify that $\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} + 3\frac{\partial g}{\partial z} = 0$.
Interpret this result in terms of the normal vectors to the surface *S*. **[3]**

(iv) Find the equation of the tangent plane to the surface *S* at the point P(2, 10, 20). **[3]**

(v) The point $\mathrm{Q}(2 + \delta x, 10 + \delta y, 20 + \delta z)$ is a point on the surface *S* close to P.
Find an approximate expression for δz in terms of δx and δy. **[4]**

(vi) R (*a*, 7, *c*) is a point on the surface *S* at which $\frac{\partial g}{\partial x} = 0$.
Show that the tangent plane at R has equation $3y - z = 6$. **[6]**

Option 3: Differential Geometry

3 A curve has parametric equations $x = a(1-\cos^3\theta)$, $y = a\sin^3\theta$, for $0 \le \theta \le \frac{1}{2}\pi$, where a is a positive constant.

(i) Find the length of this curve. **[6]**

(ii) Show that, when this curve is rotated through 2π radians about the y-axis, the curved surface area generated is $\frac{9}{5}\pi a^2$. **[6]**

(iii) Show that the radius of curvature at a general point on the curve is $3a\sin\theta\cos\theta$. **[6]**

(iv) Find the centre of curvature corresponding to the point on the curve where $\theta = \frac{1}{3}\pi$. **[6]**

Option 4: Groups

4 The set $G = \{1, 3, 7, 9, 11, 13, 17, 19\}$ is a group under the binary operation of multiplication modulo 20.

(i) Give the combination table for G. **[4]**

(ii) State the inverse of each element of G. **[3]**

(iii) Find the order of each element of G. **[3]**

(iv) List all the subgroups of G.
Identify those subgroups which are isomorphic to one another. **[6]**

(v) Show hat the subgroups of G obey Lagrange's theorum. **[3]**

(vi) For each of the following, state, giving reasons, whether or not the given set and binary operation is a group. If it is a group, state, giving a reason, whether or not it is isomorphic to G.

(*A*) $J = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under multiplication modulo 8

(*B*) $K = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under addition modulo 8 **[5]**

Option 5: Markov chains

5 Four security cameras are mounted on the walls of a building as shown in Fig. 5. The cameras are connected to a single monitor. The monitor shows pictures from one camera for exactly a minute, and then switches to a different camera for a minute, and so on indefinitely. The camera to which the monitor switches is determined by a computer program.

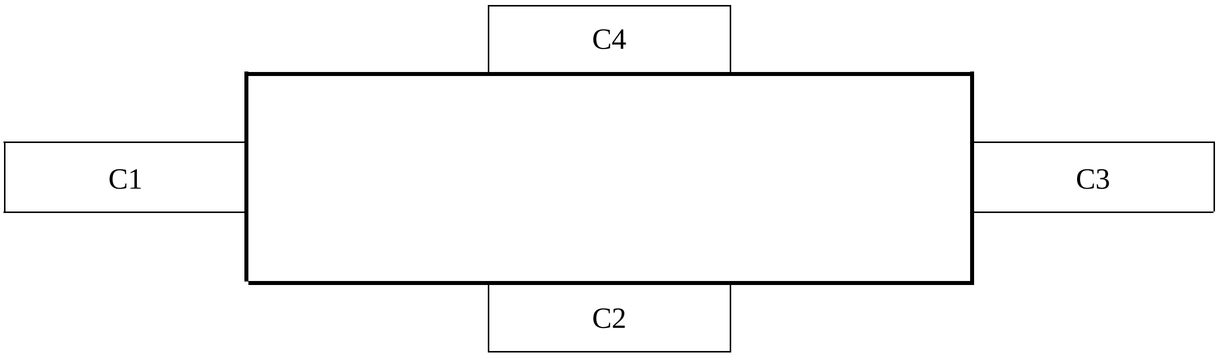


Fig. 5

(i) The system is programmed so that at the end of each minute the monitor switches from the current camera to one of its two neighbours, each being equally likely. (So, for example, when C1 is being monitored, C2 and C4 are equally likely to be monitored in the next minute.).

Write down the transition matrix of the Markov Chain that models this process. Show that the four possible states are periodic and state their period. **[5]**

(ii) A bug develops in the computer program and, as a consequence, once C4 is monitored it continues to be monitored. That is, no further switches take place.

Modify the transition matrix to represent this new situation. Determine the nature of the four states of the process now. **[6]**

(iii) Given that C1 is being monitored during the first minute, determine the probability that C4 is being monitored during the sixth minute. **[3]**

(iv) Given that C1 is being monitored during the first minute, determine the time by which it is 95% certain that C4 is being monitored. **[5]**

(v) Determine the time by which it is 99% certain that C4 is being monitored, given that the camera monitored during the first minute is equally likely to be C1, C2 or C3. **[5]**

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
FURTHER APPLICATIONS OF ADVANCED MATHEMATICS, FP3 **4757**

MARK SCHEME

Qu	Answer	Mark	Comment
Option 1: Vectors			
1(i)(*A*)	Area $=\frac{1}{2}\left\|\overrightarrow{\mathbf{AB}}\times\overrightarrow{\mathbf{AC}}\right\|=\frac{1}{2}\left\|\begin{pmatrix}3\\5\\-4\end{pmatrix}\times\begin{pmatrix}7\\-5\\24\end{pmatrix}\right\|$	M1	
	$=\frac{1}{2}\left\|\begin{pmatrix}100\\-100\\-50\end{pmatrix}\right\|$	M1,A1	
	$=75$	A1 **[4]**	
1(i)(*B*)	Volume $=\frac{1}{6}\left\|\left(\overrightarrow{\mathbf{AB}}\times\overrightarrow{\mathbf{AC}}\right).\overrightarrow{\mathbf{AD}}\right\|$	M1	
	$=\frac{1}{6}\left\|\begin{pmatrix}100\\-100\\-50\end{pmatrix}.\begin{pmatrix}10\\k-5\\20\end{pmatrix}\right\|$	M1,A1	
	$=\frac{50}{3}\|k-5\|$	A1 **[4]**	
1(i)(*C*)	$V=\frac{1}{3}Ah$	M1	
	so $h=\dfrac{3V}{A}=\frac{2}{3}\|k-5\|$	A1 **[2]**	
1(i)(*D*)	Distance is $\overrightarrow{\mathbf{AC}}\cdot\hat{\mathbf{n}}$ where	M1	
	$\mathbf{n}=\overrightarrow{\mathbf{AB}}\times\overrightarrow{\mathbf{CD}}=\begin{pmatrix}3\\5\\-4\end{pmatrix}\times\begin{pmatrix}3\\k\\-4\end{pmatrix}$	M1	
	$=\begin{pmatrix}4k-20\\0\\3k-15\end{pmatrix}$	A1	
	$=(k-5)\begin{pmatrix}4\\0\\3\end{pmatrix}$		
	so $\hat{\underset{\sim}{\mathbf{n}}}=\frac{1}{5}\begin{pmatrix}4\\0\\3\end{pmatrix}$		
	Distance is $\frac{1}{5}\begin{pmatrix}7\\-5\\24\end{pmatrix}\cdot\begin{pmatrix}4\\0\\3\end{pmatrix}=20$	M2 A1 A1 **[7]**	

Qu	Answer	Mark	Comment
Option 1: Vectors (continued)			
1(ii)	When $k = 5$, AB and CD are parallel, we require shortest distance from A(0, 5, 0) to line CD: $\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 24 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix}$	B1	
	$\dfrac{\lvert\overrightarrow{\mathbf{AC}}\times\overrightarrow{\mathbf{CD}}\rvert}{\lvert\overrightarrow{\mathbf{CD}}\rvert} = \dfrac{1}{\sqrt{50}}\left\lvert\begin{pmatrix} 7 \\ -5 \\ 24 \end{pmatrix}\times\begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix}\right\rvert$	M1 A1,A1	A1 for $\sqrt{50}$, A1 for vectors
	Distance is $= \dfrac{1}{\sqrt{50}}\left\lvert\begin{pmatrix} -100 \\ 100 \\ 50 \end{pmatrix}\right\rvert$		
	$= 15\sqrt{2}\ \ (\approx 21.2)$	M2,A1 **[7]**	
Option 2: Multi-Variable Calculus			
2(i)	$g = (10-2)(2+20-20)^2 - 32 = 0$	M1,E1 **[2]**	
2(ii)	$\dfrac{\partial g}{\partial x} = -(x+2y-z)^2 + (y-x)2(x+2y-z)$	M1	
	$= (x+2y-z)(-x-2y+z+2y-2x)$ $= (x+2y-z)(-3x+6y-z)$	A1	
	$\dfrac{\partial g}{\partial y} = (x+2y-z)^2 + (y-x)2(x+2y-z)(2)$	M1,A1	
	$= (x+2y-z)(x+2y-z+4y-4x)$ $= (x+2y-z)(-3x+6y-z)$	A1	
	$\dfrac{\partial g}{\partial z} = -2(y-x)(x+2y-z)$	B1 **[6]**	
2(iii)	$\dfrac{\partial g}{\partial x} + \dfrac{\partial g}{\partial y} + 3\dfrac{\partial g}{\partial z}$ $= (x+2y-z)(z-3x-3x+6y-z-6y+6x)$ $= 0$	B1	
	All normal vectors are perpendicular to $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$	B2 **[3]**	*Or equivalent*
2(iv)	At P, $\dfrac{\partial g}{\partial x} = 28,\ \dfrac{\partial g}{\partial y} = 68,\ \dfrac{\partial g}{\partial z} = -32$	M1,A1	
	Tangent plane is $7x+17y-8z = 14+170-160$	M1	For $7x+17y-8z$
	$7x+17y-8z = 24$	A1 **[4]**	cao
2(v)	$\partial g \approx 28\partial x + 68\partial y - 32\partial z$ (and $\partial g = 0$)	M1,A1	ft Or $7(2+\partial x)+17(10+\partial y)-8(20+\partial z) \approx 24$
	so $\partial z \approx \frac{1}{8}(7\partial x + 17\partial y)$	A1 **[3]**	ft

Qu	Answer	Mark	Comment
Option 2: Multi-Variable Calculus (continued)			
2(vi)	Since $\frac{\partial g}{\partial x}=0,\ \ c=3a$	B1	
	Since R lies on S, $(7-a)(a+14-3a)^2-32=0$	M1	
	$4(7-a)^3-32=0$		
	$7-a=2$		
	$a=5$	A1	Either a or c correct
	$c=15$		
	$\frac{\partial g}{\partial y}=48,\ \ \frac{\partial g}{\partial z}=-16$	M1	Or, using part **(ii)**, $\frac{\partial g}{\partial y}=-3\frac{\partial g}{\partial z}$
	Tangent plane is $3y-z=21-15$	M1	
	$3y-z=6$	A1 **[6]**	
Option 3: Differential Geometry			
3(i)	$\frac{dx}{d\theta}=3a\cos^2\theta\sin\theta\,,\ \frac{dy}{d\theta}=3a\sin^2\theta\cos\theta$	B1	
	$\frac{ds}{d\theta}=\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}$	M1	
	$=3a\sin\theta\cos\theta\sqrt{\cos^2\theta+\sin^2\theta}=3a\sin\theta\cos\theta$	M1	For finding $\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2$
	Arc length is $\int_0^{\frac{1}{2}\pi}3a\sin\theta\cos\theta d\theta$	A1	Correct integral expression (inc limits)
	$=\left[\frac{3}{2}a\sin^2\theta\right]_0^{\frac{1}{2}\pi}$	B1	$\sin\theta\cos\ \theta$ correctly integrated
	$=\frac{3}{2}a$	A1 **[6]**	
3(ii)	Curved surface area is $\int 2\pi x ds$	M1	
	$=\int_0^{\frac{1}{2}\pi}2\pi a(1-\cos^3\theta)(3a\sin\theta\cos\theta)d\theta$	A1	ft Integral expression (limits required)
	$=6\pi a^2\left[-\frac{1}{2}\cos^2\theta+\frac{1}{5}\cos^5\theta\right]_0^{\frac{1}{2}\pi}$	M1	Method for integrating $\sin\theta\cos^4\theta$
		A1	For $-\frac{1}{2}\cos^2\theta$
		A1	For $\frac{1}{5}\cos^5\theta$
	$=6\pi a^2\left(\frac{1}{2}-\frac{1}{5}\right)=\frac{9}{5}\pi a^2$	A1 **[6]**	

Qu	Answer	Mark	Comment
Option 3: Differential Geometry (continued)			
3(iii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\div\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\tan\theta$	M1,A1	
	$\tan\psi=\tan\theta$, so $\psi=\theta$	M1	Or $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{\sec^2\theta}{3a\cos^2\theta\sin\theta}$
	$\rho=\dfrac{ds}{d\psi}=\dfrac{ds}{d\theta}$	M1,M1	Or $\rho=(1+\tan^2\theta)^{\frac{3}{2}}\,x\,\dfrac{3a\cos^2\theta\sin\theta}{\sec^2\theta}$
	$=3a\sin\theta\cos\theta$	A1	
	(OR:		
	$\dot{x}\ddot{y}-\ddot{x}\dot{y}=\ldots=9a^2\sin^2\theta\cos^2\theta$	**(**M1,M1	
		A1	
	$\rho=\dfrac{(3a\sin\theta\cos\theta)^3}{9a^2\sin^2\theta\cos^2\theta}$	M1,M1	
	$=3a\sin\theta\cos\theta$ **)**	A1**)** **[6]**	
3(iv)	When $\theta=\frac{1}{3}\pi,\ \rho=3a\left(\frac{1}{2}\sqrt{3}\right)\left(\frac{1}{2}\right)=\dfrac{3}{4}\sqrt{3}a$	B1	
	$\hat{\mathbf{n}}=\begin{pmatrix}-\sin\psi\\ \cos\psi\end{pmatrix}=\begin{pmatrix}-\frac{1}{2}\sqrt{3}\\ \frac{1}{2}\end{pmatrix}$	M1,A1	
	$x=\frac{7}{8}a,\ \ y=\dfrac{3}{8}\sqrt{3}a$		
	Centre of curvature is		
	$\begin{pmatrix}\frac{7}{8}a\\ \frac{3}{8}\sqrt{3}a\end{pmatrix}+\frac{3}{4}\sqrt{3}a\begin{pmatrix}-\frac{1}{2}\sqrt{3}\\ \frac{1}{2}\end{pmatrix}=\begin{pmatrix}-\frac{1}{4}a\\ \frac{3}{4}\sqrt{3}a\end{pmatrix}$	M1 A1 A1 **[6]**	
Option 4: Groups			
4(i)	$\begin{array}{c\|cccccccc} & 1 & 3 & 7 & 9 & 11 & 13 & 17 & 19\\ \hline 1 & 1 & 3 & 7 & 9 & 11 & 13 & 17 & 19\\ 3 & 3 & 9 & 1 & 7 & 13 & 19 & 11 & 17\\ 7 & 7 & 1 & 9 & 3 & 17 & 11 & 19 & 13\\ 9 & 9 & 7 & 3 & 1 & 19 & 17 & 13 & 11\\ 11 & 11 & 13 & 17 & 19 & 1 & 3 & 7 & 9\\ 13 & 13 & 19 & 11 & 17 & 3 & 9 & 1 & 7\\ 17 & 17 & 11 & 19 & 13 & 7 & 1 & 9 & 3\\ 19 & 19 & 17 & 13 & 11 & 9 & 7 & 3 & 1\end{array}$	B4 **[4]**	Give B1 for 16 entries correct B2 for 32 entries correct B3 for 48 entries correct
4(ii)	$\begin{array}{c\|cccccccc} x & 1 & 3 & 7 & 9 & 11 & 13 & 17 & 19\\ \hline x^{-1} & 1 & 7 & 3 & 9 & 11 & 17 & 13 & 19\end{array}$	B3 **[3]**	Give B1 for 1, 9 Give B1 for 11, 19 Give B1 for rest
4(iii)	$\begin{array}{c\|cccccccc} x & 1 & 3 & 7 & 9 & 11 & 13 & 17 & 19\\ \hline \text{order} & 1 & 4 & 4 & 2 & 2 & 4 & 4 & 2\end{array}$	B3 **[3]**	Give B1 for 1 Give B1 for 9, 11, 19 Give B1 for 3, 7, 13, 17

Qu	Answer	Mark	Comment
Option 4: Groups (continued)			
4(iv)	$\{1\}$, $\{1,9\}$, $\{1,11\}$, $\{1,19\}$	B2	Give B1 for 2 correct
	$\{1,3,7,9\}$, $\{1,9,13,17\}$, $\{1,9,11,19\}$, G	B2	Give B1 for 2 correct (G not required)
	$\{1,9\}$, $\{1,11\}$, $\{1,19\}$ are isomorphic	B1	
	$\{1,3,7,9\}$, $\{1,9,13,17\}$ are isomorphic	B1 **[6]**	Fully correct, dependent on all subgroups of orders 2 and 4 correctly listed, and no spurious IMs given
4(v)	The subgroups of G have orders 1, 2, 4, and 8 The orders are all factors of 8	M1,A1 A1 **[3]**	
4(vi)(*A*)	0 has no inverse So J is not a group	B1 B1 **[2]**	For reason
4(vi)(*B*)	K is closed and inverses of 0, 1, 2, 3, 4, 5, 6, 7 are 0, 7, 6, 5, 4, 3, 2, 1 so K is a group	B1 B1	For reason
	Different pattern (2 self-inverse) K is not isomorphic to G	B1 **[3]**	Must include a reason
Option 5: Markov Chains			
5(i)	$M=\begin{pmatrix}0 & 0.5 & 0 & 0.5\\0.5 & 0 & 0.5 & 0\\0 & 0.5 & 0 & 0.5\\0.5 & 0 & 0.5 & 0\end{pmatrix}$	B1 B1	Diagonals Rest
	$M^2=\begin{pmatrix}0.5 & 0 & 0.5 & 0\\0 & 0.5 & 0 & 0.5\\0.5 & 0 & 0.5 & 0\\0 & 0.5 & 0 & 0.5\end{pmatrix}$	B1	May be implied
	$M^3=M$	M1	Accept argument that does *not* rely on powers of the matrix
	Each state (each camera) alternates between possible and impossible. Period is 2.	A1 **[5]**	

Qu	Answer	Mark	Comment
Option 5: Markov Chains (continued)			
5(ii)	$M = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	B1 B1	Entry 1 Other entries
	Look at M^n	M1	Or argue convincingly from physical considerations.
	C1, C2, C3 are periodic with period 2. C4 is an absorbing state.	A1,A1 A1 **[6]**	
5(iii)	Five transitions from 1st to 6th minute. Top right entry in M^5 is 0.875.	M1 M1 A1 **[3]**	
5(iv)	Calculate M^n for various *n*. Top right entry exceeds 0.95 for the first time in M^9. That is, 9 transitions. So 10th minute.	M1 M1 A1 A1 A1 **[5]**	
5(v)	Define *A* = 0.333333 0.333333 0.333333 0 Calculate *A***M*^*n* for various *n.* Right-most entry exceeds 0.99 for the first time at $n = 13$. So 13 transitions: i.e. in the 14 minute.	M1 A1 M1 A1 A1 **[5]**	
			Total: 72

AO	Range	Total	Question Number				
			1		2		3
1	42-54	51	13	12	10	11	5
2	42-54	50	10	12	11	11	6
3	0-12	6	-	-	-	-	6
4	0-12	6	1	-	-	2	3
5	0-12	7	-	-	3	-	4
	Totals	**120**	24	24	24	24	24

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

DIFFERENTIAL EQUATIONS, DE

4758

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **three** questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.
- Unless otherwise specified, the value of g should be taken to be exactly 9.8ms^{-2}.

1 A solution is sought to the differential equation $\ddot{y} + 3\dot{y} + 2y = \mathrm{e}^{kt} \sin t$, where k is a constant.

(i) In the case $k = 0$, find the general solution. **[8]**

Find also the particular solution for which $y = \dot{y} = 0$ when $t = 0$ **[4]**

(ii) In the case $k = 1$, verify that $y = \dfrac{1}{10}\mathrm{e}^{t}(\sin t - \cos t)$ is a particular integral for the differential equation.

Write down the general solution. **[5]**

(iii) Compare the behaviour of the solutions in the two cases $k = 0$ and $k = 1$ for large values of t.

In the case $k = -1$, what would you expect the behaviour of the solution to be for large valuesof t?

Explain your answer. Is it true for all initial conditions? **[7]**

2 The size, w, of a rabbit population at time t years on an island with a plentiful food supply is modelled, in the absence of predators, by the differential equation $\frac{dw}{dt} = 2w$, with $w = 2000$ when $t = 0$.

(i) Solve the differential equation to find w in terms of t. **[4]**
Find the value of w when $t = 0.1$ and when $t = 0.2$. **[2]**
Describe the behaviour of the solution and say whether this is likely to describe the actual situation. **[2]**

Foxes are introduced to the island. The foxes kill rabbits, but also compete with each other if the rabbit population is too small. The size of the fox population at time t years is x.
The situation is now modelled by the equations

$$\frac{dw}{dt} = 2w - 80x$$

$$\frac{dx}{dt} = 0.2x\left(1 - \frac{100x}{w}\right),$$

with $w = 2000$ and $x = 25$ when $t = 0$.

(ii) Without solving the equations, find the range of values of $\frac{w}{x}$ (i.e. the ratio of rabbits to foxes) for which:

(*A*) the rabbit population increases, **[2]**

(*B*) the fox population increases, **[2]**

(*C*) the rabbit population increases while the fox population decreases. **[1]**

A numerical solution to the equations is sought using a step-by-step method.
The algorithm is given by
$t_{r+1} = t_r + h,$
$w_{r+1} = w_r + hf(w_r, x_r),$
$x_{r+1} = x_r + hg(w_r, x_r)$
where $f(w, x) = \frac{dw}{dt}$ and $g(w, x) = \frac{dx}{dt}$.

The table shows the initial values and the results of the first iteration.

t	***w***	***x***
0	2000	25
0.1	2200	24.875
0.2		

(iii) **(*A*)** Verify the entries for $t = 0.1$. **[4]**

(*B*) Calculate the entries for $t = 0.2$. **[2]**

(*C*) Compare your answers to parts **(iii)(*A*)** and **(iii)(*B*)** to those for $t = 0.1$ and $t = 0.2$ in the original model in part **(i)**. Explain the differences. **[2]**

(iv) Explain briefly why the calculated values of x *cannot* be the actual numbers of foxes at these times.
What aspect of the model has led to this inaccuracy? **[3]**

3 A parachutist of mass 80 kg falls vertically from rest from a stationary helicopter.
At a distance x m below the helicopter her velocity is v ms^{-1}.
The forces acting on her are her weight and air resistance of magnitude kv^2 N, where k is a constant.
Her terminal velocity is 70 ms^{-1}.

(i) Show that the motion may be modelled by the differential equation $v\frac{dv}{dx} = 9.8 - 0.002v^2$. **[4]**

(ii) Solve this differential equation to show that $v = 70(1 - e^{-0.004x})^{\frac{1}{2}}$. **[6]**

When the parachutist's velocity reaches 99% of its terminal value, she has fallen a distance h m.

(iii) Calculate h. **[2]**

She then opens her parachute.
The magnitude of the resistance force now changes instantly to $80v$ N.

(iv) Find her velocity in terms of t, the time in seconds since the parachute opened.
Sketch a graph of v against t. **[8]**

(v) Calculate t when her velocity is 10ms^{-1}.
Calculate how far she falls in this time. **[4]**

4 A solution is sought to the differential equation $\frac{dy}{dt} + 2y = e^{-2t}$.

(i) Find the complementary function. **[2]**

(ii) Explain why an expression of the form ae^{-2t} cannot be a particular integral of this differential equation.
Find a particular integral of this differential equation. **[5]**

An alternative method for solving this equation is by using an integrating factor.

(iii) Use this method to find the general solution of the differential equation.
Hence show that the particular integral found in part **(ii)** is correct. **[8]**

(iv) When $t = 0, y = y_0$. Show that the maximum value of y is $\frac{1}{2}e^{2y_0 - 1}$.
State the range of values of y_0 for which this maximum occurs at a positive value of t. **[9]**

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

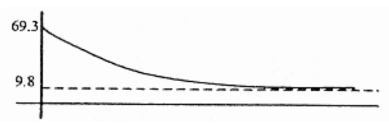
MEI STRUCTURED MATHEMATICS
DIFFERENTIAL EQUATIONS, DE

4758

MARK SCHEME

Qu	Answer	Mark	Comment
1(i)	$\alpha^2+3\alpha+2=0$	M1	
	$\alpha=-1$ or -2	A1	
	CF $y=Ae^{-t}+Be^{-2t}$	F1	CF for their roots (y in terms of t)
	PI $y=a\sin t+b\cos t$	B1	
	$(-a\sin t-b\cos t)+3(a\cos t-b\sin t)$ $+2(a\sin t+b\cos t)=\sin t$	M1	Differentiate twice and substitute
	$-a-3b+2a=1,\quad -b+3a+2b=0$	M1	Compare coefficients
	$a=\frac{1}{10}, b=-\frac{3}{10}$	A1	
	$y=Ae^{-t}+Be^{-2t}+\frac{1}{10}\sin t-\frac{3}{10}\cos t$	F1	
	$A+B-\frac{3}{10}=0$	B1	Equation for A, B from their y
	$\dot{y}=-Ae^{-t}-2Be^{-2t}+\frac{1}{10}\cos t+\frac{3}{10}\sin t$	M1	Differentiate
	$-A-2B+\frac{1}{10}=0$ and so $A=\frac{1}{2}, B=-\frac{1}{5}$	M1	Substitute $t=0$ and solve
	$y=\frac{1}{2}e^{-t}-\frac{1}{5}e^{-2t}+\frac{1}{10}\sin t-\frac{3}{10}\cos t$	A1	
		[12]	
1(ii)	$\dot{y}=\frac{1}{10}e^{t}(\sin t-\cos t)+\frac{1}{10}e^{t}(\sin t-\cos t)$	M1	Differentiate (or use PI of correct form)
	$=\frac{1}{10}e^{t}(2\sin t)$		
	$\ddot{y}=\frac{1}{10}e^{t}(2\sin t+2\cos t)$	A1	
	LHS $=\frac{1}{10}e^{t}(2\cos t+2\sin t+6\sin t+2\sin t-2\cos t)$	M1	Substitute in DE
	$=e^{t}\sin t=\text{RHS}$	E1	
	$y=Ae^{-t}+Be^{-2t}+\frac{1}{10}e^{t}(\sin t-\cos t)$	B1	General solution with their CF
		[5]	
1(iii)	*either* $k=0\Rightarrow$ bounded oscillations	B1	For two marks, must describe (not just sketch) oscillatory behaviour and (un)boundedness
	$k=1\Rightarrow$ unbounded oscillations	B1	
	[*or* both oscillate	[B1	[Accept 'growing exponentially' for 'unbounded' but not just 'increasing']
	bounded for $k=0$, unbounded for $k=1$]	B1]	
	$k=-1\Rightarrow$ solution tends to 0	B1	
	Solution is y = CF + PI		
	CF $=Ae^{-t}+Be^{-2t}\to 0$ as $t\to\infty$	B1	
	PI has form $e^{-t}(P\cos kt+Q\sin kt)$	M1	
	and $\to 0$ since $e^{-t}\to 0$ as $t\to\infty$	A1	
	Initial conditions affect A and B only so true for all initial conditions	B1	
		[7]	

Qu	Answer	Mark	Comment
2(i)	Solve by separating variables or CF $w = Ae^{2t}$ $t = 0, w = 2000$ $\Rightarrow w = 2000e^{2t}$ t = 0.1, w = 2443 t = 0.2, w = 2984 Population grows exponentially *either* unlikely as growth will be limited by (e.g.) space/disease *or* likely as long as (e.g.) sufficient space/no disease	M1 A1 M1 A1 B1 B1 B1 B1 **[8]**	 Use conditions Indicate more than just 'grows'
2(ii)*A*)	$\dot{w} > 0 \Rightarrow 2w - 80x > 0$ $\Rightarrow \frac{w}{x} > 40$	M1 A1 **[2]**	Attempt to solve $\dot{w} > 0$
2(ii)(*B*)	$\dot{x} > 0 \Rightarrow 1 - \frac{100x}{w} > 0$ $\Rightarrow \frac{w}{x} > 100$	M1 A1 **[2]**	Attempt to solve $\dot{x} > 0$
2(ii)(*C*)	$40 < \frac{w}{x} < 100$	B1 **[1]**	Correct or consistent with previous answers
2(iii)(*A*)	$w = 2000 + 0.1(2 \times 2000 - 80 \times 25)$ $= 2200$ $x = 25 + 0.1(0.2 \times 25(1 - \frac{100 \times 25}{2000}))$ $= 24.875$	M1 E1 M1 E1 **[4]**	Demonstrate use of algorithm Demonstrate use of algorithm
2(iii)(*B*)	$\dot{w} = 2410,\ \dot{x} = -0.6501...$ $w = 2441, x = 24.81$	M1 A1 **[2]**	Use alogrithm again for w and x
2(iii)(*C*)	They are smaller: 2200 to 2443 2441 to 2984 Some of the rabbits are being eaten by the foxes	B1 B1 **[2]**	
2(iv)	Actual number of foxes must be integers but values are not. Modelled as continuous change, whereas actual changes are discrete.	 B1 B1 B1 **[3]**	

Qu	Answer	Mark	Comment
3(i)	$mv\frac{\mathrm{d}v}{\mathrm{d}x} = mg - kv^2$	M1	N2L
		M1	$a = v\frac{\mathrm{d}v}{\mathrm{d}x}$
	$k \times 70^2 = mg \Rightarrow k = 0.002m (= 0.16)$	M1	Calculate k
	$\Rightarrow v\frac{\mathrm{d}v}{\mathrm{d}x} = 9.8 - 0.002v^2$	E1	Clearly shown
		[4]	
3(ii)	$\int \frac{v\mathrm{d}v}{9.8 - 0.002v^2} = \int dx$	M1	Separate variables
	$-\frac{1}{0.004}\ln\lvert 9.8 - 0.002v^2 \rvert = x + c$	M1	Integrate
		A1	All correct, including constant
	$\Rightarrow v^2 = 4900 - Ae^{-0.004x}$	M1	Rearranging
	$x = 0, v = 0 \Rightarrow A = 4900$	M1	Calculate constant
	$v = 70(1 - \mathrm{e}^{-0.004x})^{\frac{1}{2}}$	E1	Clearly shown
		[6]	
3(iii)	$(1 - \mathrm{e}^{-0.004h})^{\frac{1}{2}} = 0.99$	M1	
	$\Rightarrow h \approx 979$	A1	
		[2]	
3(iv)	$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - 80v$	M1	N2L
	$\int \frac{\mathrm{d}v}{g - v} = \int \mathrm{d}t$	M1	Separate variables
	$-\ln\lvert g - v \rvert = t + c_2$	M1	Integrate
	$\Rightarrow v = g - B\mathrm{e}^{-t}$	A1	v in terms of t
	$t = 0, v = 0.99 \times 70 \Rightarrow B = -59.5$	M1	Calculate constant from $v(0) = 69.3$ (or 70)
	$v = 9.8 + 59.5\mathrm{e}^{-t}$	A1	cao
		B1 B1	Intercept and shape Asymptote labelled
		[8]	
3(v)	$v = 10 \Rightarrow 10 = 9.8 + 59.5\mathrm{e}^{-t}$		
	$t = -\ln\left(\frac{0.2}{59.5}\right)$	M1	
	$= 5.70$	A1	
	$x = \int_0^{5.70} (9.8 + 59.5\mathrm{e}^{-t})\mathrm{d}t$	M1	Integrate v between limits
	$\approx 115\,\mathrm{m}$	A1	cao
		[4]	

Qu	Answer	Mark	Comment
4(i)	$\alpha + 2 = 0 \Rightarrow \alpha = -2$	M1	Solve auxiliary equation
	CF $y = A\mathrm{e}^{-2t}$	F1	
		[2]	
4(ii)	It is the same form as the CF so satisfies homogenous equation, hence will not satisfy the non-homogenous equation.	B1	Justifies that it will not satisfy DE (may use substitution)
	$y = at\mathrm{e}^{-2t}$	B1	Correct PI
	in DE: $a\mathrm{e}^{-2t} - 2at\mathrm{e}^{-2t} + 2at\mathrm{e}^{-2t} = \mathrm{e}^{-2t} \Rightarrow a = 1$	M1,A1	Differentiate, substitute and compare coefficients
	PI $y = t\mathrm{e}^{-2t}$	A1	
		[5]	
4(iii)	$I = \exp(\int 2dt)$	M1	attempt integrating factor
	$= \mathrm{e}^{2t}$	A1	
	$\mathrm{e}^{2t}\frac{\mathrm{d}y}{\mathrm{d}t} + 2\mathrm{e}^{2t}y = 1$	M1,A1	multiply
	$y\mathrm{e}^{2t} = \int \mathrm{d}t$	M1	Integrate
	$y\mathrm{e}^{2t} = t + A$	A1	
	$y = t\mathrm{e}^{-2t} + A\mathrm{e}^{-2t}$	A1	
	i.e. CF $A\mathrm{e}^{-2t}$, PI $t\mathrm{e}^{-2t}$ as before	E1	Correctly identify PI
		[8]	
4(iv)	condition $\Rightarrow A = y_0$	M1	Calculate constant
	$y = (t + y_0)\mathrm{e}^{-2t}$	F1	Particular solution
	$0 = \frac{\mathrm{d}y}{\mathrm{d}t} = (1 - 2t - 2y_0)\mathrm{e}^{-2t}$	M1,A1	Set derivative to zero
	$\Rightarrow t = \frac{1}{2} - y_0$	M1	Solve for t
	$t < \frac{1}{2} - y_0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} > 0, t > \frac{1}{2} - y_0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} < 0$ hence maximum	E1	Or alternative justification
	$y_{\max} = (\frac{1}{2} - y_0 + y_0)\mathrm{e}^{-2(\frac{1}{2} - y_0)} = \frac{1}{2}\mathrm{e}^{2y_0 - 1}$	M1,E1	Clearly shown
	$\frac{1}{2} - y_0 > 0 \Rightarrow y_0 < \frac{1}{2}$	B1	
		[9]	
			Total: 72

AO	Range	Total	Question Number				CourseWork
			1	2	3	4	
1	22-35	26	9	1	4	9	3
2	22-35	30	8	3	4	12	3
3	28-40	34	2	9	14	-	9
4	11-23	14	5	5	-	3	1
5	5-18	10	-	6	2	-	2
	Totals	**114**	24	24	24	24	18

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

MECHANICS 1, M1

4761

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.
- Unless otherwise specified, the value of g should be taken to be exactly 9.8ms^{-2}.

Section A (36 marks)

1

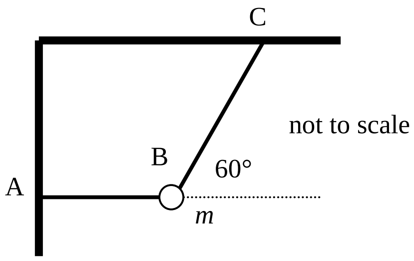


Fig.1

As shown in Fig.1, an object of mass m kg at B is held in equilibrium by two light strings AB and BC.
String AB is horizontal and fixed at A, string BC is at 60° to the horizontal and is fixed at C.
The tension in string BC is 10 N.

(i) **(*A*)** Draw a diagram showing all the forces acting on the object at B. **[1]**

(*B*) Calculate the tension in the string section AB. **[2]**

(ii) Calculate the value of m. **[3]**

2 In this question the unit of length is the metre and the time is in seconds.
An object has initial position $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and initial velocity $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$.
It has a constant acceleration of $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

(i) Calculate the initial speed of the object. **[2]**

(ii) Calculate the object's velocity and position after four seconds. **[4]**

3 A model truck of mass 5 kg is being pulled by a light string along a straight path.

The resistance to its motion is 8 N.

In one situation, the string and the path are horizontal, as shown in Fig.3.1.

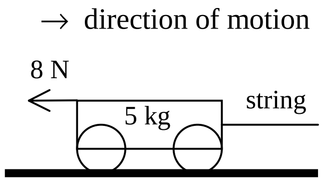


Fig.3.1

(i) Given that the acceleration of the truck is 4 ms^{-2}, calculate the tension in the string. **[3]**

In another situation, the path is horizontal and the string is inclined at 30° to the horizontal, as shown in Fig.3.2.

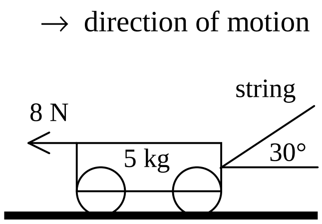


Fig.3.2

(ii) Given that the tension in the string is 40 N, calculate the acceleration of the truck. **[3]**

4

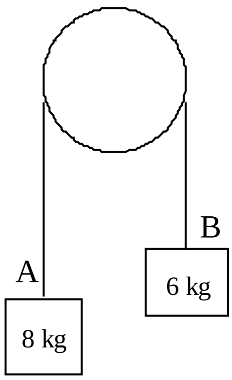


Fig. 4

A light inextensible string AB passes over a smooth peg.
Particles of mass 8 kg and 6 kg are attached to the ends A and B of the string and hang vertically, as shown in Fig.4.
The system is released from rest.

(i) Draw separate diagrams showing the forces acting on the particles at A and at B. **[1]**

(ii) **(*A*)** Write down the equation of motion for the particle at A and the equation of motion for the particle at B. **[3]**

(*B*) Show that the acceleration of the system is 1.4 ms^{-2}. **[2]**

5 A particle has a velocity, $\mathbf{v}$ ms^{-1}, given by $\mathbf{v} = (t^2 - t)\,\mathbf{i} + (t-1)\,\mathbf{j}$ where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors due east and north respectively, t is the time in seconds and the unit of length is the metre.

(i) Find the acceleration when $t = 2$. **[2]**

(ii) Determine the time(s), if any, when the particle is:

(*A*) at rest,

(*B*) moving due south. **[4]**

6

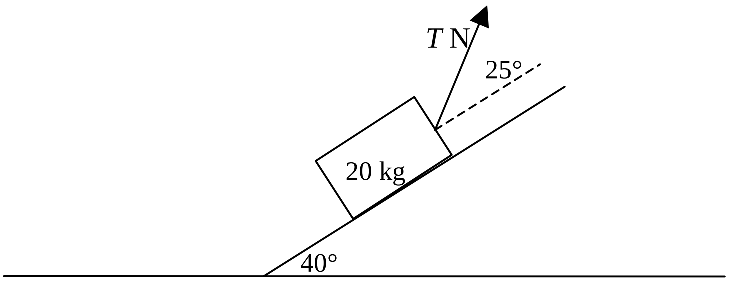


Fig. 6

A rough plane is at 40° to the horizontal. A force of T N at 25° to the greatest slope of the plane acts on a block of mass 20 kg on a plane, as shown in Fig. 6.

(i) Draw a diagram showing all the forces acting on the block. **[1]**

(ii) Given that the block is in equilibrium, calculate the frictional force between the block and the plane when $T = 172$. **[3]**

(iii) For what values of T will the frictional force on the block act up the plane? **[2]**

Section B (36 marks)

7 A car starts from rest and travels along a straight road.
Its speed, v ms^{-1}, at time t seconds is modelled by

$$\begin{array}{ll} v = 4t - 0.2t^2 & 0 \le t \le 10\,, \\ v = \text{constant} & 10 \le t \le 15\,, \\ v = 8 + 0.8t & t \ge 15\,. \end{array}$$

(i) Calculate the speed of the car at $t = 0$, $t = 10$, $t = 15$ and $t = 20$. **[3]**

(ii) Find the values of the acceleration at:

(***A***) $t = 7$,

(***B***) $t = 12$,

(***C***) $t = 16$. **[4]**

(iii) Calculate the distance the car travels in the interval $10 \le t \le 20$. **[6]**

(iv) Calculate the distance the car travels in the interval $0 \le t \le 10$. **[5]**

8 In this question, air resistance should be neglected.

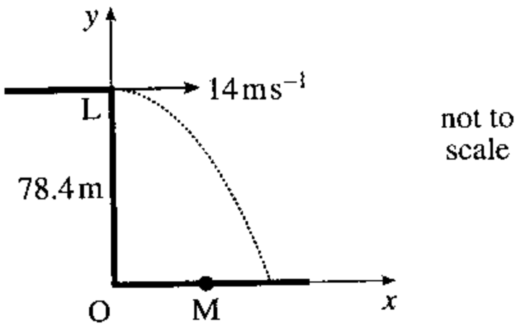


Fig. 7

Fig.7 shows a small stone being projected horizontally at a speed of 14 ms^{-1} from the point L at the top of a vertical cliff.
The cliff is 78.4 m above horizontal ground.
Coordinate axes are drawn through the origin O on the horizontal ground vertically below the point of projection.

(i) **(*A*)** Show that, t seconds after projection, the height, y m, of the stone is given by $y = 78.4 - 4.9t^2$. **[3]**

(*B*) Write down an expression in terms of t for the horizontal distance, x m, of the stone from O. **[2]**

(ii) **(*A*)** Calculate the time it takes the stone to hit the ground. **[2]**

(*B*) Calculate also the horizontal distance travelled by the stone. **[1]**

(iii) Show that the equation of the trajectory of the stone is $40y = 3136 - x^2$. **[2]**

On another occasion the stone is projected from L as before.
At the same time, a second small stone is projected vertically upwards at speed u ms^{-1} from a point M on the horizontal ground 35 m from O. The stones collide.

(iv) Show that the collision takes place just less than 48 m above the ground, 2.5 seconds after projection. **[4]**

(v) Calculate the value of u. **[4]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

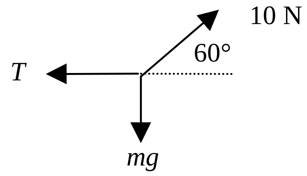
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

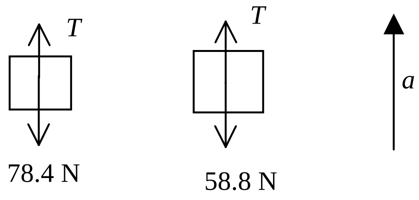
MEI STRUCTURED MATHEMATICS

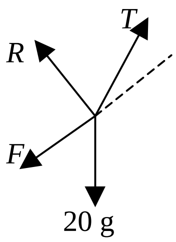
MECHANICS 1, M1

4761

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1(i)(*A*)		B1	All forces correctly labelled with arrows. Angle not required. Accept T_1, T_2, W etc. No extra forces
1(i)(*B*)	Resolve ← $T - 10\cos 60 = 0$ *T* = 5 so 5N	M1 A1 **[3]**	Attempt at horiz resolution. No extra forces
1(ii)	Resolve ↓ $mg = 10\sin 60$ *m* = 0.8836… so 0.884 (3 s.f.)	M1 A1 A1 **[3]**	Attempt at vertical resolution. No extra forces. Allow $m = 10\sin 60$ and $m = 10\cos 60$ Any reasonable accuracy
2(i)	$\sqrt{(-1)^2 + 4^2} = \sqrt{17}$ ms^{-1}	M1 A1 **[2]**	Use of Pythagoras
2(ii)	$\mathbf{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 24 \end{pmatrix}$ ms^{-1} $\mathbf{s} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + 4\begin{pmatrix} -1 \\ 4 \end{pmatrix} + 8\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 14 \\ 55 \end{pmatrix}$ m	M1,A1 M1 A1 **[4]**	 Must attempt all terms [If integration used M1 for integration attempted plus attempt at initial condition]
3(i)	N2L → $T - 8 = 5 \times 4$ *T* = 28 so 28 N	M1 A1 A1 **[3]**	Use of N2L. Accept *mga*. All forces present. No extras Accept sign errors LHS
3(ii)	N2L → $40\cos 30 - 8 = 5a$ *a* = 5.3287… so 5.33 ms^{-2} (3 s.f.)	M1 B1 A1 **[3]**	N2L. Must be *ma*. All terms present. No extras $40\cos 30$

Qu	Answer	Mark	Comment
Section A (continued)			
4(i)		B1 **[1]**	Accept any form for weight. Arrows required. Accn not required. Accept different tensions only if shown equal later. Accept single equivalent diagram. No spurious forces
4(ii)(*A*)	For A, using N2L $8\times 9.8 - T = 8a$ For B, using N2L $T - 6\times 9.8 = 8a$	M1 A1 A1	N2L. Allow '$F = mga$' and sign errors; condone one force missing. LHS correct. Accept $T - 8\times 9.8$ Must be consistent with equation for A Signs consistent, all forces present and '$F=ma$' used. Elimination of T or a.
4(ii)(*B*)	Solve a = 1.4 so 1.4 ms^{-1}	M1 E1 **[5]**	
5(i)	$\mathbf{a} = (2t-1)\mathbf{i} + \mathbf{j}$ $\mathbf{a}(2) = 3\mathbf{i} + \mathbf{j}$	M1 A1 **[2]**	Differentiation
5(ii)(*A*)	**i** component of **v** zero when $t^2 - t = 0$ so $t = 0$ or $t = 1$ **j** cpt zero when $t = 1$ At rest when both cpts zero so $t = 1$	M1 A1 A1	Finding when either cpt of **v** is zero. Do not accept **a** or **s.** All three times correct ft their values
5(ii)(*B*)	Travelling south when **i** cpt zero so $t = 0$	A1 **[4]**	ft their values

Qu	Answer	Mark	Comment
Section A (continued)			
6(i)		B1 **[1]**	All forces present. No extras. All labelled and with arrows. *F* up or down plane. No angles required. Accept *W*, *mg*, 196 N
6(ii)	$172\cos 25 = 20g\sin 40 + F$	M1	Resolving parallel to the plane. All forces present. At least one force resolved. Accept $\pm F$
		B1	Weight term
	F = 29.89 … so 29.9 N (3 s.f.)	A1 **[3]**	Accept negative only if consistent with the diagram
6(iii)	We need $T\cos 25 < 20g\sin 40$	M1	
	So T < 139.01.. so 139 N (3 s.f.)	A1 **[2]**	
			Section A Total: 36

Qu	Answer	Mark	Comment
Section B			
7(i)	t 0 10 15 20 v 0 20 20 24	B1 B1 B1 **[3]**	$t=0$ $t=10$ $t=20$ and $t=15$ (FT on their $t=10$)
7(ii)(*A*)	$a=4-0.4t$ so $4-0.4\times 7=1.2\ \text{ms}^{-2}$	M1 A1	Differentiating with one term correct
7(ii)(*B*)	$0\ \text{ms}^{-2}$	B1	
7(ii)(*C*)	$0.8\ \text{ms}^{-2}$	B1 **[4]**	
7(iii)	$10\le t\le 15\quad 5\times 20=100$ m $15\le t\le 20\quad 20\times 5+0.5\times 0.8\times 25$	M1 B1 M1 A1	Recognise need to split into 2 sections '*uvast*' or integrate from $t=15$ to $t=20$ Correct subst into *uvast* or correct integration (neglect limits). If *uvast* ft only $v(15)$, $v(20)$ from part **(i)** and $a(16)$ from part **(ii)(C)**
	= 110 m Total is 210 m	A1 A1 **[6]**	cao ft dep on both B1 and M1 awarded [If single rule applied from $t=10$ to 20: Using *uvast*. FT *u*, *v* and $a\neq 0$. Allow sign errors. SC1 If integration of $v=8+0.8t$ attempted and integration correct SC1]

Qu	Answer	Mark	Comment
Section B (continued)			
7(iv)	$\int_0^{10}(4t-0.2t^2)\,dt$	M1	Integration; must see evidence. Neglect limits. M0 for use of const accn
	$=\left[2t^2-\frac{2}{30}t^3\right]_0^{10}$	A1	At least one term correct. Neglect limits
	$=200-\frac{2000}{30}$	M1	Dependent on 1st M1. Subst correct limits in definite integral or correct subst for arb constant. Need $\int_0^{10}$ or $[\ ]_0^{10}$ or evidence of $t=0$ substituted
	$=133\frac{1}{3}$ m or 133 m (3 s.f.)	A1 B1 **[5]**	Correct limits or arb constant At least 3 s.f. accuracy. Award if seen [SC M1 for correct attempt at numerical integration (i.e. find area under curve) M1 for attempt at trapezia with strips ≤ 1 s A2 only if accurate to 3 s.f.]

Qu	Answer	Mark	Comment
Section B (continued)			
8(i)(*A*)	Distance dropped is $0 + .5 \times 9.8t^2$	M1,A1	Must have ± 9.8 or ± 10 and initial speed zero
	so $y = 78.4 - 4.9t^2$	E1	Must be fully shown
8(i)(*B*)	$x = 14t$	B1 **[4]**	Allow if seen later
8(ii)(*A*)	$y = 0$ gives $4.9t^2 = 78.4$	M1	Setting $y = 0$
	so $t^2 = 16$ and $t = 4$	A1	Only positive *t* need be considered
8(ii)(*B*)	$x = 14 \times 4 = 56$ so 56 m	M1	
		A1 **[4]**	ft *t* only
8(iii)	$y = 78.4 - 4.9 \times \left(\frac{x}{14}\right)^2$	M1	Substitute in correct expression to eliminate *t*
	giving $40y = 3136 - x^2$	E1 **[2]**	Fully shown
8(iv)	1st stone takes $\frac{35}{14} = 2.5$ s to reach $x = 35$	M1	
	2nd stone is at *y* s.t.	E1	
	$40y = 3136 - 35^2$	M1	Use of this equation or equivalent method
	so $y = 47.775$	E1 **[4]**	
8(v)	2nd stone is 47.775 m high after 2.5 s		
	so $47.775 = 2.5u - 4.9 \times 2.5^2$	M1	An appropriate choice of *uvast*(s) for the motion of the 2nd stone
		B1	s = 47.775 or 48 and t = 2.5 used
		A1	Condone s = 48
	and u = 31.36 so 31.4 ms^{-1} (3 s.f.) (31.45… if s = 48 used)	A1 **[4]**	cao
			Section B Total: 36
			Total: 72

AO	Range	Total	Question Number							
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	14-22	22	1	-	2	3	1	1	8	6
2	14-22	16	1	3	1	1	1	1	4	4
3	18-26	20	2	3	2	1	1	2	4	5
4	7-15	9	1	-	-	1	3	2	1	1
5	3-11	5	1	-	1	-	-	-	1	2
	Totals	**72**	6	6	6	6	6	6	18	18

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
MECHANICS 2, M2

4762

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.
- Unless otherwise specified, the value of g should be taken to be exactly 9.8 ms^{-2}.

1 Two young skaters, Percy of mass 55 kg and Queenie of mass 45 kg, are moving on a smooth horizontal plane of ice. You may assume that there are no external forces acting on the skaters in this plane.

Percy and Queenie are moving with speeds of 2 ms^{-1} and $\frac{4}{3}$ ms^{-1} respectively **towards** one another in the same line of motion. When they meet they embrace.

(i) Calculate the common velocity of the two skaters after they meet and the magnitude and direction of the impulse on Percy in the collision. **[6]**

Percy and Queenie, still together, collide directly with a moving skater, Roger, of mass 60 kg. The coefficient of restitution in the collision is 0.2.
After the collision, Percy and Queenie have a speed of 0.1 ms^{-1} in the same direction as before the collision.

(ii) Calculate Roger's velocity before the collision and his velocity after it. **[7]**

While moving at 0.1 ms^{-1} horizontally, Percy drops a small ball. The ball has zero vertical speed initially and drops 0.4 m onto the ice. The coefficient of restitution in the collision between the ball and the ice is 0.5.

(iii) At what angle to the horizontal does the ball leave the ice as it bounces? **[5]**

2 A parcel of mass 20 kg is pushed **up** a slope at 30° to the horizontal against a constant sliding resistance of 50 N at a steady speed of 4 ms^{-1}.

(i) Calculate the power developed by the pushing force. **[3]**

The parcel now slides **down** a slope at 35° to the horizontal that produces a different resistance to its motion. Its speed increases from 4 ms^{-1} to 6 ms^{-1} while sliding a distance of 5 m down the slope.

(ii) Calculate the work done against the resistance to motion. **[4]**

(iii) Assuming that a constant frictional force between the parcel and the slope is the only resistance to motion, show that the coefficient of friction between the parcel and the slope is 0.45, correct to two significant figures. **[4]**

(iv) For what value of the coefficient of friction would the parcel slide down the slope at a constant speed? **[2]**

The parcel is sliding down the slope and the coefficient of friction is 0.45.
A force, applied parallel to the slope, does 520 J of work and brings the parcel to rest from 6 ms^{-1} in x m.

(v) Calculate the value of x. **[5]**

3

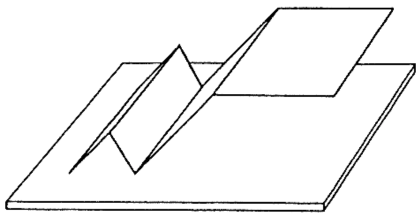


Fig. 3.1

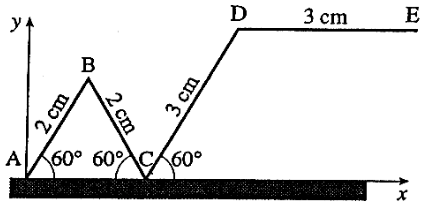


Fig. 3.2

A uniform, rectangular lamina of mass 25 kg is folded and placed on a horizontal floor, as shown in Fig. 3.1.
Fig 3.2 shows the cross-section ABCDE of the folded lamina.
The dimensions and angles of the cross-section are given in Fig. 3.2 and DE is horizontal.

(i) Show that the x-coordinate of the centre of mass of the lamina is 2.725, referred to the axes shown in Fig 3.2.
Calculate also the y-coordinate, referred to the same axes, giving your answer correctly to three decimal places. **[6]**

(ii) Explain briefly why the lamina cannot be in equilibrium in the position shown without the application of an additional force. **[2]**

(iii) What is the least vertical force that must be applied to the lamina at A so that it will stay in equilibrium in the position shown? **[4]**

Instead of applying the vertical force at A, a horizontal force is applied to the lamina at E.
The lamina does not slide on the floor.

(iv) Calculate the least value of the horizontal force at E for the lamina to be in equilibrium. **[3]**

(v) Calculate the greatest value the horizontal force at E can take without the lamina turning *anti*-clockwise. **[3]**

4

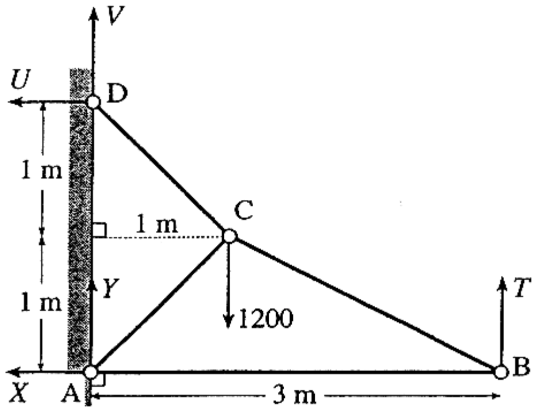


Fig. 4

Fig. 4 shows a light framework ABCD freely pin-jointed together at A, B and C and freely attached to a vertical wall at A and D.
There is a load of 1200N at C and a vertical force of *T*N acts at B.
The other external forces *U, V, X* and *Y* N and essential geometrical information are marked in the diagram.
The framework is in equilibrium.

(i) Show that $X = -U$ and that $U = \frac{1}{2}(1200 - 3T)$. **[3]**

(ii) By considering the equlibirium at D, show that $U = V$. **[2]**

(iii) Show that $Y = \frac{1}{2}(1200 + T)$ and find expressions in terms of *T* for the internal forces in each of the rods AB, BC, AC and CD. **[9]**

(iv) As *T* increases from zero through positive values, show that one of the rods changes from being in tension to being in thrust.
For what value of *T* is there no internal force in this rod?
Describe what happens to the forces in the rods as *T* decreases from zero through negative values. **[4]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
MECHANICS 2, M2

4762

MARK SCHEME

Qu	Answer	Mark	Comment
1(i)	Before P → ← Q 2 ms^{-1} $^4/_3$ ms^{-1} After PQ → v ms^{-1} PCLM $55\times 2 - 45\times\frac{4}{3} = 100v$	M1 B1	PCLM applied Signs correct and consistent with the question
	$v = 0.5$ so 0.5 ms^{-1} in original direction of Percy $\rightarrow\ 55(0.5-2) = -82.5$ Ns	A1 F1 M1 A1 **[6]**	 Either explicit or implied by diagram Attempt at impulse Must have direction explicit (diagram will do)
1(ii)	Before PQ → R → 0.5 ms^{-1} v ms^{-1} After PQ → R → 0.1 ms^{-1} v' ms^{-1}		
	PCLM $50 + 60v = 10 + 60v'$ $3v' - 3v = 2$	M1 A1	PCLM Any Form
	NEL $\frac{v'-0.1}{v-0.5} = -0.2$ $v' + 0.2v = 0.2$	M1 A1	Including consistent use of signs Any form
	Solving $v = \frac{7}{18},\ v' = \frac{5}{18}$	M1	
	So before, $-\frac{7}{18}$ ms^{-1} (opp direction to PQ) after, $\frac{5}{18}$ ms^{-1} (same direction as PQ)	A1 A1 **[7]**	Award max A1 for final answers unless directions both specified or implied by diagram
1(iii)	Ball hits ice at vert speed $\sqrt{2\times 0.4\times 9.8}$ = 2.8 ms^{-1}	M1 A1	
	Linear momentum conserved horiz NEL on vert cpt gives 1.4 ms^{-1} up so after bounce 0.1 ms^{-1} horiz and 1.4 ms^{-1} up	M1 B1	May be implied e.g. in diagram
	Angle is $\arctan(\frac{1.4}{0.1}) \approx 86°$	A1 **[5]**	

Qu	Answer	Mark	Comment
2(i)	$(20g\sin 30+50)\times 4$ = 592 W	M1 B1 A1 **[3]**	Use of $P=Fv$ Weight term
2(ii)	$20\times 9.8\times 5\times \sin 35-\frac{1}{2}\times 20\times (6^2-4^2)$ = 362.104.. so 362 J (3s.f.)	M1 B1 B1 A1 **[4]**	Difference in GPE and KE GPE term Either KE term Accept 2 s.f.
2(iii)	$5F$ = 362.104… so F = 72.4209… $R=20\times 9.8\times \cos 35$ μ = 0.4510… so 0.45 (2s.f.)	B1 B1 M1 E1 **[4]**	 Use of $F=\mu R$
2(iv)	$\mu mg\cos 35=mg\sin 35$ μ = 0.70 (2s.f.)	M1 A1 **[2]**	 Accept WW
2(v)	$72.2492..\times x+520-20gx\sin 35$ $=\frac{1}{2}\times 20\times 6^2$ x = 3.982… so 3.98m (2 s.f.)	M1 B1 A1 A1 A1 **[5]**	Use of work-energy Equation contains GPE term All terms present Signs correct (dependent on A1 above)
3(i)	$10\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=2\begin{pmatrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{pmatrix}+2\begin{pmatrix}\frac{3}{2}\\ \frac{\sqrt{3}}{2}\end{pmatrix}+3\begin{pmatrix}2.75\\ \frac{3\sqrt{3}}{4}\end{pmatrix}+3\begin{pmatrix}5\\ \frac{3\sqrt{3}}{2}\end{pmatrix}$ (2.725, 1.516)	M1 B1 B1 B1 E1,A1 **[6]**	Appropriate method Correct masses At least two x cpts correct At least two y cpts correct
3(ii)	cm gives a clockwise moment about C Reaction at A cannot give an a.c. moment	E1 E1 **[2]**	Considering moments Complete argument
3(iii)	Moments about C $2w=25g\times 0.725$ w = 88.8125 so about 88.81 N	M1 A1 B1 A1 **[4]**	 Use of weight

Qu	Answer	Mark	Comment
3(iv)	Moments about C $3\frac{\sqrt{3}}{2}F = 25g \times 0.725$ $F = 68.367\ldots$ so 68.3 N (3 s.f.)	M1 A1 A1 **[3]**	 Any reasonable accuracy
3(v)	Moments about A $3\frac{\sqrt{3}}{2}F = 25g \times 2.725$ F = 256.968.. so about 257 N	M1 A1 A1 **[3]**	 Any reasonable accuracy
4(i)	$\rightarrow \quad U + X = 0 \Rightarrow x = -U$ Â $\quad 2U + 3T = 1200$ so $-X = U = \frac{1200 - 3T}{2}$	E1 M1 E1 **[3]**	 Moments about A or D
4(ii)	$\uparrow \quad V = T_{CD} \cos 45$ $\rightarrow \quad U = T_{CD} \cos 45$ so $U = V$	M1 E1 **[2]**	Resolving in each direction Clearly shown

Qu	Answer	Mark	Comment
4(iii)	$\uparrow$ For the whole system $V+Y+T=1200$ so $Y=1200-T-\dfrac{(1200-3T)}{2}=\dfrac{1200+T}{2}$ Consider all the struts in tension and consider the equilibria at pin-joints at D $\rightarrow\quad T_{\text{CD}}\cos 45=U$ so $T_{\text{CD}}=\dfrac{1200-3T}{\sqrt{2}}$ at A $\uparrow\quad Y+T_{\text{AC}}\cos 45=0$ so $T_{\text{AC}}=-\dfrac{(1200+T)}{\sqrt{2}}$ $\rightarrow\quad X=T_{\text{AC}}\cos 45+T_{\text{AB}}$ so $=T_{\text{AB}}=-\dfrac{(1200-3T)}{2}+\dfrac{(1200+T)}{2}=2T$ at B $\uparrow\quad T_{\text{CB}}\times\dfrac{1}{\sqrt{5}}+T=0$ so $T_{\text{CB}}=-\sqrt{5}T$	B1 E1 M1 M1 A1 A1 F1 M1 A1 **[9]**	 Must be clearly derived Considering equilibrium at a pin-joint At least two equilibrium equations attempted Attempt to find angle [For forces in struts, FT according to order they are determined]
4(iv)	When T increases Only CD can change sign for $T>0$. There is zero force in CD when $T=400$ When T decreases BC, CD remain in tension AB remains in thrust CA changes from thrust to tension when $T<-1200$	 E1 E1 B1 B1 **[4]**	 Identifying CD $T=400$
			Total: 72

AO	Range	Total	Question Number			
			1	2	3	4
1	14-22	17	2	7	5	3
2	14-22	21	7	2	3	9
3	18-26	18	5	5	4	4
4	7-15	7	3	-	2	2
5	3-11	9	1	4	4	-
	Totals	**72**	18	18	18	18

OCR RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

MECHANICS 3, M3

4763

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.
- Unless otherwise specified, the value of g should be taken to be exactly 9.8ms^{-2}.

1 **(i)** Write down the dimensions of velocity, acceleration and force. **[3]**

(ii) Use the definitions of work, kinetic energy and change in gravitational potential energy to show that these quantities have the same dimensions. **[3]**

The tension in a stretched wire is given by $T = \frac{YAx}{l_0}$, where A is the cross-sectional area of the wire, l_0 is the natural length of the wire, x is the extension and Y is a quantity called Young's modulus which depends on the material from which the wire is made.

(iii) Determine the dimensions of Young's modulus. **[3]**

The energy stored in the stretched wire is given by the equation $E = cY^{\alpha}\left(\frac{A}{l_0}\right)^{\beta} x^{\gamma}$ where c is a dimensionless constant.

(iv) Use dimensional analysis to determine the value of α and to find a relationship between β and γ. **[4]**

(v) Use the standard formulae for tension and energy in terms of stiffness and extension to determine the values of β and γ and the constant c. **[5]**

2 A weighing machine is being designed. It consists of a square platform of mass 2.5 kg supported by a number of identical springs each of stiffness 25 000 N m^{-1}, which are attached to a fixed horizontal base as shown in Fig. 2.

Throughout this question assume that the platform remains horizontal.

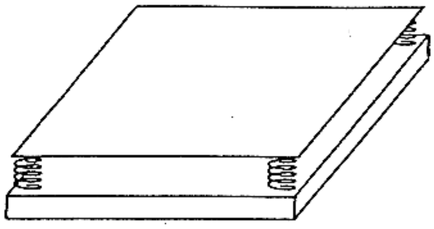


Fig. 2

Initially the designer uses four springs and the system is in equilibrium.

(i) Calculate the compression in each spring before any object is placed on the platform. **[2]**

A child of mass 30 kg is standing on the platform, which is at rest.

(ii) Calculate the compression in each of the four springs. **[2]**

(iii) Calculate the minimum number of additional springs required to reduce the compression to less than 0.002m. **[4]**

The 30 kg child is standing on the platform supported by four springs as in the original design. The child's father lifts her off quickly, allowing the platform to oscillate freely in a vertical direction.

(iv) The displacement of the platform *below* the equilibrium position at time t seconds is y metres. Write down the equation of motion for the platform.

Hence show that the platform performs simple harmonic motion of period $\frac{1}{100}\pi$ s.

Calculate the maximum speed of the platform. **[10]**

3 A hollow, circular cylinder of radius 35 cm is rotating about its axis, which is vertical, at a constant rate of 2π radians per second. A small object of mass m on the inside of the cylinder is rotating in a horizontal circle with the same angular speed as the cylinder i.e. it does not slip. The coefficient of friction between the object and the cylinder is μ.. This situation is shown in Fig. 3.1.

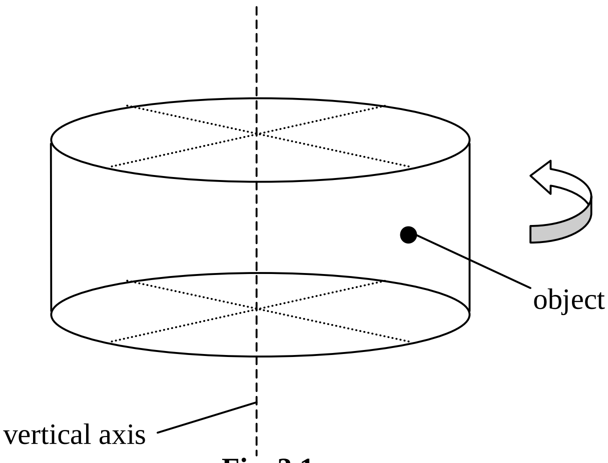


Fig. 3.1

(i) State what forces act on the object and explain briefly why the frictional force acts vertically upwards.
Write down an equation for the vertical equilibrium and also an equation for the radial motion of the object.
Hence deduce that μ is at least about 0.71. **[8]**

The same cylinder is now made to rotate, **with its axis horizontal**, at a constant speed of ω radians per second. A small object of mass m on the inside of the cylinder is now rotating in a vertical circle without slipping. The situation when the object has turned through an angular distance θ is shown in Fig. 3.2, where F is the frictional force and R the normal reaction acting on the object.

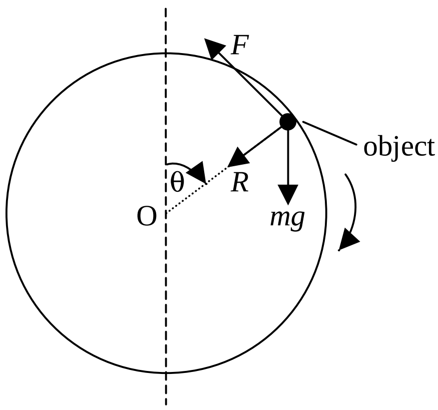


Fig. 3.2

(ii) Show that $F = mg\sin\theta$. **[2]**

(iii) Write down an equation for the radial motion of the object and deduce that

$\omega^2 \geq \frac{28}{\mu}(\sin\theta + \mu\cos\theta)$ if the object does not slip. **[8]**

4 A uniform solid hemisphere of radius r is formed by rotating the region in the first quadrant within the curve $x^2 + y^2 = r^2$ through 2π radians about the x-axis, as shown in Fig. 4.1.

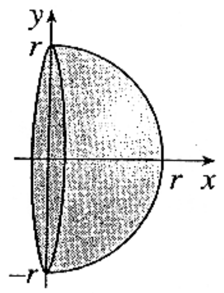


Fig. 4.1

(i) Find, by integration, the volume of the hemisphere and show that the centre of mass of the hemisphere has coordinates $\left(\frac{3}{8}r, 0\right)$. **[8]**

A hemisphere of radius kr (where $0 < k < 1$) is removed from a hemisphere of radius r to leave a uniform hemispherical shell of constant thickness, as shown in cross-section in Fig. 4.2.

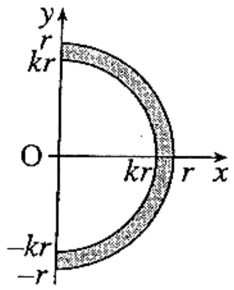


Fig. 4.2

(ii) Show that the x-coordinate of the centre of mass of the shell is $\frac{3}{8}r\left(\frac{1-k^4}{1-k^3}\right)$. **[5]**

(iii) By writing $k = 1 - \varepsilon$ where ε is small, show that $1 - k^3 \approx 3\varepsilon$.
Find a similar expression for $1 - k^4$.
Hence, or otherwise, show that the centre of mass of a hemispherical shell of negligible thickness is at the midpoint of the axis of symmetry of the shell. **[5]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
MECHANICS 3, M3

4763

MARK SCHEME

Qu	Answer	Mark	Comment
1(i)	[velocity] = LT^{-1} [acceleration] = LT^{-2} [force] = MLT^{-2}	B1,B1 B1 **[3]**	
1(ii)	[work done] = [F.d] = MLT^{-2} .L = ML^2T^{-2} [KE] = $\left[\frac{1}{2}mv^2\right]$ = $M(LT^{-1})^2$ = ML^2T^{-2} [GPE] = [mgh] = $M.LT^{-2}.L$ = ML^2T^{-2}	B1 B1 B1 **[3]**	Must be shown, not just stated
1(iii)	$Y = \frac{Tl_0}{Ax}$ $[Y] = \frac{MLT^{-2}.L}{L^2.L}$ $= ML^{-1}T^{-2}$	M1 M1 A1 **[3]**	Rearranging Sub. dimensions
1(iv)	$ML^2T^{-2} = (ML^2T^{-2})^{\alpha}L^{\beta}L^{\gamma}$ $\alpha = 1$ $-1 + \beta + \gamma = 2$ $\beta + \gamma = 3$	M1 A1 M1 A1 **[4]**	Sub. dimensions Equating powers of L or equivalent
1(v)	$T = kx \Rightarrow k = \frac{YA}{l_0}$ $E = \frac{1}{2}kx^2 = \frac{1}{2}\frac{YA}{l_0}x^2$ $\Rightarrow \beta = 1, \gamma = 2, c = \frac{1}{2}$	B1 M1 B1 B1 B1 **[5]**	Formulae for tension and energy Making k subject Eliminating k β and γ for c
2(i)	$4(25\ 000x) = 2.5g$ $x = 0.000\ 245$ (m)	M1 A1 **[2]**	Use of Hooke's Law
2(ii)	$4(25\ 000x) = 32.5g$ $x = 0.003\ 185$ (m)	M1 A1 **[2]**	Use of Hooke's Law
2(iii)	$n(25\ 000x) = 32.5g$ $x < 0.02 \Rightarrow n > 6.37$ $\Rightarrow$ minimum number is 7	M1 M1 A1 A1 **[4]**	Equilibrium equation involving n (or equivalent) Solving 6.37

Qu	Answer	Mark	Comment
2(iv)	$2.5\ddot{y} = 2.5g - 4 \times 25\ 000(y + 0.000\ 245)$	M1	Newton's 2nd Law
		M1,A1	Linear expression for force in spring
		B1	Weight
		A1	All correct with consistent signs
	$\ddot{y} = -40\ 000y \Rightarrow$ SHM	E1	
	Period $= \dfrac{2\pi}{200} = \dfrac{\pi}{100}$	E1	
	Ampl $= 0.003185 - 0.000245 = 0.00294$	B1	
	Max. speed $= 0.00294 \times 200 = 0.588$	M1,A1	
		[10]	
3(i)	Weight, Friction, Normal Reaction	B1	All (accept centripetal force as an extra force **but not instead of** the Normal reaction)
	No transverse component of acceleration	B1	Accept 'tangential' (allow 'only acceleration is towards the centre')
	Vertically $F = mg$	B1	
	Radially $R = m \times 0.35 \times (2\pi)^2$	M1	'Force = $mr\omega^2$'
		A1	No need to simplify
	$F \le \mu R$	M1	
	$\mu \ge 0.709 \ldots \approx 0.71$	E1	Substituted and clearly shown
			(Accept use of equality throughout)
		E1	Inequality established
		[8]	
3(ii)	No transverse component of acceleration so resolving in transverse direction		
	$F = mg\sin\theta$	E1	No transverse acceleration
		E1	Clearly shown (accept 'equilibrium in transverse direction')
		[2]	
3(iii)	In radial direction	M1	
	$R + mg\cos\theta = m \times 0.35\omega^2$	B1	LHS
		B1	RHS
	Using $F \le \mu R$ and $F = mg\sin\theta$ to eliminate R and F	M1	
	$mg\sin\theta \le \mu(0.35m\omega^2 - mg\cos\theta)$	A1	Accept =
	$0.35\omega^2 \ge \dfrac{g}{\mu}\sin\theta + g\cos\theta$	M1,A1	Make ω^2 subject
	$\omega^2 \ge \dfrac{28}{\mu}(\sin\theta + \mu\cos\theta)$	E1	Clearly shown including inequality
			(accept verbal argument)
		[8]	

Qu	Answer	Mark	Comment
4(i)	$V=\int_0^r \pi(r^2-x^2)\mathrm{d}x=\left[r^2x-\frac{1}{3}x^3\right]_0^r$	M1,A1	
	$=\frac{2}{3}\pi r^3$	A1	
	$V\overline{x}=\int_0^r \pi x(r^2-x^2)\mathrm{d}x$	M1	Use of formula
	$=\pi\left[\frac{1}{2}r^2x^2-\frac{1}{4}x^4\right]_0^r$	A1	Limits (dependent on previous M marks)
		A1	For $\frac{1}{4}\pi r^4$
	$\overline{x}=\dfrac{\frac{1}{4}\pi r^4}{\frac{2}{3}\pi r^3}=\frac{3}{8}r$	E1	
	$\overline{y}=0$ by symmetry	E1	
		[8]	
4(ii)	$\overline{x}=\dfrac{\frac{3}{8}r.\frac{2}{3}\pi r^3-\frac{3}{8}kr.\frac{2}{3}\pi(kr)^3}{\frac{2}{3}\pi r^3-\frac{2}{3}\pi(kr)^3}$	M1	$\dfrac{\sum mx}{\sum m}$ or moments
		A1,A1 M1	Numerator each term Denominator
	$=\frac{3}{8}r\left(\dfrac{1-k^4}{1-k^3}\right)$	E1	
		[5]	
4(iii)	$(1-\varepsilon)^3=1-3\varepsilon+3\varepsilon^2-\varepsilon^3$	M1	Binomial expansion
	$1-k^3\approx 1-(1-3\varepsilon)=3\varepsilon$	E1	
	$1-k^4=1-\left(1-4\varepsilon+6\varepsilon^2-4\varepsilon^3+\varepsilon^4\right)\approx 4\varepsilon$	B1	
	$\overline{x}=\frac{3}{8}r\left(\dfrac{1-k^4}{1-k^3}\right)\approx\frac{3}{8}r\left(\dfrac{4\varepsilon}{3\varepsilon}\right)$	M1	Substituting
	$=\frac{1}{2}r$	A1	
		[5]	
			Total: 72

AO	Range	Total	Question Number			
			1	2	3	4
1	14-22	18	5	2	2	9
2	14-22	22	5	5	7	5
3	18-26	21	8	5	5	3
4	7-15	7	-	3	3	1
5	3-11	4	-	3	1	-
	Totals	**72**	18	18	18	18

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

MECHANICS 4, M4

4764

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.
- Unless otherwise specified, the value of g should be taken to be exactly 9.8 ms^{-2}.

Section A (24 marks)

1 A pulley is modelled as a circular disc of radius r whose plane is vertical.
It can turn freely about a horizontal axis through its centre and the moment of inertia of the axis is I.
Particles of mass m_1 and m_2, where $m_2 > m_1$, are attached to the ends of a light rough string which hangs vertically over the pulley, as shown in Fig.1.
T_1 and T_2 are the tensions in the hanging part of the string and the string is inextensible.
During the motion the string does not slip on the pulley.

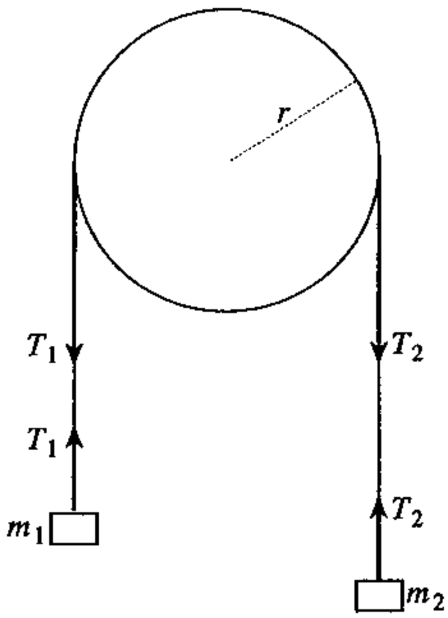


Fig. 1

(i) Write down the equations of motion of the two masses and the pulley. **[4]**

(ii) Hence find the acceleration of the masses and the tensions T_1 and T_2. **[4]**

(iii) In the case where $m_1 = m$, $m_2 = 2m$, the disc is uniform and has mass $4m$, find the kinetic energy of the system, in terms of m and g, two seconds after it is released from rest. **[6]**

2

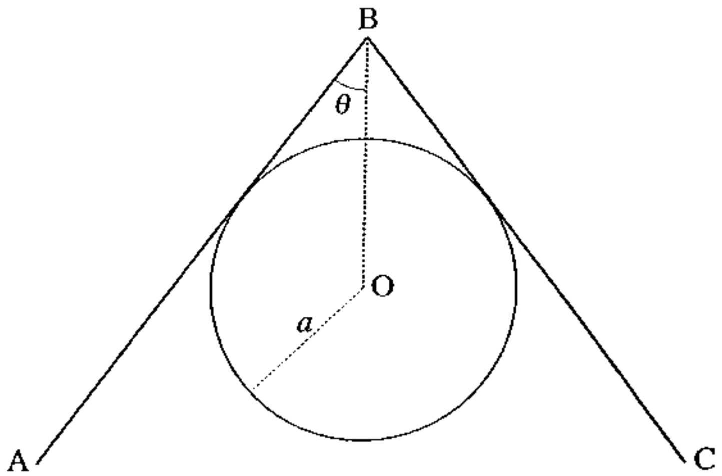


Fig.2

Two light rods AB and BC, each of length *l*, are smoothly jointed at B and are placed on a smooth fixed cylinder, as shown in Fig.2.
The radius of the cylinder is *a* and its axis is horizontal.
The rods each carry a mass *m* at their free ends A and C, and B moves along the vertical line through O.
In a general position, the angle OBA is equal to θ .

(i) (***A***) Show that the potential energy V , relative to O, can be written $V = 2mg\left(\frac{a}{\sin\theta} - l\cos\theta\right)$. **[2]**

(***B***) Show that a position of equilibrium occurs where $a\cos\theta = l\sin^3\theta$. **[2]**

(***C***) Explain, graphically or otherwise, why this equation has only one solution for $0 < \theta < \frac{\pi}{2}$. **[1]**

(***D***) Show further that the position of equilibrium is stable. **[3]**

(ii) Show that if the rods are in equilibrium with $\theta = \frac{\pi}{4}$, then $l = 2a$. **[2]**

Section B (48 marks)

3 The speed limiter on a test vehicle operates by reducing the driving force F as the speed increases. The force is given by: $F = mk\left(A^2 - v^2\right)$ where v is the speed, m is the mass and k, A are constants. When moving on level ground, the resistance to motion is Bmv^2, where B is a constant. The greatest speed that the vehicle can reach is V_0.

(i) Write down the equation of motion for the vehicle. **[3]**

(ii) Show that $V_0^2 = \dfrac{kA^2}{k+B}$ and deduce that the equation of motion can be written as:

$\dfrac{\mathrm{d}v}{\mathrm{d}t} = c\left(V_0^2 - v^2\right)$, where $c = k + B$ and t is time. **[4]**

The vehicle starts from rest at $t = 0$ and after a time t has moved a distance x.

(iii) Show that the speed v at time t is given by: $v = V_0\left(\dfrac{\mathrm{e}^{cV_0t} - \mathrm{e}^{-cV_0t}}{\mathrm{e}^{cV_0t} + \mathrm{e}^{-cV_0t}}\right)$ **[9]**

(iv) Hence, or otherwise, show that $x = \dfrac{1}{c}\ln\left[\dfrac{1}{2}\left(\mathrm{e}^{V_0ct} + \mathrm{e}^{-V_0ct}\right)\right]$ **[5]**

(v) Show that after a long time $x \approx V_0\,t - D$ where D is to be determined. **[3]**

4 A particle of initial mass M falls from rest under gravity through a stationary cloud.
The particle picks up mass from the cloud at a rate equal to mkv, where m and v are the mass and speed of the particle at time t and k is a constant.
Resistance to motion can be neglected.

(i) Write down differential equations which describe:

(*A*) the increase in mass of the particle; **[1]**

(*B*) the motion of the particle. **[2]**

(ii) Hence show that the speed satisfies the diffential equation $v\dfrac{dv}{dx} + kv^2 = g$ where x is the distance fallen. **[5]**

(iii) By solving the equation in part **(i)** find v in terms of g, k, and x.
Deduce that the speed tends to the limiting value $\sqrt{\dfrac{g}{k}}$. **[9]**

(iv) Show that $\dfrac{dm}{dx} = km$.
Hence, show that the mass of the particle is $2M$ when its speed is a fraction $\dfrac{\sqrt{3}}{2}$ of its limiting value. **[7]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
MECHANICS 4, M4

4764

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1(i)	$T_1 - m_1 g = m_1 a$ $m_2 g - T_2 = m_2 a$ $(T_2 - T_1)a = I\ddot{\theta}$	M1 A1 M1,A1 **[4]**	Attempt at these 2 equations Both correct Rotation equation
1(ii)	$\ddot{\theta} = \frac{a}{r}$ $\Rightarrow T_2 - T_1 = \frac{Ia}{r^2}$ $a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{I}{r^2}}$ $T_1 = \frac{m_1 g\left(2m_2 + \frac{I}{r^2}\right)}{m_1 + m_2 + \frac{I}{r^2}}$ $T_2 = \frac{m_2 g\left(2m_1 + \frac{I}{r^2}\right)}{m_1 + m_2 + \frac{I}{r^2}}$	M1 A1 B1 B1 **[4]**	Eliminating $\ddot{\theta}$ Finding a without tensions
1(iii)	$I = 0.5mr^2$ $a = \frac{mg}{3m + \frac{0.5mr^2}{r^2}} = \frac{g}{5}$ After 2s $v = \frac{2g}{5}$ $\dot{\theta} = \frac{2g}{5r}$ KE is $0.5 \times 4mr^2 \times \frac{4g^2}{25r^2} + 0.5 \times (m + 2m) \times \frac{4g^2}{25}$ $= \frac{8mg^2}{25} + \frac{6mg^2}{25} = \frac{14mg^2}{25}$	B1 F1 F1 F1 M1 A1 **[6]**	MI of disc ft from a ft from a and v Attempt at both of the KE terms cao

Qu	Answer	Mark	Comment
Section A (continued)			
2(i)(*A*)	$\text{PE} = mg(\text{OB} - \text{AB}\cos\theta)$ rel to 0 for each mass $V = 2mg\left(\dfrac{a}{\sin\theta} - l\cos\theta\right)$	M1 A1 **[2]**	
2(i)(*B*)	$V' = 2mg\left(\dfrac{a\cos\theta}{\sin^2\theta} + l\sin\theta\right)$ $= 0$ when $l\sin^3\theta = a\cos\theta$	M1 A1 **[2]**	
2(i)(*C*)	Compare curves $y = \cos\theta$ and $y = \dfrac{1}{a}\sin^3\theta$ to show one solution only	B1 **[1]**	
2(i)(*D*)	$V'' = 2mg\left(l\cos\theta + a\left(\dfrac{\sin^3\theta + 2\sin\theta\cos^2\theta}{\sin^4\theta}\right)\right)$ >0 since all terms are >0 in $0 < \theta < \dfrac{\pi}{2}$ hence stable equilibrium	M1 M1 A1 **[3]**	
2(ii)	when $\theta = \dfrac{\pi}{4},\ V' = 0\ \therefore \dfrac{a}{\sqrt{2}} = \dfrac{1}{2\sqrt{2}} \Rightarrow l = 2a$	M1,A1 **[2]**	
			Section A Total: 24
Section B			
3(i)	$mk\left(A^2 - v^2\right) - mBv^2 = m\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1 A1 **[3]**	Equation Either term on LHS cao
3(ii)	$\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0$ when $v = V_0$ $\Rightarrow V_0^2 = \dfrac{kA^2}{k+B}$ Substitute in equation $c = k + B$	M1 A1 M1 A1 **[4]**	

Qu	Answer	Mark	Comment
Section B (continued)			
3(iii)	$\int \frac{dv}{V_0^2 - v^2} = \int c\,dt$	M1	Separation
	$= \frac{1}{2V_0} \int \left[\frac{1}{V_0 - v} + \frac{1}{V_0 + v} \right] dv$	M1	(pf)
		A1,A1	
	$\Rightarrow \frac{1}{2V_0} \ln\left(\frac{V_0 + v}{V_0 - v} \right) = c(t + t_0)$	M1	Integrating partial fractions
		A1	Correct
	when $t = 0,\ v = 0 \Rightarrow t_0 = 0$	E1	Consider limits
	Take exponentials	M1	
	$v = V_0 \left(\frac{e^{V_0 ct} - e^{-V_0 ct}}{e^{V_0 ct} + e^{-V_0 ct}} \right)$	A1	
		[9]	
3(iv)	$x = \int v\,dt$	M1	
	$x = \frac{1}{c} \ln\left[\frac{1}{2}\left(e^{V_0 ct} + e^{-V_0 ct} \right) \right]$	M1	Recognising ln integral
		A1	Correct
		M1	Evaluating limits
		A1	
		[5]	
3(v)	$x \approx \frac{1}{c} \ln\left[\frac{1}{2} e^{V_0 ct} \right]$	M1	
	$= \frac{1}{c}\left[\ln e^{V_0 ct} - \ln 2 \right]$	M1	
	$= V_0 t - \frac{\ln 2}{c}$		
	$D = \frac{\ln 2}{c}$	A1	
		[3]	
4(i)(*A*)	$\frac{dm}{dt} = kmv$	B1	
		[1]	
4(i)(*B*)	$\frac{d}{dt}(mv) = mg$	M1,A1	
		[2]	
4(ii)	$m\frac{dv}{dt} + v\frac{dm}{dt} = mg$	M1	
	$m\frac{dv}{dt} + kmv^2 = mg$	M1,A1	
	$\frac{dv}{dt} = v\frac{dv}{dx}$, and hence $v\frac{dv}{dx} + kv^2 = g$	M1,B1	
		[5]	

Qu	Answer	Mark	Comment
Section B (continued)			
4(iii)	$\int \frac{v\,dv}{g-kv^2} = \int dx$	M1,A1	
	Hence $\frac{-1}{2k}\ln\left(g-kv^2\right) = x + \text{constant}$,	M1,A1	
	and $g - kv^2 = A\mathrm{e}^{-2kx}$.	A1	
	When $x = 0$, $v = 0$ $\therefore A = g$	M1,A1	
	$\therefore v^2 = \frac{g}{k}\left(1-\mathrm{e}^{-2kx}\right)$	A1	
	As $x \to \infty$, $v^2 \to \frac{g}{k}$ $\therefore v \to \sqrt{\frac{g}{k}}$	A1	
		[9]	
4(iv)	$\frac{\mathrm{d}m}{\mathrm{d}t} = kmv = km\frac{\mathrm{d}x}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}m}{\mathrm{d}x} = km$	M1,A1	
	$\therefore \ln m = kx + \text{constant} \Rightarrow m = M\mathrm{e}^{kx}$	M1,A1	
	$v = \frac{1}{2}\sqrt{3}\sqrt{\frac{g}{k}} \Rightarrow \mathrm{e}^{2kx} = 4$ $\therefore \mathrm{e}^{2kx} = 2$	M1,A1	
	$\therefore m = 2M$ as required	A1	
		[7]	
			Section B Total: 48
			Total: 72

AO	**Range**	**Total**	**Question Number**			
			1	**2**	**3**	**4**
1	14-22	21	2	-	11	8
2	14-22	19	6	3	6	4
3	18-26	21	4	3	6	8
4	7-15	7	1	2	1	3
5	3-11	4	1	2	-	1
	Totals	**72**	14	10	24	24

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

STATISTICS 1 S1

4766

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

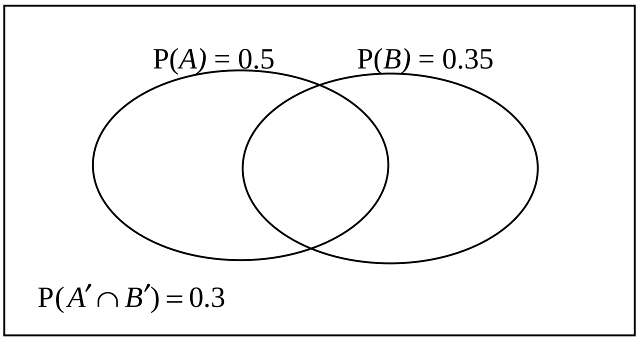
- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (30 marks)

1 The diagram illustrates the occurrence of two events A and B.

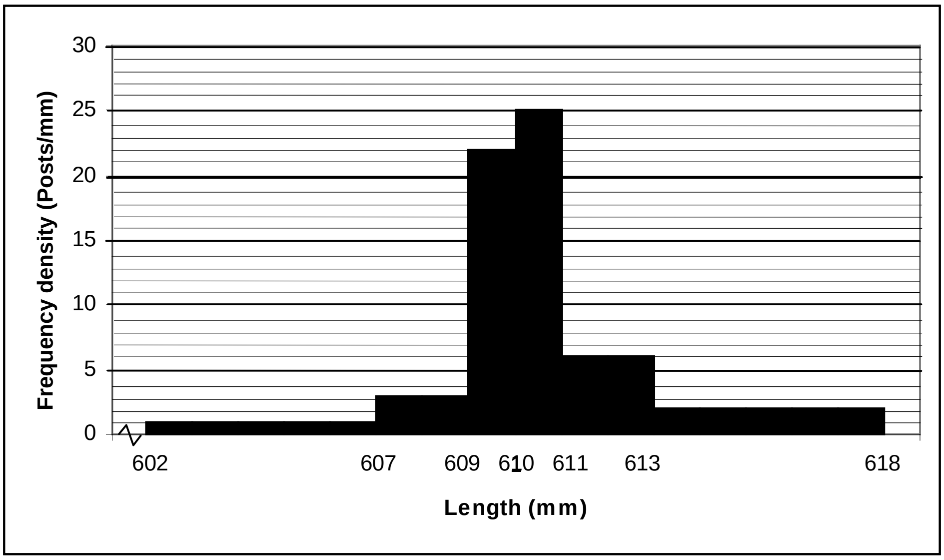


You are given these probabilities:

that A occurs	0.5,
that B occurs	0.35,
that neither A nor B occurs	0.3.

Find the probability that both A and B occur. **[3]**

2 A sawmill cuts wooden posts which should be 610 mm long.
They measure the lengths of a sample of 80 posts.
Their lengths are illustrated in the histogram below.



(i) State the number of posts in each of the classes used in the histogram. **[3]**

(ii) What can you say about the range of the lengths of the posts in the sample? **[1]**

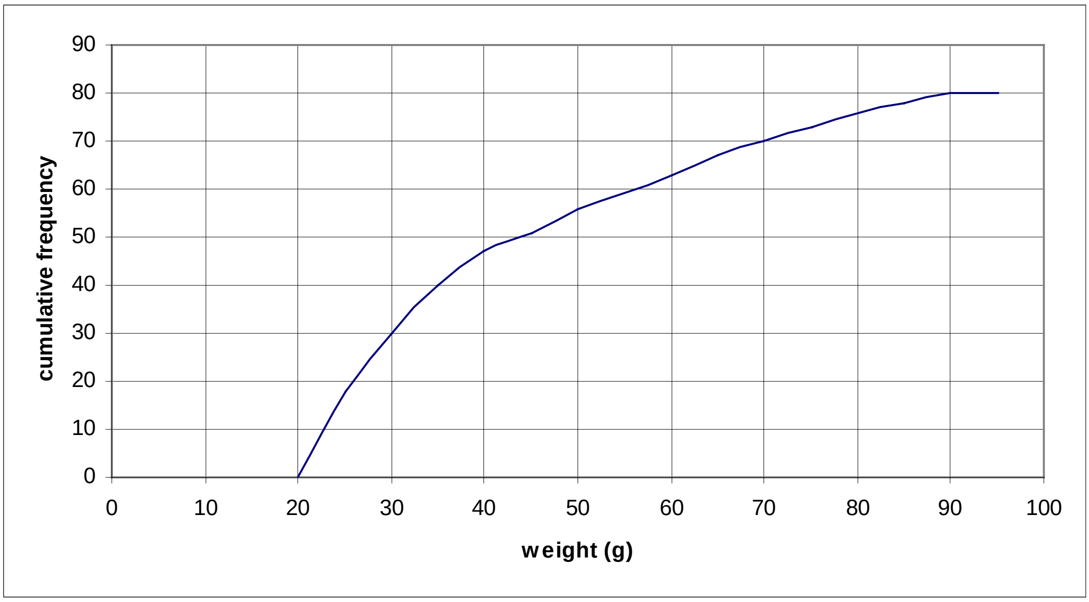
(iii) Without doing any further calculations, explain why an estimate of the mean will be greater than 610 mm. **[2]**

3 In a group of 36 blood donors, 16 are male and 20 are female.
Four of these people are chosen at random for an interview.

(i) In how many ways can they be chosen? **[2]**

(ii) Find the probability that they are all of the same sex. **[3]**

4 As part of a survey of fish stocks in a river, 80 specimens of a particular type of fish are trapped and weighed.
The results are shown on the cumulative frequency graph below.



(i) Find the median and quartiles of the distribution. **[2]**

(ii) Draw a box and whisker plot to illustrate the distribution. **[2]**

(iii) Comment on the shape of the distribution and draw a rough sketch of it. **[3]**

5 A train operating company does a survey of the time-keeping of a particular train over the working days in two weeks.
The results for this sample are shown in Table 5.1 below.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Week 1	0	2	3	0	5
Week 2	6	1 early	32	0	3

Table 5.1: Minutes late

(i) Calculate:

(*A*) the mean; **[1]**

(*B*) the root mean square deviation; **[2]**

(*C*) the standard deviation. **[1]**

of these data.

(ii) Use your results from part **(i)** to justify classifying the figure for Week 2 Wednesday as an outlier. **[2]**

(iii) The delay on Week 2 Wednesday was caused by a security alert.
The train operating company says this was not their fault and so removes the outlier from the data set.
What effect does this have on the mean and standard deviation? **[2]**

6 The number, X, of occupants of cars coming into a city centre is modelled by the probability distribution $\mathrm{P}(X = r) = \frac{k}{r}$ for r = 1, 2, 3, 4.

(i) Tabulate the probability distribution and determine the value of k. **[3]**

(ii) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **[4]**

Section B (36 marks)

7 Wendy is an amateur weather forecaster.
She classifies the weather on a day as either wet or fine.
From past records she suggests that:

- if a day is wet then the probability that the next day is wet is 0.5,
- if a day is fine then the probability that the next day is fine is 0.8.

In a particular week, it is wet on Monday.

(i) Draw a probability tree diagram for wet or fine days on Tuesday, Wednesday and Thursday. **[4]**

(ii) Find the probability that Tuesday, Wednesday and Thursday all have the same weather. **[3]**

(iii) Find the probability that the weather is wet on Thursday. **[4]**

(iv) Find the probability that it is fine on Tuesday and wet on Thursday. **[3]**

(v) Given that it is wet on Thursday, find the conditional probability that it was fine on Tuesday. **[3]**

8 A police road-safety team examines the tyres of a large number of commercial vehicles. They find that 17% of lorries and 20% of vans have defective tyres.

(i) Six lorries are stopped at random by the road-safety team. Find the probability that:

(*A*) none of the lorries has defective tyres, **[2]**

(*B*) exactly two lorries have defective tyres, **[3]**

(*C*) more than two lorries have defective tyres. **[3]**

Following a road-safety campaign to reduce the proportion of vehicles with defective tyres, 18 vans are stopped at random and their tyres are inspected.
Just one of the vans has defective tyres.
You are to carry out a suitable hypothesis test to examine whether the campaign appears to have been successful.

(ii) State your hypotheses clearly, justifying the form of the alternative hypothesis. **[3]**

(iii) Carry out the test at the 5% significance level, stating your conclusions clearly. **[4]**

(iv) State, with a reason, the critical value for the test. **[2]**

(v) Give a level of significance such that you would come to the opposite conclusion for your test. Explain your reasoning. **[2]**

Oxford Cambridge and RSA Examinations

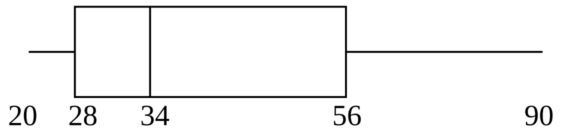
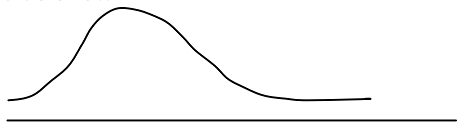
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
STATISTICS 1, S1

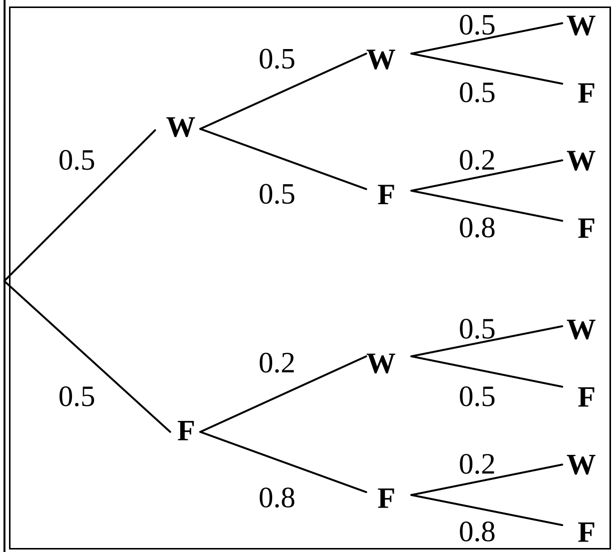
4766

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	$P(A \cup B) = 1 - 0.3 = 0.7$ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= 0.5 + 0.35 - 0.7$ $= 0.15$	B1 M1 A1 **[3]**	
2(i)	<u>Length</u> <u>Frequency</u> 602 to 607 5 607 to 609 6 609 to 610 22 610 to 611 25 611 to 613 12 613 to 618 <u>10</u> **Total 80**	 B1 B1 B1 **[3]**	 For 5 and 10 For 6 and 12 For figures with total 80
2(ii)	The range lies between 6 and 16.	B1 **[1]**	
2(iii)	Mean is estimated as $\sum \frac{(\text{Mid-point} \times \text{Frequency})}{\text{Total}}$ The intervals are symmetrically placed either side of 410 but in each case the frequency on the right is greater	 B1 B1 **[2]**	 Allow 1 mark for each of two Sensible statements
3(i)	Number of ways 4 may be chosen from 36 $= {}^{36}C_4 = 58905$	M1 A1 **[2]**	${}^{36}C_4$ term
3(ii)	P(All of same sex) = P(All male) + P(All female) $= \frac{16}{36} \times \frac{15}{35} \times \frac{14}{34} \times \frac{13}{33} + \frac{20}{36} \times \frac{19}{35} \times \frac{18}{34} \times \frac{17}{33}$ $= 0.113$ (3 s.f.)	M1 M1 A1 **[3]**	 Attempt at correct numbers cao

Qu	Answer	Mark	Comment
Section A (continued)			
4(i)	Median = 34 Upper quartile = 56 Lower quartile = 26	B1 B1 **[2]**	Median Quartiles
4(ii)		B1 B1 **[2]**	Box Whiskers
4(iii)	Positive skew 	B1 B1 B1 **[3]**	1 mark for skew 1 mark for positive Sketch
5(i)(*A*)	$\overline{x} = \frac{50}{10} = 5$	B1	
5(i)(*B*)	$\sum (x-\overline{x})^2 = 858 \Rightarrow rmsd = \sqrt{\frac{858}{10}} = 9.26$	B1 B1	For 858 seen cao
5(i)(*C*)	$s = \sqrt{\frac{858}{9}} = 9.76$	B1 **[4]**	For division by 9
5(ii)	$\overline{x} + 2s = 5 + 2\times 9.76 = 24.52$ Since $32 > 24.52$, 32 may be classified as an outlier.	M1 E1 **[2]**	
5(iii)	Without the 32, $\overline{x} = \frac{18}{9} = 2,\ s = \sqrt{\frac{48}{8}} = 2.45$ Both the mean and standard deviation are much reduced	 B1 B1 **[2]**	 One mark both

Qu	Answer	Mark	Comment
Section A (continued)			
6(i)	$\begin{array}{c\|cccc} r & 1 & 2 & 3 & 4 \\ \hline \mathrm{P}(X=r) & k & \frac{1}{2}k & \frac{1}{3}k & \frac{1}{4}k \end{array}$	M1	Tabulation (SO1)
	Now $k+\frac{1}{2}k+\frac{1}{3}k+\frac{1}{4}k=1$	M1	
	$\Rightarrow k=\frac{12}{25}=0.48$	A1	Value of k
		[3]	
6(ii)	$\mathrm{E}(X)=1\times0.48+2\times0.24+3\times0.16+4\times0.12$ $=1.92$	B1	$\mathrm{E}(X)$ (provided $\sum p=1$)
	$\mathrm{E}(X^2)=1\times0.48+4\times0.24+9\times0.16+16\times0.12$	M1	$\mathrm{E}(X^2)$ ($\sum p=1$)
	Hence $\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$ $=4.8-1.92^2$	M1	Positive variance
	$=4.8-3.6864$ $= 1.1136$ *or* 1.11 (to 3 s.f.)	A1	cao
		[4]	
			Section A Total: 36

Qu	Answer	Mark	Comment
Section B			
7(i)		B1 B1 B1 B1 **[4]**	Overall shape 1st pair branches 2nd set branches 3rd set branches
7(ii)	P(same weather on Tuesday, Wednesday, and Thursday) $= 0.5^3 + 0.5 \times 0.8^2 = 0.445$	M1 M1 A1 **[3]**	2 triple products Sum of products cao
7(iii)	P(wet Thursday) $= 0.5^3 + 0.5^2 \times 0.2 + 0.5^2 \times 0.2 + 0.5 \times 0.8 \times 0.2$ $= 0.305$	M1 A1 M1 A1 **[4]**	4 triples Correct triples Sum of products cao
7(iv)	P(fine Tuesday and wet Thursday) $= 0.5 \times 0.2 \times 0.5 + 0.5 \times 0.8 \times 0.2$ $= 0.13$	M1 A1 A1 **[3]**	2 triples
7(v)	P(fine Tuesday | wet Thursday) Use of $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ $= \dfrac{0.13}{0.305}$ $= 0.426$ (3 s.f.) *or* $\dfrac{26}{61}$	M1 A1 A1 **[3]**	 Numerator and denominator cao

Qu	Answer	Mark	Comment
Section B (continued)			
8(i)(*A*)	P(no lorries have defective tyres) $= 0.83^6 = 0.327$ (3 s.f.) $= 0.33$ (2 s.f.)	M1 A1 **[2]**	 cao
8(i)(*B*)	P(exactly 2 lorries have defective tyres) $= {}^6C_2 \times 0.17^2 \times 0.83^4$ $= 0.206$ (to 3 s.f.) $= 0.21$ (2 s.f.)	M1 M1 A1 **[3]**	For $0.17^2 \times 0.83^4$ For ${}^6C_2 \times \ldots$ cao
8(i)(*C*)	P(1 lorry has defective tyres) $= {}^6C_1 \times 0.17 \times 0.83^5$ $= 0.402$ (to 3 s.f.) P(more than 2 lorries have defective tyres) $= 1 - (0.327 + 0.402 + 0.206)$ $= 0.065(5)$	 B1 M1 A1 **[3]**	
8(ii)	$H_0 : P = 0.2$ $H_1 : P < 0.2$ H_1 takes this form because we are looking for a *reduction* in the proportion of defective tyres.	B1 B1 E1 **[3]**	Null hypothesis Alternative hyp. Explanations
8(iii)	Let $X \sim B(18, 0.2)$ $P(X \le 1) = 0.0991$ Since $0.0991 > 0.05$, do not reject H_0 (*or* accept H_0) There is not enough evidence to suggest that there has been a (significant) reduction in the proportion of defective tyres *or* 'campaign appears to have been successful'	 B1 M1 A1 **[4]**	 Tail probablity Comparison Conclusion in words
8(iv)	The critical value for the test is 0, since $P(X \le 0)$ $[= 0.018] < 0.05$	B1 B1 **[2]**	Critical value Reason
8(v)	The opposite conclusion would be reached provided the significance level was above 9.91%, e.g. 10%	B1 E1 **[2]**	Suitable percentage Explicit comparison with 9.91%
			Section B Total: 36
			Total: 72

AO	Range	Total	Question Number							
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	14-22	19	1	1	2	2	1	4	4	4
2	14-22	18	1	2	1	3	1	3	4	3
3	18-26	21	-	-	2	-	2	-	8	9
4	7-15	8	-	3	-	2	2	-	-	1
5	3-11	6	1	-	-	-	2	-	1	2
	Totals	**72**	3	6	5	7	8	7	17	19

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

STATISTICS 2, S2

4767

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

1 A medical statistician wishes to carry out a hypothesis test to see if there is any correlation between the head circumference and body length of newly-born babies.

(i) State appropriate null and alternative hypotheses for the test. **[2]**

A random sample of 20 newly-born babies have had their head circumference, x cm, and body length, y cm, measured. This bivariate sample is illustrated in Fig. 1.

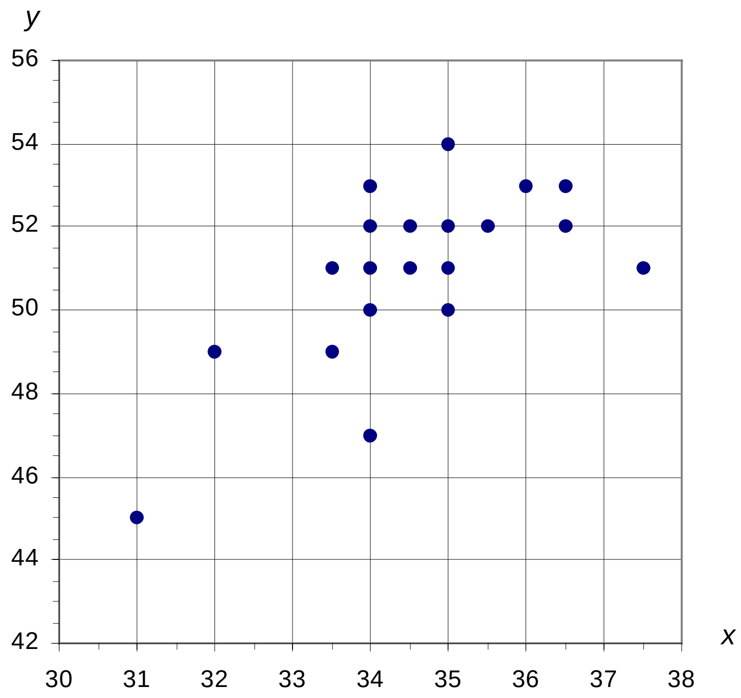


Fig. 1

Summary statistics for this data set are as follows.

$n = 20 \quad \sum x = 691 \quad \sum y = 1018 \quad \sum x^2 = 23\,917 \quad \sum y^2 = 51\,904 \quad \sum xy = 35\,212.5$

(ii) Calculate the product-moment correlation coefficient for the data.
Carry out the hypothesis test at the 1% significance level, stating the conclusion carefully.
What assumption is necessary for the test to be valid? **[10]**

Originally, the point $x = 34,\ y = 51$ had been recorded incorrectly as $x = 51,\ y = 34$.

(iii) Calculate the values of the summary statistics if this error had gone undetected.
Use the uncorrected summary statistics to show that the value of the product-moment correlation coefficient would be negative. **[4]**

(iv) How is it that this one error produces such a large change in the value of the correlation coefficient and also changes its sign? **[2]**

2 *Extralite* are testing a new long-life bulb. The life-times, in hours, are assumed to be Normally distributed with mean μ and standard deviation σ.

After extensive tests, they find that 19% of bulbs have a life-time exceeding 5000 hours, while 5% have a life-time under 4000 hours.

(i) Illustrate this information on a sketch. **[2]**

(ii) Show that $\sigma = 396$ and find the value of μ. **[5]**

In the remainder of this question take μ to be 4650 and σ to be 400.

(iii) Find the probability that a bulb chosen at random has a life-time between 4250 and 4750 hours. **[3]**

(iv) Find the probability that a bulb has a life-time of over 5450 hours. **[1]**

(v) *Extralite* wish to quote a life-time which will be exceeded by 99% of bulbs. What time, correct to the nearest 100 hours, should they quote? **[3]**

A new school classroom has 6 light-fittings, each fitted with an *Extralite* long-life bulb.

(vi) Find the probability that no more than one bulb needs to be replaced within the first 4250 hours of use. **[4]**

3 The numbers of goals per game scored by teams playing at home and away in the Premier League are modelled by independent Poisson distributions with means 1.63 and 1.17 respectively.

(i) Find the probability that, in a game chosen at random,

(*A*) the home team scores at least 2 goals, **[4]**

(*B*) the result is a 1-1 draw, **[3]**

(*C*) the teams score 5 goals between them. **[4]**

(ii) Give **two** reasons why the proposed model might not be suitable. **[2]**

The number of goals scored per game at home by *Rovers* is modelled by the Poisson distribution with mean 1.63. In a season they play 19 home games.

(iii) Use a suitable approximating distribution to find the probability that *Rovers* will score more than 35 goals in their home games. **[5]**

4 **(a)** The length of metal rods used in an engineering structure is specified as being 40 cm. It does not matter if they are slightly longer, but they should not be any shorter. These rods are made by a machine in such a way that their lengths are Normally distributed with standard deviation 0.2 cm. The mean, μ cm, of the lengths is set to a value slightly above 40 cm to give a margin for error.

To examine whether the specification is being met, a random sample of 12 rods is taken. Their lengths, in cm, are found to be:

40.43	40.49	40.19	40.36	40.81	40.47
40.46	40.63	40.41	40.27	40.34	40.54

It is desired to test whether $\mu = 40.5$.

(i) State a suitable alternative hypothesis for the test. **[1]**

(ii) Carry out the test at the 5% level of significance, stating your conclusion carefully. **[8]**

(b) Data are extracted from the medical records of a random sample of patients of a large general practice, showing for part of a particular year the frequencies of contracting or not contracting influenza for patients who had or had not had influenza inoculations.

		Influenza	
		Yes	**No**
Inoculated	**Yes**	8	18
	No	35	17

State null and alternative hypotheses for a suitable test for independence of inoculation and occurrences of influenza.
Carry out the test at the 5% level of significance. **[9]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

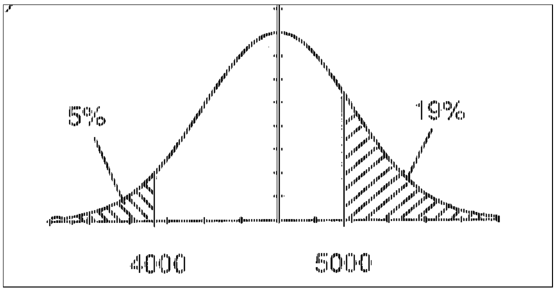
MEI STRUCTURED MATHEMATICS

STATISTICS 2, S2

4767

MARK SCHEME

Qu	Answer	Mark	Comment
1(i)	$H_0 : \rho = 0$, $H_1 : \rho \neq 0$ [where ρ is the population correlation coefficient]	B1 B1 **[2]**	For H_0 For H_1
1(ii)	$S_{xy} = \Sigma xy - n\overline{xy} = 35\ 212.5 - 20 \times 34.55 \times 50.9 = 40.6$	B1	S_{xy} or covariance
	$S_{xx} = \Sigma x^2 - n\overline{x}^2 = 23\ 917 - 20 \times 34.55^2 = 42.95$	B1	S_{xx}
	$S_{yy} = \Sigma y^2 - n\overline{y}^2 = 51\ 904 - 20 \times 50.9^2 = 87.8$	B1	S_{yy}
	$r = \dfrac{40.6}{\sqrt{42.95 \times 87.8}}$ *or*	M1	Structure of r
	$\dfrac{2.03}{\sqrt{2.1475}\sqrt{4.39}} = 0.66$ (2 s.f.)	A1	cao
	For $n = 20$, 1% critical value = 0.5614	M1,A1	Critical value
	Since $0.5614 < 0.661$ we reject H_0:	M1	Comparison
	There is sufficient evidence at the 1% significance level to suggest there is correlation between head circumferences and lengths of babies.	A1	Conclusion in words in context
	Background population is *bivariate Normal*.	E1 **[10]**	Explanation
1(iii)	$\sum x = 708$, $\sum y = 1001$, $\sum x^2 = 25\ 362$, $\sum y^2 = 50\ 459$, $n = 20$, $\sum xy = 35\ 212.5$	B3	All 6 correct (B2 for any 4 correct, B1 for any 2 correct)
	$\Rightarrow S_{xy} = -222.9$ and so $\rho < 0$.	B1 **[4]**	Or $\rho = -0.681$
1(iv)	The incorrect pair produce an *extreme* point to the *right and* *below* existing cluster, producing a negative correlation.	E1 E1	Extreme point Relative position
	(Or There will be a large change in the summary statistics, which will make the covariance negative.**)**	**(**E1 E1**)** **[2]**	For large change For negative cov.

Qu	Answer	Mark	Comment
2(i)		B1 B1 **[2]**	Correct overall shape Tails with area of right-hand tail *larger* than the left-hand tail area
2(ii)	$\mathrm{P}(X > 5000) = 0.19 \Rightarrow 5000 = \mu + 0.8779\sigma$ $\mathrm{P}(X < 4000) = 0.05 \Rightarrow 4000 = \mu - 1.645\sigma$ Solving: $1000 = 2.523\sigma$ $\Rightarrow \sigma = \dfrac{1000}{2.523} = 396$ (3 s.f.) Hence: $\mu = 4000 + 1.645 \times 396 = 4650$ (3 s.f.)	B1 M1 M1 B1 A1 **[5]**	Both z-values Attempt at one equation with z-value Attempt at finding σ σ μ
2(iii)	$\mathrm{P}(4250 < X < 4750) = \mathrm{P}(-1 < Z < 0.25)$ $= 0.5987 - (1 - 0.8413)$ $= 0.4400$	M1 M1 A1 **[3]**	Standardisations Probability calculations cao
2(iv)	$\mathrm{P}(X > 5450) = \mathrm{P}(Z > 2)$ $= 1 - 0.9772 = 0.0228$	B1 **[1]**	cao
2(v)	$\mathrm{P}(Z > -2.326) = 0.99$ $\Rightarrow x = 4650 - 2.326 \times 400 = 3719.6$ hence should quote 3700 hours	B1 M1 A1 **[3]**	± 2.326 Calculation cao
2(vi)	P(0 or 1 bulbs need replacing) $= 0.8413^6 + 6 \times 0.8413^5 \times 0.1587$ $= 0.76$ (2 s.f.)	M1 M1,A1 A1 **[4]**	0 or 1 Sum of 2 terms cao

Qu	Answer	Mark	Comment
3(i)(*A*)	$P(X \geq 2) \; = 1 - P(X \leq 1)$ $= 1 - e^{-1.63}(1 + 1.63)$ $= 1 - 0.515 = 0.485 \;$ (3 s.f.)	M1 M1,A1 A1 **[4]**	Sum of 2 probs. 1 – sum of 2 probs.
3(i)(*B*)	$P(X = 1) \times P(Y = 1)$ $= (e^{-1.63} \times 1.63) \times (e^{-1.17} \times 1.17)$ $= 0.116 \;$ (3 s.f.)	M1 M1 A1 **[3]**	2 probabilities Product
3(i)(*C*)	Using $\lambda = 1.63 + 1.17 = 2.8$: $P(X + Y = 5) = 0.9349 - 0.8477 = 0.087 \;$ (2 s.f.) **(***or* $P(X + Y = 5) = e^{-2.8} \times \dfrac{2.8^5}{5!} = 0.087 \;$ (2 s.f.) **)**	M1,A1 M1 A1 **[4]**	$\lambda = 2.8$ For calculation cao
3(ii)	Two reasons why proposed model might not be suitable: Poisson parameter unlikely to be same for each team; lack of independence between the variables.	E1 E1 **[2]**	For one reason For second reason
3(iii)	$\lambda = 19 \times 1.63 = 30.97$, hence suitable approximating distribution is N(30.97, 30.97) P(more than 35 goals in a season) $= P(X > 35.5) = P(Z > \dfrac{35.5 - 30.97}{\sqrt{30.97}})$ $= P(Z > 0.814)$ $= 1 - 0.792$ $= 0.208 \;$ (3 s.f.)	M1,A1 B1 M1 A1 **[5]**	Use of Normal approx. Continuity corr. Calculation

Qu	Answer	Mark	Comment
4(a)(i)	$H_1 : \mu < 40.5$	B1 **[1]**	Hypothesis
4(a)(ii)	$n = 12, \sum x = 485.4 \Rightarrow \bar{x} = 40.45$	M1,A1	Mean value
	Test statistic is $\dfrac{40.45 - 40.5}{\frac{0.2}{\sqrt{12}}} = -0.866$	M1 M1 A1	Numerator Denominator
	Since $-0.866 > -1.645$, the result is not significant, and it is reasonable to accept that $\mu = 40.5$	B1 M1 A1 **[8]**	'1.645' Comparison Conclusion in words
4(b)	H_0: There is no association between inoculation and the occurrence of influenza	B1	
	H_1: There is an association between inoculation and the occurrence of influenza	B1	
	Expected frequencies: (table below)	M1,A1	Expected frequencies
	$X^2 = \dfrac{(8-14.333)^2}{14.333} + \dfrac{(18-11.667)^2}{11.667} + \dfrac{(35-28.667)^2}{28.667} + \dfrac{(17-23.333)^2}{23.333}$	M1	Calculation of the test statistics
	$= 9.35$ (3 s.f.)	A1	cao
	Since $9.35 > 3.84$, the result is significant, and therefore it seems there is association between incidence of inoculation and influenza	B1 M1 A1 **[9]**	3.84 Comparison Conclusion in words
			Total: 72

Expected frequencies:

		Influenza: Yes	Influenza: No	*Total*
Inoculated	Yes	14.333	11.667	**26**
	No	28.667	23.333	**52**
Totals		**43**	**35**	**78**

AO	Range	Total	Question Number 1	2	3	4
1	14-22	15	7	1	2	5
2	14-22	16	2	6	4	4
3	18-26	20	5	9	5	1
4	7-15	12	2	-	4	6
5	3-11	9	2	2	3	2
	Totals	**72**	18	18	18	18

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

STATISTICS 3, S3

4768

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

1 An insurance company is investigating the amounts of money paid out for each claim on a certain type of insurance policy. It uses the continuous random variable X as a model for the amounts paid out per claim (measured in thousands of pounds), where X has probability density function

$$\mathrm{f}(x) = x\mathrm{e}^{-x} \text{ for } x \geq 0.$$

(i) Use integration by parts to find the cumulative distribution function of X and hence show that, for $t \geq 0$,

$$\mathrm{P}(X > t) = \mathrm{e}^{-t}(1+t)$$ **[5]**

(ii) Evaluate $\mathrm{P}(X > 2.5)$. **[1]**

(iii) Verify that the median amount paid out per claim is, to a good approximation, £1 680. **[3]**

(iv) Use the result that $\int_0^\infty y^n \mathrm{e}^{-y} \mathrm{d}y = n!$ for $n = 0, 1, 2, \ldots$ to find the mean and variance of X. **[5]**

(v) A manager decides to investigate the Normal distribution with mean 2 and variance 2 as a model for the amounts (in thousands of pounds) paid out per claim. Find the probability given by this model that an individual pay-out will exceed £2 500. **[2]**

(vi) Fig. 1 is a sketch of the graph of $\mathrm{f}(x)$.
Make a rough copy of this sketch and draw on the same axes a rough sketch of the probability density function of the N (2, 2) distribution. Indicate clearly the areas that correspond with the probabilities calculated in parts **(ii)** and **(iv)**. **[2]**

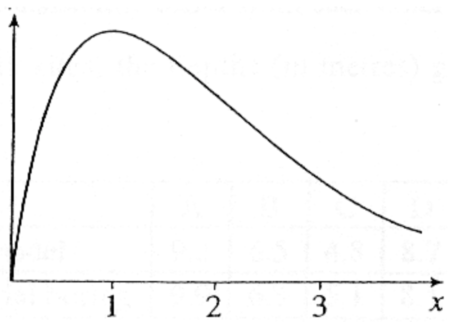


Fig. 1

2 A commuter's train journey to work is scheduled to take 52 minutes. Having noticed that he is *always* late, even when the trains are running normally, he decided to keep records for a random sample of ten journeys. On two of these occasions, there were major signal failures leading to severe disruption and complete suspension of services. He therefore decided to eliminate these two occasions from his records. On the other eight occasions, his journey times in minutes were as follows.

65 61 62 60 59 62 61 57

(i) Carry out a two-sided 5% test of the hypothesis that his overall mean lateness is 10 minutes. State the required distributional assumption underlying your analysis. **[8]**

(ii) Provide a 99% confidence interval for the mean journey time.
Hence comment on the railway company's policy of offering refunds for journeys that are more than 15 minutes late. **[6]**

(iii) Comment on the commuter's decision not to include the two occasions when there were major signal failures. **[4]**

3 A construction company operating at many sites uses a computer model to assess the depth of bedrock at each site. Trial borings are also made at some sites to help check the model. Neither the model nor the trial borings can be expected to give completely accurate answers, but it is important that they do not consistently differ from each other. For a random sample of six sites, the depths (in metres) given by the model and by the trial borings are as follows:

Site	**A**	**B**	**C**	**D**	**E**	**F**
Result from model	9.2	6.5	4.8	8.7	9.6	12.5
Result from trial boring	9.9	6.3	5.1	8.1	9.5	13.0

(a) Use an appropriate *t* test, at the 5% level of significance, to examine whether the mean difference between the depths given by the model and by the trial borings is zero.
State the required distributional assumption. **[10]**

(b) Investigate the situation using the Wilcoxon paired sample test, again using a 5% significance level. **[8]**

4 It is thought that the time (in hours) between minor breakdowns on a computer network might be modelled by the exponentially distributed random variable X with probability density function

$$f(x) = \lambda e^{-\lambda x} \text{ for } x > 0,$$

where λ is a parameter ($\lambda > 0$). A random sample of 80 times between minor breakdowns is summarised by the following frequency distribution. In this random sample, $\bar{x} = 20$ in hours.

time x hours	$0 < x \le 10$	$10 < x \le 20$	$20 < x \le 30$	$30 < x \le 40$	$40 < x \le 50$	$x > 50$
frequency	26	16	9	10	9	10

(i) Use the result that, for $0 < a < b$, $P(a < X \le b) = e^{-\lambda a} - e^{-\lambda b}$ and the estimate $\hat{\lambda} = \frac{1}{\bar{x}}$ to calculate the expected frequency corresponding to the (0, 10) cell of the above table. **[5]**

(ii) The remaining expected frequencies are as follows:

cell	$10 < x \le 20$	$20 < x \le 30$	$30 < x \le 40$	$40 < x \le 50$	$x > 50$
expected frequency	19.09	11.58	7.02	4.26	6.57

The (40, 50) cell has expected frequency *less* than 5. Suggest why, despite this, it should perhaps *not* be grouped with another cell or cells when conducting a χ^2 goodness of fit test. **[3]**

(iii) Carry out a χ^2 goodness of fit test, keeping *all* the cells. Use a 5% significance level. **[8]**

(iv) Discuss briefly your conclusions. **[2]**

Extra specimen question

5 The Reverend Thomas, a clergyman in the north of England who is also a keen statistician, has been monitoring the lengths of his sermons. He aims for each sermon to be between 10 and 15 minutes long, but in fact the sermons' lengths are given by the random variable X which is Normally distributed with mean 13½ minutes and standard deviation 2 minutes. The lengths of different sermons are independent of one another.

(i) Find the probability that an individual sermon lasts between 10 and 15 minutes. **[4]**

(ii) During a particular week, Rev. Thomas gives four sermons. Find the probability that their total length is more than an hour. **[3]**

(iii) Rev. Thomas is asked to provide a series of sermons to be broadcast in religious radio programmes but is instructed that he must reduce their length. Suppose he is successful to the extent that the random variable giving the sermons' lengths is now $\frac{1}{2}X$. Find the time interval required in a radio programme to ensure that, with probability 0.9, there is time for a sermon. **[7]**

(iv) Because of other variable elements in the radio programmes, the time available for a reduced-length sermon is itself a random variable, Normally distributed with mean 8 minutes and standard deviation 0.5 minutes. Find the probability that there is time for a sermon. **[4]**

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
STATISTICS 3, S3

4768

MARK SCHEME

Qu	Answer	Mark	Comment
1(i)	$\mathrm{f}(x) = x\mathrm{e}^{-x}, x \geq 0,$ (x in thousands of pounds). C.d.f. $\mathrm{F}(t) = \int_0^t x\mathrm{e}^{-x}\mathrm{d}x$ $= \left[-x\mathrm{e}^{-x}\right]_0^t + \int_0^t \mathrm{e}^{-x}\mathrm{d}x$ $= \left[-t\mathrm{e}^{-t}\right] - [0] + \left[-\mathrm{e}^{-x}\right]_0^t$ $= -t\mathrm{e}^{-t} - \mathrm{e}^{-t} + \mathrm{e}^0$ $= 1 - \mathrm{e}^{-t} - t\mathrm{e}^{-t}$ $\therefore \mathrm{P}(X > t) = 1 - (1 - \mathrm{e}^{-t} - t\mathrm{e}^{-1}) = \mathrm{e}^{-t}(1+t)$	M1 M1 A1 A1 A1 **[5]**	Set up required integral including limits Reasonable attempt to integrate by parts Successful integration to $-x\mathrm{e}^{-x} - \mathrm{e}^{-x}$ Limits used to obtain correct cdf cao cao ANSWER GIVEN
1(ii)	$\mathrm{P}(X > 2.5) = 3.5\mathrm{e}^{-2.5} = 0.2873$	B1 **[1]**	
1(iii)	Median of X, m, is given by $\frac{1}{2} = \mathrm{e}^{-m}(1+m)$ Inserting $m = 1.68$ in RHS gives $2.68\mathrm{e}^{-1.68} = 0.4995 \approx 0.5$ as required	M1 M1 A1 **[3]**	Definition of median Convincingly shown
1(iv)	$\mathrm{E}(X) = \int_0^\infty x^2\mathrm{e}^{-x}\mathrm{d}x$ = 2! (from the given result) = 2 $\mathrm{E}(X^2) = \int_0^\infty x^3\mathrm{e}^{-x}\mathrm{d}x = 3! = 6$ $\therefore \mathrm{Var}(X) = \mathrm{E}(X^2) - \{\mathrm{E}(X)\}^2 = 6 - 4 = 2$	M1 A1 M1 M1 A1 **[5]**	Set up required integral with limits cao Set up required integral for $\mathrm{E}(X^2)$ Evidence of intention to use the definition of variance ft c's E(X) only
1(v)	Using N(2,2), $\mathrm{P}(X > 2.5) = \mathrm{P}(\mathrm{N}(0,1) > \frac{2.5-2}{\sqrt{2}} = 0.3535(5))$ $= 1 - 0.6381 = 0.3619$	 M1 A1 **[2]**	 Accept use of $z = 0.353$ or 0.354 leading to probabilities in the range 0.3617 to 0.3621
1(vi)	Sketch showing $\mathrm{f}(x)$ and N(2, 2)	B1 B1 **[2]**	N(2, 2) near enough correct BOTH areas *clearly* marked

Qu	Answer	Mark	Comment
2(i)	$\bar{x} = 60.875$ $s_{n-1}^2 = 5.5536,\ s_{n-1} = 2.3566$	B1	For both Allow $s_n^2 = 4.8594,\ s_n = 2.2044$ but ONLY if correctly used in sequel
	Assumption of underlying Normality.	B1	For the assumption (about population, not data; e.g. do not allow just 'it's Normal')
	Test statistic is $\dfrac{60.875-62}{\frac{2.3566}{\sqrt{8}}}$	M1	Numerator might be given as $8.875-10$ Allow M1A0 then ft for $\mu - \bar{x}$ in numerator Allow $s_n/\sqrt{7}$ (see above)
	$= -1.35(024)$	A1	cao Correct answer ww scores 2/2
	Refer to t_7	M1	May be awarded even if test statistic is wrong. Must see evidence of intention to use t distribution. But no ft if v is wrong
	Double tail 5% point is 2.365	A1	No ft if wrong. May be +ve or −ve
	Not significant	B1	For comparison (pi) and simple conclusion (pi) consistent with c's t and critical value
	Accept hypothesis that mean lateness = 10 minutes. (Or that mean journey time = 62)	B1 SC	Consistent contextual conclusion t_8 and 2.306 or t_7 and 1.895 used can score max B1 for either form of conclusion seen N.B. ZERO OUT OF 4 if not same distribution as used for test. Same wrong distribution can score max M1B0M1A0. BUT allow recovery to t_7 for possible 4/4
		[8]	
2(ii)	99% C.I. given by: $60.875 \pm 3.499 \times \dfrac{2.3566}{\sqrt{8}} = 60.875 \pm 2.915(3)$	M1	For $\bar{x} \pm \ldots$ Allow c's $\bar{x}$ from part **(i)** or $\bar{x} - 52$
		B1	For 3.499 (from t_7)
	$= (57.960, 63.790)$	M1	For $s/\sqrt{8}$. Allow c's s_{n-1} from part **(i)** Also allow $s_n/\sqrt{7}$ (see above)
		A1	BOTH. cao Accept correct ww for 4/4 Must be an interval. Min 2 dp required

s	t	60.875+/-	lower	upper	Mark	Comment
2.3566	3.499	2.91531	57.9597	63.7903	Full marks	
2.2044	3.499	2.72703	58.148	63.602	M1B1M0A0	
2.3566	3.355	2.79533	58.0797	63.6703	M1B0M1A0	if t_8 in (i)
2.2044	3.355	2.6148	58.2602	63.4898	M1B0M0A0	it t_8 in (i)
						(3.355 is t_8 (1%))

Qu	Answer	Mark	Comment
	15 minutes late corresponds to journey time of 67 minutes… • this is (well) outside the interval	E1	A statistical comment: e.g. to include an explicit reference to the interval
	• so the penalty will hardly ever be invoked (despite regular lateness)	E1	A contextual comment Alternatively allow 2, 1, 0 for candidate who shows appreciation of the relationship of outliers to the mean
		[6]	

Qu	Answer	Mark	Comment
2(iii)	Seems reasonable to exclude these two occasions as they will not reflect normal normal daily conditions…	E2	Either ‘exclude’ or ‘include’, together with a reason
	… but, strictly speaking, the sample is no longer a random one.	E2	Recognise need for the sample to be random
	Full credit for answers as outlined above. The real point, of course, consists of subtle but important discussions as to exactly what the underlying population is (all the journeys, or just the normal ones?). FULL CREDIT for discussing this.	**[4]**	Allow equivalent marks for comments which address the distributional assumptions
3(i)	Must be PAIRED COMPARISON t procedure	M1	
	$\begin{array}{lcccccc} & 9.2 & 6.5 & 4.8 & 8.7 & 9.6 & 12.5 \\ & 9.9 & 6.3 & 5.1 & 8.1 & 9.5 & 13.0 \\ \hline \text{Differences} & -0.7 & 0.2 & -0.3 & 0.6 & 0.1 & -0.5 \end{array}$		
	$\overline{d} = -0.1 \quad s_{n-1}^2 = 0.236, \ \ s_{n-1} = 0.4858$	A1	For both
	Accept $s_n^2 = 0.196$, $s_n = 0.4435$, but ONLY if correctly used in sequel		
	Test statistic is $\dfrac{-0.1-0}{\dfrac{0.4858}{\sqrt{6}}}$	M1	
	$= -\,0.50(42)$	A1	
	Refer to t_5 May be awarded even if test statistic is wrong, but NO f.t. if wrong	1	
	Dt 5%pt is 2.571 (NO f.t. if wrong)	1	
	Not significant.	1	
	Seems no overall mean difference between model and trial borings.	1	
	Needs Normality of <u>differences</u>.	2 **[10]**	1 for Normality, 1 for differences
3(ii)	Use of differences	M1	
	Ranks of $\|d\|$ are 6 2 3 5 1 4	M1	For clear attempt to rank $\|d\|$
	( ) denotes a negative d	A1	If all correct
	$T = 8$ or 13	A1	(Correct answer from candidate’s ds)
	Refer smaller value to appropriate table	M1	
	Dt 5% pt for $n = 6$ is ZERO [note for examiner – st 5% pt is 2]	1	
	Result is not significant	1	
	Seems on the whole model and trial borings give ‘the same’ results	1 **[8]**	

Qu	Answer	Mark	Comment
4(i)	$\hat{\lambda} = \frac{1}{\bar{x}} = \frac{1}{20} = 0.05$	B1	
	$P(0 < X \le 10) = e^{-0} - e^{-0.5} = 1 - 0.6065 = 0.3935$	M1,A1	
	$\therefore$ Expected frequency $= 80 \times 0.3935 = 31.48$	M1,A1 **[5]**	
4(ii)	Expected frequency < 5 is only a rule of thumb and not a hard-and-fast law and might include points such as	E1	
	• 4.26 is not much less than 5		
	• Some other expected frequencies are not much more than 5 – arbitrary and unsatisfactory to treat them differently		
	• This cell might turn out to contain important information – unsatisfactory to sacrifice it		
	• There are not many cells anyway – unsatisfactory to reduce their number still further	E1 E1 **[3]**	
4(iii)	$X^2 = 0.95395 + 0.5002 + 0.5748 + 1.2650 + 5.2741 + 1.7907$	M1 B1	For at least 4 values correct
	$= 10.36$ [10.3587]	A1	
	Refer to χ_4^2	B2	Allow B1 for χ_5^2, but no ft.
	Upper 5% point is 9.488	B1	
	Significant	E1	
	Suggests model does not fit data	E1 **[8]**	
4(iv)	The main point is that the data are 'heavy in the tail' and 'light near the origin'.	E2 **[2]**	
			Total: 72

Mark scheme for extra specimen question 5

Qu	Answer	Mark	Comment
5(i)	$X \sim \mathrm{N}(13\frac{1}{2}, 2^2)$		
	$\mathrm{P}(10 < X < 15) = \mathrm{P}\left(\frac{10-13\frac{1}{2}}{2} < Z < \frac{15-13\frac{1}{2}}{2}\right)$	M1	Award ONCE here or elsewhere for standardising. Condone $\mu - \overline{x}$ in numerator
	$= \mathrm{P}(-1.75 < Z < 0.75)$		
	$= 0.7734 - 0.0401$	A1,A1	Both ϕ values, not necessarily subtracted. Accept unsimplified forms, e.g. $1-0.9599$ for 0.0401
	$= 0.7333$	A1 **[4]**	cao Expect 3 d.p. or better
5(ii)	$T = X_1 + X_2 + X_3 + X_4$	B1	Mean
	$\sim \mathrm{N}(4 \times 13\frac{1}{2} = 54,\ \sigma^2 = 4+4+4+4 = 16)$	B1	Variance (or $\sigma = 4$, provided it is clear)
	$\mathrm{P}(T > 60) = \mathrm{P}\left(Z > \frac{6}{4} = 1.5\right)$		
	$= 1 - 0.9332 = 0.0668$	A1F **[3]**	ft incorrect mean but not variance
5(iii)	$\frac{1}{2}X \sim \mathrm{N}\left(\frac{1}{2} \times 13\frac{1}{2} = 6.75,\ \sigma^2 = \left(\frac{1}{2}\right)^2 \times 4 = 1\right)$	B1 B1	Mean Variance (or $\sigma = 1$, provided it is clear)
	Require t such that $0.9 = \mathrm{P}\left(\frac{1}{2}X < t\right)$		
	$= \mathrm{P}\left(Z < \frac{t-6.75}{1}\right)$	M1	Formulation of the requirement as a one-sided inequality
	But $0.9 = \mathrm{P}(Z < 1.282)$	M1 B1 SC	Sensible attempt to use Normal tables 1.28(2) cao After M0 for a two-sided inequality allow B1 for + and − 1.645 seen
	$\therefore t - 6.75 = 1.282$, $\therefore t = 8.032$	M1,A1F **[7]**	ft incorrect mean (but not variance) or c's 1.282
5(iv)	Time available $Y \sim \mathrm{N}(8, 0.5^2)$	M1	Formulation of the requirement as
	We want $\mathrm{P}(Y - \frac{1}{2}X > 0)$		$\mathrm{P}(Y - \text{c's}\frac{1}{2}X > 0)$ $\mathrm{P}(Y >$ ans part **(iii)**$)$ scores 0/4
	$= \mathrm{P}\left(\mathrm{N}\left(1.25,\ \sigma^2 = \frac{1}{4} + 1 = \frac{5}{4}\right) > 0\right)$	B1F B1F	Mean ft for 8 – mean part **(iii)** Variance (or σ, provided it is clear) ft for 0.25 + Var part **(iii)**
	$= \mathrm{P}\left(Z > \frac{-1.25}{\sqrt{\frac{5}{4}}} = -1.118\right) = 0.868(1)$	A1 **[4]**	cao

AO	Range	Total	Question Number			
			1	**2**	**3**	**4**
1	14-22	18	6	2	5	5
2	14-22	15	4	1	7	3
3	18-26	21	3	10	2	6
4	7-15	8	3	2	2	1
5	3-11	10	2	3	2	3
	Totals	**72**	18	18	18	18

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

STATISTICS 4, S4

4769

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **three** questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Option 1: Estimation

1 The random variable X is distributed as $N(0,\sigma^2)$ so that its probablity density function is

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x^2}{2\sigma^2}}.$$

A random sample $x_1, x_2, ..., x_n$ is available.

(i) Write down the likelihood of this sample and hence show that the maximum likelihood estimate of σ^2 is

$$\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}x_i^2$$

(you should verify that this is a maximum). **[9]**

You are now *given* the following results:

(1) for the underlying random variables $X_1, X_2,..., X_n$ the distribution of $\sum_{i=1}^{n}X_i^2$ is $\sigma^2 X_n^2$;

(2) the mean of a chi-squared distribution is equal to the number of degrees of freedom;

(3) the variance of a chi-squared distribution is equal to twice the number of degrees of freedom.

(ii) Use these results to show that $\hat{\sigma}^2$ is an unbiased estimator of σ^2 and find its standard error. **[4]**

(iii) Now, consider a more general estimator of σ^2 of the form $T=k\sum_{i=1}^{n}X_i^2$ where k depends on n.

Show that $E[T]=kn\sigma^2$ and deduce that the bias of T as an estimator of σ^2 is $(kn-1)\sigma^2$.

Find the mean square error of T as an estimator of σ^2.

Hence, find the value of k that minimises the mean square error of T. **[11]**

Option 2: Generating Functions

2 [You may in this question use without proof the 'linear transformation' result for moment generating functions:

If X has moment generating function $\mathrm{M}(t)$ and $Y = aX + b$ (where a and b are constants), then Y has moment generating function $\mathrm{e}^{bt}\mathrm{M}(at)$.]

The random variable X has the Poisson distribution with parameter λ.

(i) Show that the probability generating function for X is $\mathrm{e}^{-\lambda}\mathrm{e}^{\lambda t}$. **[3]**

(ii) Hence, obtain the mean and variance of X. **[5]**

(iii) Write down the moment generating function for X. **[1]**

$X_1, X_2, ..., X_n$ are independent random variables each distributed as X.
Their sum is $T = \sum X_i$ and their mean is $\overline{X} = \frac{1}{n}\sum X_i = \frac{1}{n}T$.

(iv) State the mean and variance of $\overline{X}$. **[2]**

(v) Write down the moment generating function for T and hence show that the moment generating function for $\overline{X}$ is

$$\exp(-n\lambda)\exp(n\lambda \mathrm{e}^{t/n}).$$ **[6]**

[$\exp(x)$ is an alternative notation for e^x.]

(vi) The 'standardised mean' is $Z = \dfrac{\overline{X} - \lambda}{\sqrt{\lambda / n}}$.

Show that the moment generating function for Z is $\exp(-t\sqrt{n\lambda} - n\lambda + n\lambda \mathrm{e}^{t/\sqrt{n\lambda}})$.

Show that the logarithm of this function tends to $\frac{1}{2}t^2$ as $n \to \infty$. **[5]**

(vii) Given that the moment generating function for the N(0,1) random variable is $\exp(\frac{1}{2}t^2)$, what do you conclude about the distribution of Z as $n \to \infty$? **[2]**

Option 3: Inference

3 A company makes heavy-duty waterproof clothing. Part of the manufacturing process consists of spraying a polymer onto a synthetic fibre. The water-absorbent quality of the fibre after this spraying is routinely measured during the manufacturing process. Low values of this measure are desirable.

In the existing process, it is found that the behaviour of the measure is well modelled by the Normal distribution with mean 48.6 and standard deviation 2.4.

An experimental process is being developed. It has been established that the corresponding model for this process is again Normal and with the same standard deviation, but its mean μ is as yet unknown. It is required to examine the null hypothesis $H_0 : \mu = 48.6$ against the alternative hypothesis $H_1 : \mu < 48.6$, using the customary significance test based on the mean $\bar{X}$ of a random sample of size *n*. To avoid unnecessary costs of changing from the existing process, it is required that the probability of rejecting H_0 if in fact μ is 48.6 should be at most 3%. If on the other hand μ is in fact 45.0, it is required that the probability of accepting H_0 should be at most 2%.

(i) Find an expression for the critical value of $\bar{X}$ and show that the least sample size that will meet the requirements is 7. **[16]**

(ii) Taking $n = 7$, derive an expression for the power function of the test in the form

$$\mathrm{P}(Z < a - b\mu)$$

where $Z \sim \mathrm{N}(0,1)$ and *a* and *b* are constants to be determined.
Hence verify that the requirement when $\mu = 45.0$ is met. **[8]**

Option 4: Design and Analysis of Experiments

4 **(i)** State the usual model for the one-way analysis of variance for a situation having k treatments with n_i observations on the ith treatment, with x_{ij} denoting the jth observation on the ith treatment ($i = 1, 2, \ldots, k$; $j = 1, 2, \ldots, n_i$).
Interpret the parameters in the model.
State the usual assumptions about the term representing experimental error. **[7]**

(ii) State carefully the null and alternative hypotheses that are customarily tested in the analysis of variance. **[2]**

At a process development laboratory, engineers are investigating five methods for igniting gas in a cylinder. The percentage of the gas that remains unburnt is measured four times for each method, with the following results.

Method A	11.2	10.8	10.7	10.1
Method B	9.4	9.9	9.6	9.1
Method C	9.2	8.6	8.8	8.4
Method D	12.1	12.3	12.7	11.9
Method E	13.6	12.4	13.1	12.9

[The sum of these data items is 216.8 and the sum of their squares is 2403.86]

(iii) Draw up the usual analysis of variance table and report your conclusions. **[10]**

(iv) Suppose now that the 20 individual runs in this experiment had *not* all been carried out on the same cylinder, but that four different cylinders had been used with 5 runs in each.

Name the experimental design that should have been used in setting up the experiment.
Explain briefly why the design would have been appropriate. **[5]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
STATISTICS 4, S4

4769

MARK SCHEME

Qu	Answer	Mark	Comment
Option 1: Estimation			
1(i)	$L = \frac{1}{(2\pi)^{\frac{n}{2}}\sigma^n} e^{\frac{-\sum x_i^2}{2\sigma^2}}$	B1	Any equivalent product form
	$\ln L = -\frac{n}{2}\ln(2\pi) - n\ln\sigma - \frac{1}{2\sigma^2}\sum x_i^2$	M1,A1	
	$\frac{\mathrm{d}\ln L}{\mathrm{d}\sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum x_i^2$	M1,A1	
	$= 0 \to \hat{\sigma}^2 = \frac{1}{n}\sum x_i^2$	A1	[Beware printed answer!]
	Check this is max: $\frac{\mathrm{d}^2\ln L}{\mathrm{d}\sigma^2} = \frac{n}{\sigma^2} - \frac{3}{\sigma^4}\sum x_i^2$	M1,A1	
	which, at $\sigma^2 = \hat{\sigma}^2$, equals		
	$\frac{n}{\hat{\sigma}^2} - \frac{3n}{\hat{\sigma}^2} < 0 \therefore$ max	A1	
		[9]	
1(ii)	We have $\hat{\sigma}^2 \sim \frac{\sigma^2}{n}\chi_n^2$	M1	
	so $\mathrm{E}\left[\hat{\sigma}^2\right] = \frac{\sigma^2}{n}\cdot n = \sigma^2$ so unbiased	A1	
	and $\mathrm{Var}\left(\hat{\sigma}^2\right) = \frac{\sigma^4}{n^2}\cdot 2n = \frac{2\sigma^4}{n}$	A1	
	so SE $= \sigma^2\sqrt{\frac{2}{n}}$	A1	
		[4]	
1(iii)	We have $T = k\sum x_i^2 \sim k\sigma^2\chi_n^2$	M1	
	$\therefore \mathrm{E}[T] = kn\sigma^2$	A1	
	$\therefore$ bias in $T = kn\sigma^2 - \sigma^2 = (kn-1)\sigma^2$	M1,A1	[METHOD must be clear – beware printed answer!]
	also, $\mathrm{Var}(T) = k^2\sigma^4.2n$	M1,A1	
	$\therefore \mathrm{MSE}[T] = \mathrm{Var} + \mathrm{bias}^2$	M1	
	$= 2k^2n\sigma^4 + (kn-1)^2\sigma^4$	A1	
	$\frac{\mathrm{dMSE}}{\mathrm{d}k} = 4kn\sigma^4 + 2n(kn-1)\sigma^4$	M1,A1	*Candidates are not required to check*
	$= 0 \to k = \frac{1}{n+2}$	A1	*but:* $\frac{\mathrm{d}^2\mathrm{MSE}}{\mathrm{d}k^2} = 4n\sigma^4 + 2n^2\sigma^4 > 0 \therefore$ min
		[11]	

Qu	Answer	Mark	Comment
Option 2: Generating Functions			
2(i)	$\text{Pgf} = \text{E}\left[t^x\right] = \sum_{x=0}^{\infty} t^x \frac{\text{e}^{-\lambda}\lambda^x}{x!}$	M1	
	$= \text{e}^{-\lambda}\sum \frac{(\lambda t)^x}{x!}$	A1	
	$= \text{e}^{-\lambda}\,\text{e}^{\lambda t}$	A1 **[3]**	
2(ii)	$\mu = \text{G}'(1)$		
	$\text{G}'(t) = \text{e}^{-\lambda}\lambda\text{e}^{\lambda t}$	M1	For attempt to differentiate G(t)
		A1	If correct
	$\therefore \mu = \text{e}^{-\lambda}\lambda\text{e}^{\lambda} = \lambda$	A1	
	$\sigma^2 = \text{G}''(1) + \mu - \mu^2$		
	$\text{G}''(t) = \lambda\text{e}^{-\lambda}.\lambda\text{e}^{\lambda t}$		
	$\therefore \text{G}''(1) = \lambda^2$	A1	
	$\therefore \sigma^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$	A1 **[5]**	
2(iii)	Mgf is pgf with t replaced by e^t:		
	$\text{e}^{-\lambda}\text{e}^{\lambda\text{e}^t}$	B1 **[1]**	
2(iv)	For $\bar{X}$: mean = λ	B1	
	Var = $\frac{\lambda}{n}$	B1 **[2]**	
2(v)	$T = X_1 + X_2 + ... + X_n$		
	By convolution theorem:	M1	Might be implicit in the candidate's work
	mgf of T = (mgf of X)^n = $\text{e}^{-n\lambda}\text{e}^{n\lambda\text{e}^t}$	A1	
	$\bar{X} = \frac{1}{n}T$.	B1	
	By linear transformation result	M1	Might be implicit in the candidate's work
	(with $a = \frac{1}{n},\ b = 0$)	M1	
	mgf of $\bar{X} = \text{e}^{-n\lambda}\text{e}^{n\lambda\text{e}^{\frac{t}{n}}}$	A1 **[6]**	

Qu	Answer	Mark	Comment
Option 2: Generating Functions (continued)			
2(vi)	$Z = \dfrac{\bar{X} - \lambda}{\sqrt{\lambda / n}} = \sqrt{\dfrac{n}{\lambda}}\bar{X} - \sqrt{n\lambda}$	B1	
	again using linear transformation result	M1	
	mgf of Z = $\mathrm{e}^{-\sqrt{n\lambda}t}.\mathrm{e}^{-n\lambda}.\mathrm{e}^{n\lambda \mathrm{e}^{\frac{t\sqrt{\frac{n}{\lambda}}}{n}}}$ as required	A1	
	ln(mgf of *Z*)		
	$= -t\sqrt{n\lambda} - n\lambda + n\lambda\left(1 + \dfrac{t}{\sqrt{n\lambda}} + \dfrac{t^2}{2n\lambda} + \dfrac{t^3}{6(n\lambda)^{\frac{3}{2}}} + \ldots\right)$ cancel cancel	A1	
	$\to \dfrac{t^2}{2}$ as $n \to \infty$ (all other terms are $\mathrm{O}\left(n^{-\frac{1}{2}}\right)$)	A1	
		[5]	
2(vii)	We have mgf of $Z \to \mathrm{e}^{\frac{t^2}{2}}$ which is mgf of N(0,1) $\therefore$ dist of Z must $\to \mathrm{N}(0,1)$	E2	
		[2]	

Qu	Answer	Mark	Comment
Option 3: Inference			
3(i)	Usual test is based on comparing $Z=\frac{\overline{X}-48.6}{\frac{2.4}{\sqrt{n}}}$ with N(0,1)	M1	Likely to be implicit in later work
	We require: $0.03=\text{P}(\text{reject } \mu=48.6 \mid \mu=48.6)$	M1	
	$=\text{P}(Z<k \mid Z\sim \text{N}(0,1))$	M1,M1	
	$=\text{P}(Z<-1.881)$	B1	
	$\Rightarrow$ reject H_0 if $\frac{\overline{x}-48.6}{\frac{2.4}{\sqrt{n}}}<-1.881$	M1	Accept write-down of this for all marks thus far
	i.e. if $\overline{x}<48.6-\frac{4.5144}{\sqrt{n}}$	A1	
	We require: $0.02=\text{P}(\text{accept } \mu=48.6 \mid \mu=45.0)$	M1	
	$=\text{P}\left(\overline{X}>48.6-\frac{4.5144}{\sqrt{n}} \,\middle|\, \overline{X}\sim \text{N}(45.0,\frac{2.4^2}{n})\right)$	M1,M1	Might be implicit
	$=\text{P}\left(\text{N}(0,1)>\frac{3.6-\frac{4.5144}{\sqrt{n}}}{\frac{2.4}{\sqrt{n}}}\right)$	M1	Standardising
	$=\text{P}\left(\text{N}(0,1)>2.054\right)$	A1	
	$\therefore \frac{3.6-\frac{4.5144}{\sqrt{n}}}{\frac{2.4}{\sqrt{n}}}=2.054$	M1	
	$\Rightarrow \sqrt{n}=2.62\dot{3}$	A1	
	$n=6.882$	A1	
	i.e. take $n=7$ (next integer **up**)	E1 **[16]**	
3(ii)	Power function $=\text{P}(\text{reject } H_0 \mid \mu)$	M1	
	$=\text{P}\left(\overline{X}<48.6-\frac{4.5144}{\sqrt{7}}=48.6-1.706=46.894 \,\middle|\, \overline{X}-\text{N}(\mu,\frac{2.4^2}{7})\right)$	M1,A1	Might be implicit
	$=\text{P}\left(Z<\frac{46.894-\mu}{\frac{2.4}{\sqrt{7}}}=51.696-1.102\mu\right)$	A1,A1	1 mark for 51.696, 1 mark for 1.102μ
	With $\mu=45$, this gives $\text{P}(Z<2.106)$	M1,A1	
	= 0.9823, i.e. > 0.98 as required	A1 **[8]**	

Qu	Answer	Mark	Comment
Option 4: Design and Analysis of Experiments			
4(i)	$x_{ij} = \mu + \alpha_i + e_{ij}$	B2	B2 if all correct, B1 if only one error/ommission
	μ is population grand mean for whole experiment	B1	Must be clear reference to
	α_i is population mean amount by which the i'th treatment differs from μ	B1	population
	$e_{ij} \sim \text{ind N}(0, \sigma^2)$	B1	Ind N (or 'uncorrelated')
		B1	Mean 0
		B1	Variance σ^2
		[7]	
4(ii)	$H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_k$	B1	Verbal statements acceptable
	H_1 : The α_i are not all equal	B1	
		[2]	
4(iii)	See working below	M1,A1 A1 M1 M1,A1	For SS For df For MS For MS ratio
	Refer to $F_{4,15}$	A1	
	– overwhelming …	A1	
	… evidence …	A1	
	… that methods are not all the same	A1	
		[10]	
4(iv)	Randomised blocks	B1	
	Recognition of blocking factor	E1,A1	
	Discussion of need for blocking	E1,A1	
		[5]	
			Total: 72

Working for 4(iii):

				Totals
11.2	10.8	10.7	10.1	**42.8**
9.4	9.9	9.6	9.1	**38.0**
9.2	8.6	8.8	8.4	**35.0**
12.1	12.3	12.7	11.9	**49.0**
13.6	12.4	13.1	12.9	**52.0**
				216.8

'Correction Factor' $= \dfrac{(216.8)^2}{20} = 2350.112$

Total SS $= 2403.86 - 2350.112 = 53.748$ with 19df

Between methods SS $= \dfrac{42.8^2}{4} + \ldots + \dfrac{52.0^2}{4} - 2350.112$
$= 2401.46 - 2350.112 = 51.348$ with 4df

Residual SS (by subtraction) $= 53.748 - 51.348 = 2.4$ with 15 df

Source of variation	SS	df	MS	MS ratio
Between methods	51.348	4	12.837	80.23
Residual	2.4	15	0.16	
Total	53.748	19		

AO	Range	Total	Question Number			
			1	**2**	**3**	**4**
1	19-29	27	8	13	2	4
2	19-29	25	13	7	4	1
3	24-34	25	-	-	11	14
4	9-19	12	3	4	2	3
5	4-15	7	-	-	5	2
	Totals	**96**	24	24	24	24

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
DECISION MATHEMATICS 1, D1

4771

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an insert for use in Question 6.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (24 marks)

1 **(a)** Vertices of the graph shown in Fig.1 represent objects.
Some arcs have been drawn to connect vertices representing objects which are the same colour.

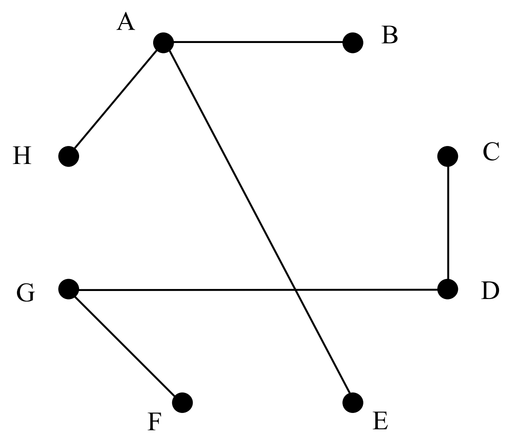


Fig.1

(i) Copy Fig.1 and draw in whichever arcs you can be sure should be added. **[2]**

(ii) How many arcs would be needed in total if you were also told that the objects represented by B and F were the same colour? **[2]**

(b) **(i)** Give two properties that a graph must have for it to be a tree. **[2]**

(ii) Draw three different trees each containing 5 vertices and 4 edges. **[2]**

2 The following six steps define an algorithm:

Step 1: Think of a positive whole number and call it X.
Step 2: Write X out in words (i.e. using letters, not numbers).
Step 3: Let Y be the number of letters used.
Step 4: If $Y = X$ then stop.
Step 5: Replace X by Y.
Step 6: Go to step 2.

(i) Apply the algorithm with $X = 62$. **[4]**

(ii) Show that for all values of X between 1 and 99 the algorithm produces the same answer.
(You may use the fact that, when written out, numbers between 1 and 99 all have twelve or fewer letters.) **[4]**

3 **(i)** Use the matrix form of Prim's algorithm, starting at A, to find a minimum connector for the network defined by the arc weights given in Table 4.

	A	**B**	**C**	**D**	**E**
A	–	12	8	7	9
B	12	–	10	–	9
C	8	10	–	4	5
D	7	–	4	–	3
E	9	9	5	3	–

Table 4 **[4]**

(ii) Draw your minimum connector and give its total weight. **[2]**

(iii) Give the order in which arcs would be included when using Kruskal's algorithm. **[2]**

Section B (48 marks)

4 Claire wants to prepare and eat her breakfast in the minimum time.
The activities involved, their immediate predecessors and their durations are shown in Table 5.

	Activity	Immediate Predecessors	Duration (mins)
F	Fill kettle	–	0.5
I	Put instant coffee in cup	–	0.5
W	Boil water	F	10
G	Grill toast	–	7
D	Dish out cereal	–	0.5
O	Fetch and open milk	–	0.5
M	Make coffee	I, W	0.5
B	Butter toast	G	0.5
E	Eat cereal and milk	D, O	3
T	Eat toast	E, B	5
C	Drink coffee	M, T	3

Table 5

(i) Draw an activity-on-arc network for these activities.
Do not take account of the fact that Claire can do only one thing at a time. **[5]**

(ii) Show on your network the early time and the late time for each event. **[4]**

(iii) Give the critical activities and the minimum time needed for Claire to complete her breakfast, again taking no account of the fact that she can do only one thing at a time. **[2]**

(iv) Activities W and G do not require Claire's attention. For all the other activities Claire can do only one thing at a time.
Starting at 7 am, at what time can Claire actually finish her breakfast, and when would she start eating her cereal and start eating her toast? **[5]**

5 A vet is treating a farm animal. He must provide minimum daily requirements of an antibiotic, a vitamin and a nutrient.
He has two types of medicine available, tablets and liquid.
Table 6 summarises what the medicines contain and the requirements.

	Antibiotic	**Vitamin**	**Nutrient**
Tablets (units per tablet)	3	2	10
Liquid (units per dose)	2	4	50
Daily requirement (units)	18	16	100

Table 6

(i) If x is the number of tablets which the vet prescribes per day, and y is the number of doses of liquid medicine, explain why $3x + 2y$ must not be less than 18.
Draw the inequality $3x + 2y \geq 18$ on a graph, with each axis labelled from 0 to 10. **[5]**

(ii) Construct inequalities in terms of x and y relating to daily vitamin and nutrient requirements. Draw these two inequalities on your graph. **[6]**

The tablets cost £0.38 each and liquid medicine costs £1 per dose. The vet wants to find the cheapest way to treat the animal.

(iii) Solve the linear programming (LP) problem, allowing x and y to take any values. **[2]**

(iv) Solve the problem when x and y must be integers. **[2]**

(v) Which solution should the vet adopt and why? **[1]**

6 *[There is an insert for use in part (v) of this question. It can be found at the back of this paper.]*

During peak periods passengers arrive to buy tickets at a station at intervals modelled by the distribution shown in Table 6.1.

Arrival interval (seconds)	5	10	15	20
Probability	0.7	0.15	0.1	0.05

Table 6.1: Passenger inter-arrival times

The distribution of the time taken to serve a passenger is modelled by the distribution in Table 6.2.

Service time (seconds)	10	15	20	25
Probability	$\frac{1}{3}$	$\frac{5}{12}$	$\frac{1}{6}$	$\frac{1}{12}$

Table 6.2: Passenger service times

(i) Give an efficient rule for using two-digit random numbers beginning with 00 to simulate passenger *inter-arrival* times. **[2]**

(ii) Use random digits from the list below to construct simulated *arrival* times for 10 passengers. (Your first passenger should arrive at the time given by your first inter-arrival time.)

29 76 01 16 56 23 96 86 56 52 29 34 10 44 34 09 **[2]**

(iii) Give an efficient rule for using two-digit random numbers beginning with 00 to simulate passenger *service* times. **[2]**

(iv) Use random digits from the list below to construct simulated *service* times for 10 passengers.

98 74 53 90 43 42 03 13 39 58 22 92 29 36 84 97 **[1]**

The station manager wants to know whether or not it will be sufficient to have two servers operating.

(v) Using Table 6.3 on the insert, simulate the arrival, service and departure of 10 passengers with two servers operating. Assume that both servers are available when the first passenger arrives, and that there is a single queue.
(If both servers are available then server 1 should be chosen in preference to server 2.) **[5]**

(vi) Calculate for your 10 passengers the mean time they wait before being served.
Find also the greatest length of queue for your 10 passengers. **[3]**

(vii) Give your advice to the manager. **[1]**

Table 6.3 [Insert for Question 6(v)]

Passenger number	Arrival time	Server	Start time (server 1)	Start time (server 2)	Service time	End time (server 1)	End time (server 2)
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

Spare copy of the table for question **6(v)**. (You do not need to use this.)

Passenger number	Arrival time	Server	Start time (server 1)	Start time (server 2)	Service time	End time (server 1)	End time (server 2)
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

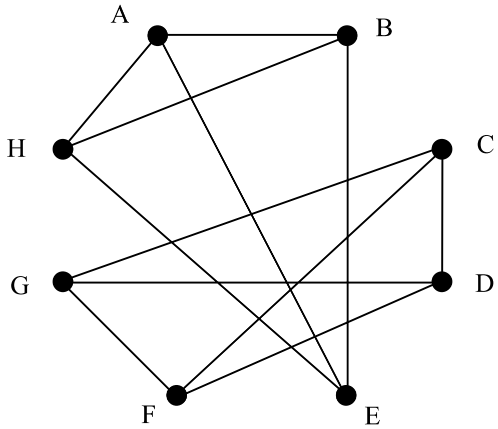
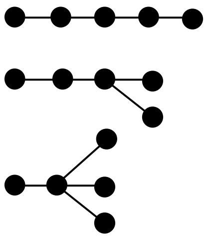
Oxford Cambridge and RSA Examinations

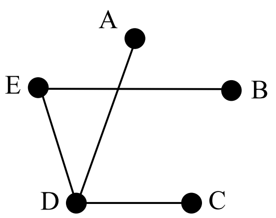
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

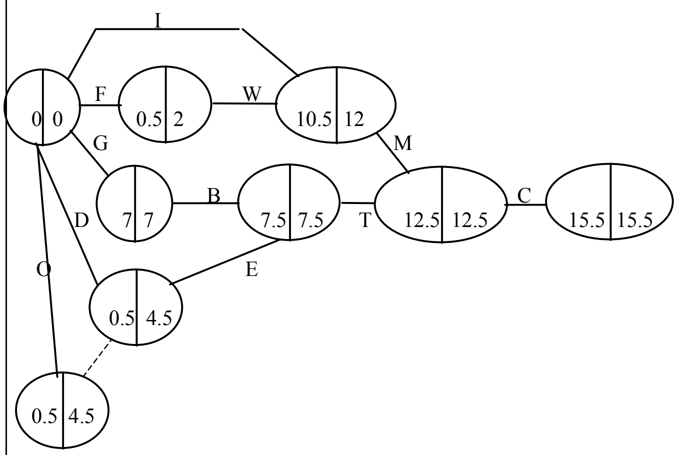
MEI STRUCTURED MATHEMATICS
DECISION MATHEMATICS 1, D1

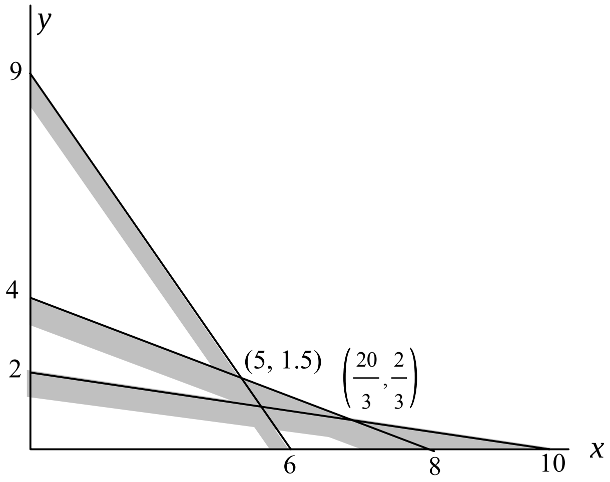
4771

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1(a)(i)		M1 A1 **[2]**	
1(a)(ii)	${}^8C_2 = 28$	M1,A1 **[2]**	
1(b)(i)	connected no cycles (or n–1 arcs)	B1 B1 **[2]**	
1(b)(ii)	e.g. 	M1 A1 **[2]**	
2(i)	sixty two 8 eight 5 five 4 four	M1 A1 A1 A1 **[4]**	 To ‘eight’ To ‘five’ To ‘four’
2(ii)	$\{1, 2, 6, 10\} \rightarrow 3, 5, 4$ $\{4, 5, 9\} \rightarrow 4$ $\{3, 7, 8\} \rightarrow 5, 4$ $\{11, 12\} \rightarrow 6, 3, 5, 4$	M1 A1 A1 A1 **[4]**	Enumeration 4, 5, 9, 3, 7, 8 1, 2, 6, 10 11, 12

Qu	Answer	Mark	Comment
Section A (continued)			
3(i)	$\begin{array}{cccccc} & \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} \\ \mathbf{A} & \cancel{-} & \cancel{12} & \cancel{8} & \cancel{7} & \cancel{9} \\ \mathbf{B} & 12 & - & 10 & - & 9 \\ \mathbf{C} & \cancel{8} & \cancel{10} & \cancel{-} & 4 & \cancel{5} \\ \mathbf{D} & 7 & \cancel{-} & \cancel{4} & \cancel{-} & \cancel{3} \\ \mathbf{E} & \cancel{9} & \cancel{9} & \cancel{5} & 3 & \cancel{-} \end{array}$	M1 A1 A1 A1 **[4]**	Matrix form of Prim Selecting arcs (circled elements) Deleting rows (except for circled elements) Order of inclusion
3(ii)	 Total weight = 23	B1 B1 **[2]**	Diagram Total weight
3(iii)	DE CD AD BE	M1,A1 **[2]**	
			Section A Total: 24

Qu	Answer	Mark	Comment
Section B			
4(i) & (ii)		M1 A1 A1 M1 A1 **[5]** M1,A1 M1,A1 **[4]**	Sca, activity-on-arc I, F, W, M G, B, T, C Dummy D, O, E Forward pass Backward pass
4(iii)	G, B, T, C 15.5 mins	B1,B1 **[2]**	
4(iv)	Can finish at 0716. Based on the schedule below she would start eating her cereal at 0702, and start eating her toast at 0707.5 0700 Put toast on to grill Fill kettle 0700.5 Put kettle on to boil Dish out cereal 0701 Put coffee in cup 0701.5 Fetch and open milk 0702 Eat cereal 0707 Butter toast 0707.5 Eat toast 0712.5 Make coffee 0713 Drink coffee	M1 A1 A1 A1 B1 **[5]**	Scheduling Cereal Toast (cao for 0716)

Qu	Answer	Mark	Comment
Section B (continued)			
5(i)	$3x + 2y =$ units of antibiotic provided Must not be less than the 18 units needed	B1 B1	
		B1 B1 B1 **[5]**	Axes labelled and scaled Antibiotic line Shading
5(ii)	$x + 2y \geq 8$ $x + 5y \geq 10$	M1 A1 A1	(Simplification not needed)
		B1 B1 B1 **[6]**	Vitamin line Nutrient line Shading
5(iii)	Finding solution to LP (£3.20 at $\left(\frac{20}{3}, \frac{2}{3}\right)$)	M1,A1 **[2]**	
5(iv)	£3.28 for 6 tablets and 1 dose	B1,B1 **[2]**	
5(v)	The integer solution, thirds of tablets being too difficult to make the saving worthwhile. (Or £3.26 for 7 tablets and 0.6 of a dose!)	B1 **[1]**	

Qu	Answer	Mark	Comment
Section B (continued)			
6(i)	00-69→5; 70-84→10; 85-94→15; 95-99→20	M1,A1 **[2]**	
6(ii)	intervals 5 10 5 5 5 5 20 15 5 5 times 5 15 20 25 30 35 55 70 75 80	M1 A1 **[2]**	
6(iii)	00-31→10; 32-71→15; 72-87→20; 88-95→25 96, 97, 98, 99 → reject	M1 A1 **[2]**	Reject some
6(iv)	times 20 15 25 15 15 10 10 15 15 10	B1 **[1]**	
6(v)	(see table below)	B1 M1 A1 M1 A1 **[5]**	Allocating to server Service starts Service ends
6(vi)	$\frac{(5+5+15+15+5+5)}{10} = 5$ seconds Longest queue is 2.	M1,A1 B1 **[3]**	
6(vii)	Looks OK, but more repetitions needed.	B1 **[1]**	
			Section B Total: 48
			Total: 72

6(v):

pass.	arrives	server	start 1	start 2	s time	end 1	end 2
1	5	1	5		20	25	
2	15	2		15	15		30
3	20	1	25		25	50	
4	25	2		30	15		45
5	30	2		45	15		60
6	35	1	50		10	60	
7	55	1	60		10	70	
8	70	1	70		15	85	
9	75	2		75	15		90
10	80	1	85		10	95	

AO	Range	Total	Question Number					
			1	2	3	4	5	6
1	14-22	20	1	4	3	4	6	2
2	14-22	20	5	2	3	2	5	3
3	18-36	19	-	1	-	9	4	5
4	7-15	8	2	1	2	1	1	1
5	3-11	5	-	-	-	-	-	5
	Totals	**72**	8	8	8	16	16	16

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

DECISION MATHEMATICS 2, D2

4772

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There are inserts for use in Questions 2 and 3.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

1 **(a)** **(i)** Draw the switching circuit representing $(a \wedge (b \vee c)) \vee (\sim a \wedge b \wedge c)$. **[4]**

Fig.1 shows a circuit for a voting machine for 3 people, A, B and C.
Person A, voting for a proposal, is represented by a.
Person A, voting against the proposal, is represented by $\sim a$.

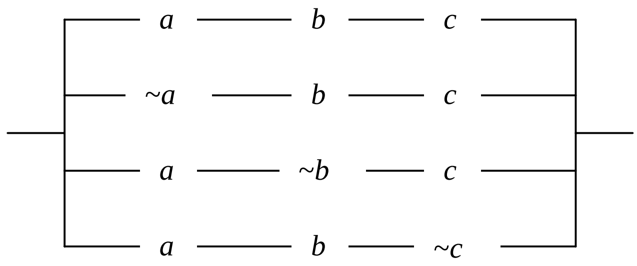


Fig.1

(ii) Show that the expression in part **(i)** is equivalent to the voting machine in Fig.1. **[3]**

(iii) Draw an equivalent circuit in which the symbols a, b and c are used, twice each, and in which the symbol ~ is not used. **[2]**

(b) Let s represent the proposition 'There is snow'.
Let n represent the proposition 'There is a north wind'.

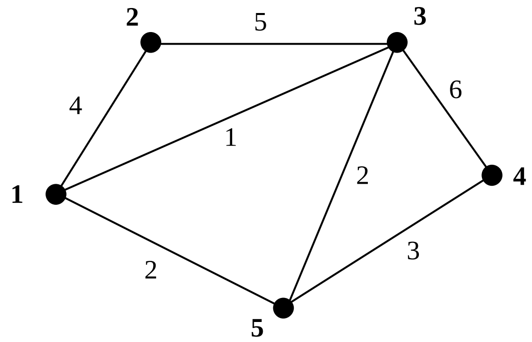
You are given that if there is no snow then there is no north wind.

(i) Express what you are given in terms of s, n and logical symbols. **[3]**

(ii) You are also given that there is a north wind.
Use a truth table to prove that there is snow **[4]**

2 *[There is an insert for use in this question. It can be found at the back of this paper.]*

The weights on the network represent distances.



(i) The insert shows the initial tables and the results of iterations 1, 2, 4 and 5 when Floyd's algorithm is applied to the network.

(*A*) Complete the two tables for iteration 3. **[6]**

(*B*) Use the final route table to give the shortest route from vertex 4 to vertex 2. **[1]**

(*C*) Use the final distance table to draw a complete network with weights representing the shortest distances between vertices. **[2]**

(ii) Using the complete network of shortest distances, find a lower bound for the solution to the travelling salesperson problem by deleting vertex **1** and its arcs, and by finding the length of a minimum connector for the remainder.
(You may find the minimum connector by inspection.) **[3]**

(iii) Use the nearest neighbour algorithm, starting at vertex 1, to produce a Hamilton cycle in the complete network. Give the length of your cycle. **[3]**

(iv) Interpret your Hamilton cycle in part **(iii)** in terms of the original network. **[2]**

3 *[There is an insert for use in this question. It can be found at the back of this paper.]*

One of three similar types of new car, A, B or C, is to be purchased. The decision is to be made on the basis of annual service and repair costs.
For each car a warranty can be purchased which insures against unexpected costs. Otherwise a chance can be taken on whether the particular car purchased turns out to be reliable or unreliable.

	Annual costs (£)			**Probabilities**	
	with extended warranty	**reliable**	**unreliable**	**reliable**	**unreliable**
A	1000	750	2100	$\frac{2}{3}$	$\frac{1}{3}$
B	1100	800	2000	$\frac{3}{4}$	$\frac{1}{4}$
C	1100	810	1800	$\frac{5}{6}$	$\frac{1}{6}$

(i) Complete the decision tree on Fig.3.1 on the insert, and give the best decision, together with its EMV. **[7]**

An alternative is to buy a cheaper second-hand car. Annual service and repair costs of second-hand cars are higher, and warranties are more expensive.
A free independent inspection of one car can be arranged. Approval by the inspector gives a good indication of the car being reliable. If the report is not favourable then a warranty will be purchased, fixing costs at £1150 per year.
The relevant probabilities are summarised on the decision tree in Fig.3.2 on the insert.

(ii) Complete the EMV calculations on the decision tree in Fig.3.2 on the insert and give the best course of action and its EMV. **[8]**

(iii) For each type of car give the value of having an inspection. **[2]**

(iv) The cost of a warranty increases.
To what value would the fixed cost of £1150 per year have to rise to change the decision in part **(ii)**? **[2]**

4 A manufacturer of garden furniture produces chairs, round tables and square tables.
There must be at least 4 chairs produced for each table. At least 100 round tables and 80 square tables must be produced.
The costs of manufacture are £4 per chair, £10 per round table and £8 per square table.

(i) Using x, y and z to represent the numbers of chairs, round tables and square tables produced respectively, formulate as a linear program the problem of deciding how many of each item to produce at minimum cost. **[5]**

(ii) The initial tableau and the final tableau for a two-stage simplex solution to the LP are shown below.

Initial tableau

Q	C	x	y	z	$s1$	$s2$	$s3$	$a2$	$a3$	RHS
1	0	0	1	1	0	–1	–1	0	0	180
0	1	–4	–10	–8	0	0	0	0	0	0
0	0	–1	4	4	1	0	0	0	0	0
0	0	0	1	0	0	–1	0	1	0	100
0	0	0	0	1	0	0	–1	0	1	80

Final tableau

Q	C	x	y	z	$s1$	$s2$	$s3$	$a2$	$a3$	RHS
1	0	0	0	0	0	0	0	–1	–1	0
0	1	0	0	0	–4	–26	–24	26	24	4520
0	0	1	0	0	–1	–4	–4	4	4	720
0	0	0	1	0	0	–1	0	1	0	100
0	0	0	0	1	0	0	–1	0	1	80

Explain the structure of the initial tableau, including the variables and the two objective functions.
Interpret the final tableau. **[10]**

(iii) Chairs are sold to retailers at £8 each, round tables at £15 each and square tables at £12 each.
Write down an expression in terms of x, y and z for the total profit. **[1]**

4 **(iv)** The manufacturer wishes to maximise the profit, P, while spending no more than £5000 on manufacturing costs.
You are given that the tableau shown below takes the solution represented by the final tableau in part **(ii)** as the starting point for this problem.
Apply the simplex algorithm to this tableau to find the most profitable production plan, pivoting on the ***s*1** column.

P	***x***	***y***	***z***	***s*1**	***s*2**	***s*3**	***s*4**	**RHS**
1	0	0	0	–4	–21	–20	0	3700
0	1	0	0	–1	–4	–4	0	720
0	0	1	0	0	–1	0	0	100
0	0	0	1	0	0	–1	0	80
0	0	0	0	4	26	24	1	480

[4]

Fig. 2.1 [Insert for Question 2(i)]

	1	2	3	4	5
1	∞	4	1	∞	2
2	4	∞	5	∞	∞
3	1	5	∞	6	2
4	∞	∞	6	∞	3
5	2	∞	2	3	∞

	1	2	3	4	5
1	1	2	3	4	5
2	1	2	3	4	5
3	1	2	3	4	5
4	1	2	3	4	5
5	1	2	3	4	5

	1	2	3	4	5
1	∞	4	1	∞	2
2	4	8	5	∞	6
3	1	5	2	6	2
4	∞	∞	6	∞	3
5	2	6	2	3	4

	1	2	3	4	5
1	1	2	3	4	5
2	1	1	3	4	1
3	1	2	1	4	5
4	1	2	3	4	5
5	1	1	3	4	1

	1	2	3	4	5
1	8	4	1	∞	2
2	4	8	5	∞	6
3	1	5	2	6	2
4	∞	∞	6	∞	3
5	2	6	2	3	4

	1	2	3	4	5
1	2	2	3	4	5
2	1	1	3	4	1
3	1	2	1	4	5
4	1	2	3	4	5
5	1	1	3	4	1

	1	2	3	4	5
1					
2					
3					
4					
5					

	1	2	3	4	5
1					
2					
3					
4					
5					

	1	2	3	4	5
1	2	4	1	7	2
2	4	8	5	11	6
3	1	5	2	6	2
4	7	11	6	12	3
5	2	6	2	3	4

	1	2	3	4	5
1	3	2	3	3	5
2	1	1	3	3	1
3	1	2	1	4	5
4	3	3	3	3	5
5	1	1	3	4	1

	1	2	3	4	5
1	2	4	1	5	2
2	4	8	5	9	6
3	1	5	2	5	2
4	5	9	5	6	3
5	2	6	2	3	4

	1	2	3	4	5
1	3	2	3	5	5
2	1	1	3	1	1
3	1	2	1	5	5
4	5	5	5	5	5
5	1	1	3	4	1

Fig. 3.1 [Insert for Question 3(i)]

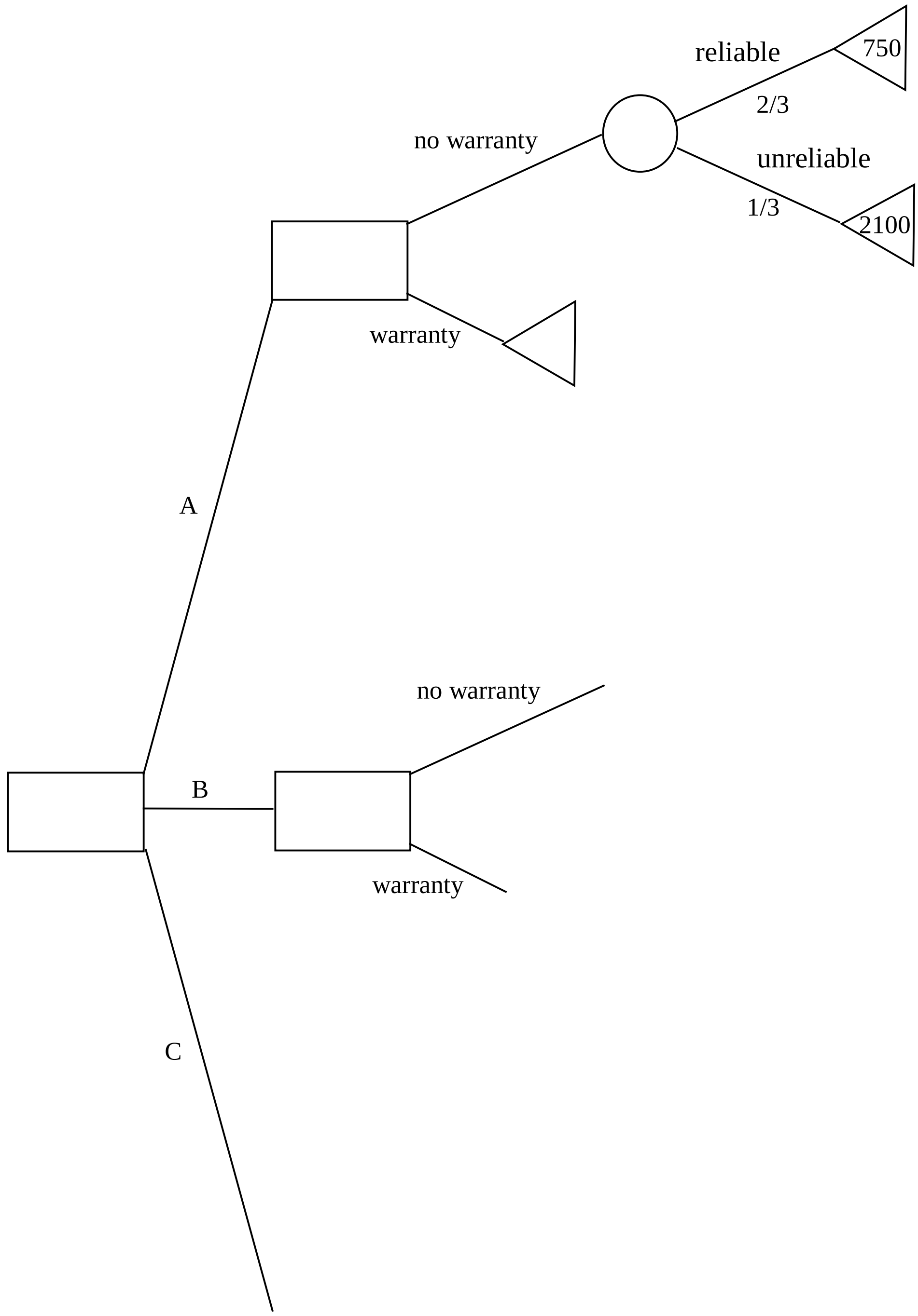
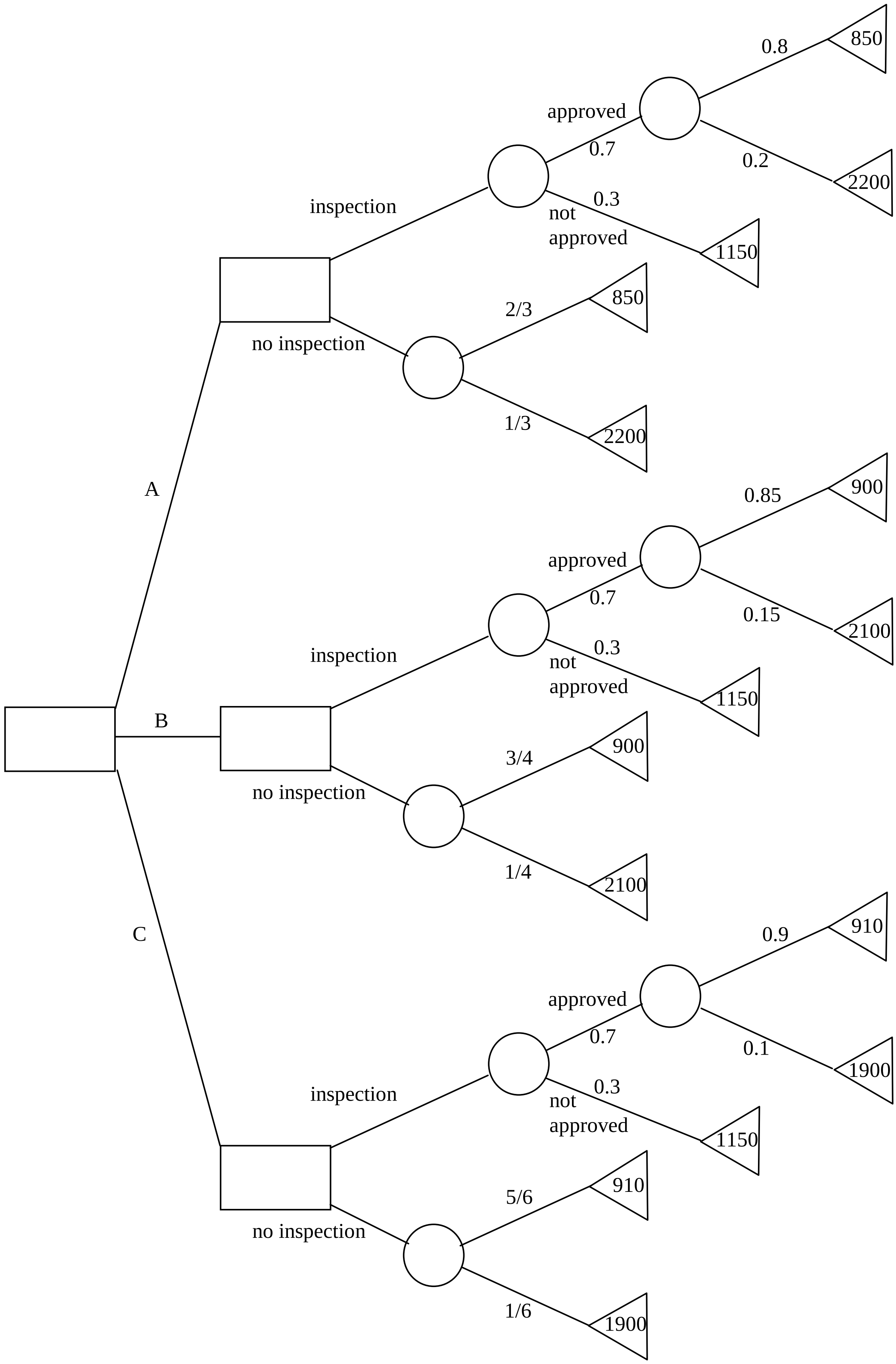


Fig. 3.2 [Insert for Question 3(ii)]



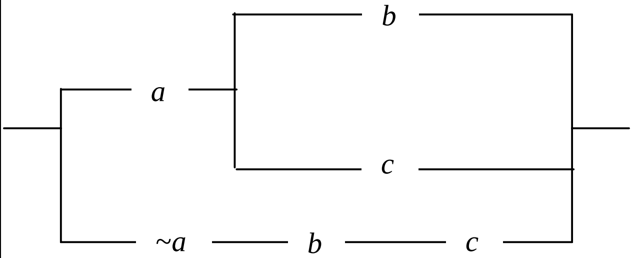
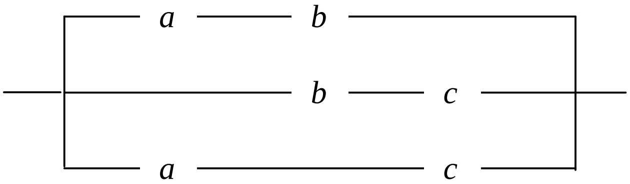
Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
DECISION MATHEMATICS 2, D2

4772

MARK SCHEME

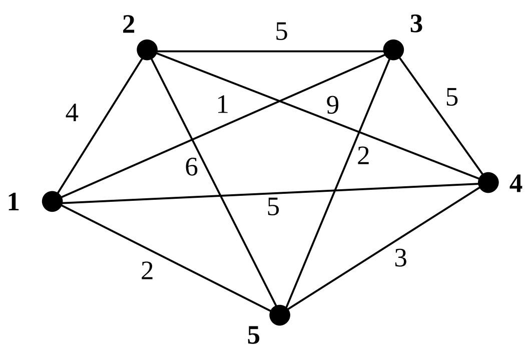
Qu	Answer	Mark	Comment
1(a)(i)		M1 A2 A1 **[4]**	Switching circuit $a \wedge (b \vee c)$ $\sim a \wedge b \wedge c$
1(a)(ii)	Argument about majority voting, or tables of outcomes/truth tables, or Boolean algebra	M1 A2 **[3]**	 1 for circuit and 1 for expression
1(a)(iii)		M1 A1 **[2]**	
1(b)(i)	$\sim s \Rightarrow \sim n$	M1 A1 A1 **[3]**	$\Rightarrow$ $2 \times \sim$ All correct
1(b)(ii)	$(\sim s \Rightarrow \sim n) \wedge n \Rightarrow s$ 1 0 1 1 0 0 0 1 0 1 0 0 0 1 0 1 1 0 0 1 1 1 0 0 0 1 1 0 1 1 0 1 1 1 1 1	M1 A3 **[4]**	4 rows (-1 each error)

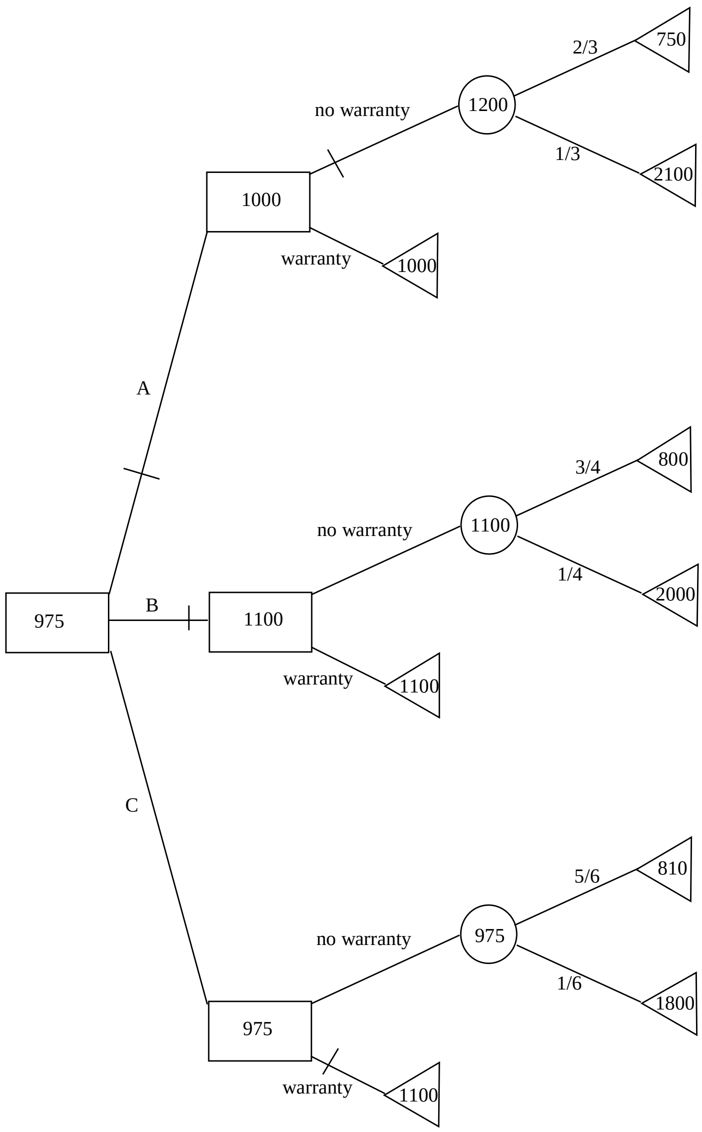
<table>
<tr><th>Qu</th><th>Answer</th><th>Mark</th><th>Comment</th></tr>
<tr><td>2(i)(A)</td><td>
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><th>1</th><td>2</td><td>4</td><td>1</td><td>7</td><td>2</td></tr>
<tr><th>2</th><td>4</td><td>8</td><td>5</td><td>11</td><td>6</td></tr>
<tr><th>3</th><td>1</td><td>5</td><td>2</td><td>6</td><td>2</td></tr>
<tr><th>4</th><td>7</td><td>11</td><td>6</td><td>12</td><td>3</td></tr>
<tr><th>5</th><td>2</td><td>6</td><td>2</td><td>3</td><td>4</td></tr>
</table>

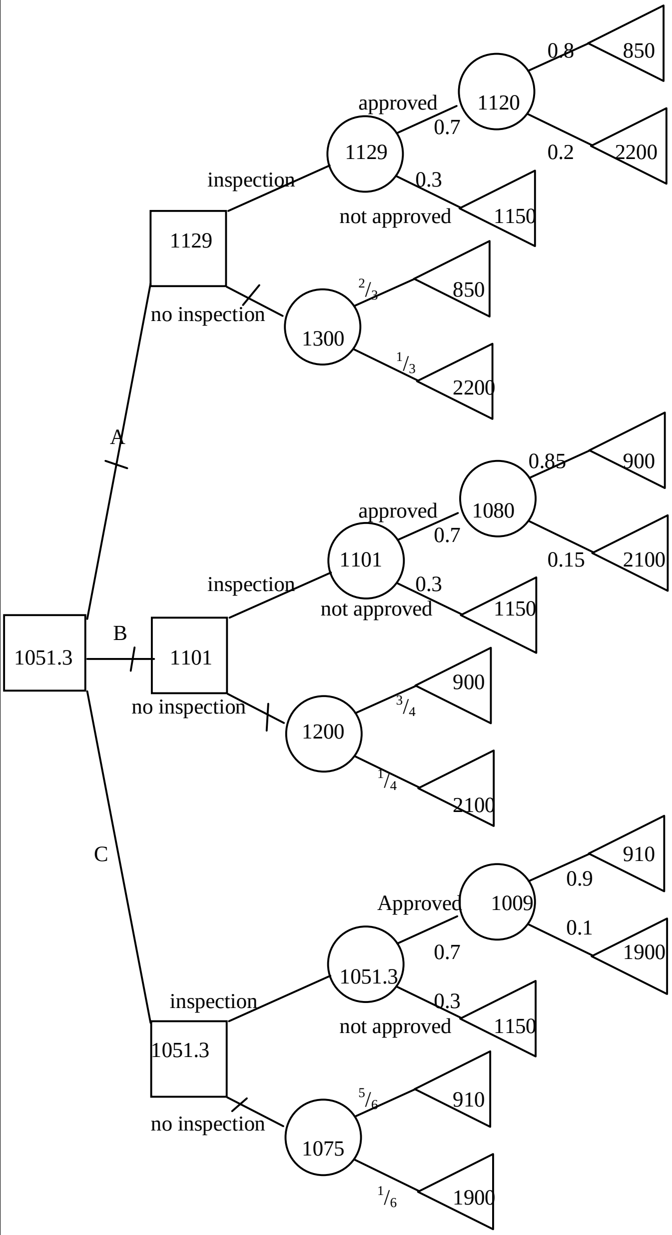
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><th>1</th><td>3</td><td>2</td><td>3</td><td>3</td><td>5</td></tr>
<tr><th>2</th><td>1</td><td>1</td><td>3</td><td>3</td><td>1</td></tr>
<tr><th>3</th><td>1</td><td>2</td><td>1</td><td>4</td><td>5</td></tr>
<tr><th>4</th><td>3</td><td>3</td><td>3</td><td>3</td><td>5</td></tr>
<tr><th>5</th><td>1</td><td>1</td><td>3</td><td>4</td><td>1</td></tr>
</table>
</td><td>M1
A2

M1
A2

[6]</td><td>(-1 each error)

(-1 each error)</td></tr>
<tr><td>2(i)(B)</td><td>$4 \to 5 \to 1 \to 2$</td><td>B1
[1]</td><td></td></tr>
<tr><td>2(i)(C)</td><td>
</td><td>M1
A1
[2]</td><td>Complete</td></tr>
<tr><td>2(ii)</td><td>$(5+2+3)+(1+2)=13$</td><td>M1
A1
A1
[3]</td><td>
$(5+2+3)$
$(1+2)$</td></tr>
<tr><td>2(iii)</td><td>$1 \to 3 \to 5 \to 4 \to 2 \to 1$ length = 19</td><td>M1
A1,A1
[3]</td><td></td></tr>
<tr><td>2(iv)</td><td>$1 \to 3 \to 5 \to 4 \to 5 \to 1 \to 2 \to 1$</td><td>M1,A1
[2]</td><td></td></tr>
</table>

Qu	Answer	Mark	Comment
3(i)		B1	Calculations
		M1 A1	B branch Calculations
		M1 A1	C branch Calculations
	Choose a car of type C and do not buy a warranty. EMV = 975	B1 B1 **[7]**	

Qu	Answer	Mark	Comment
3(ii)		M1 A2 M1 A2	Handling chance nodes (-1 each error) Handling decision nodes (-1 each error)
	Choose a second-hand car of type C and have it inspected EMV = £1051.30	B1 B1 **[8]**	
3(iii)	A: £171 B: £99 C: £23.70	M1,A1 **[2]**	
3(iv)	Require x st 0.7*1009 + 0.3*x=1075 giving x=1229	M1,A1 **[2]**	

Qu	Answer	Mark	Comment
4(i)	Min $4x+10y+8z$ s.t. $x \ge 4(y+z)$ $y \ge 100$ $z \ge 80$	M1 A1 A1 A1 A1 **[5]**	
4(ii)	First constraint has been written as $-x+4y+4z \le 0$ using slack s1 Second constraint is $y \ge 100$ using surplus s2 and artificial a2 Third constraint is $z \ge 80$ using surplus s3 and artificial a3 Q is sum of artificial variables $= a2 + a3$ Q row has been rewritten using $y - s2 + a2 = 100$ $z - s3 + a3 = 80$ C is cost $= 4x+10y+8z$ Final tableau shows $x = 720$, $y = 100$ and $z = 80$, with cost = 4520 This is feasible since Q = 0 (it is also optimal since there are no non-artificial positive numbers in the C row)	B1 B1 B1 B1 B1 B1 M1,A1 B1 B1 **[10]**	
4(iii)	$4x+5y+4z$	B1 **[1]**	
4(iv)	(tableau below) Produce 840 chairs, 100 round tables and 80 square tables	M1 A2 B1 **[4]**	(-1 each error)
			Total: 72

P	x	y	z	S1	S2	S3	S4	RHS
1	0	0	0	0	5	4	1	4180
0	1	0	0	0	2.5	2	0.25	840
0	0	1	0	0	-1	0	0	100
0	0	0	1	0	0	-1	0	80
0	0	0	0	1	6.5	6	0.25	120

AO	Range	Total	Question Number			
			1	2	3	4
1	14-22	19	4	6	5	4
2	14-22	20	5	5	7	3
3	18-26	19	4	2	5	8
4	7-15	8	3	2	2	1
5	3-11	6	-	2	-	4
	Totals	**72**	16	17	19	20

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

DECISION MATHEMATICS COMPUTATION, DC

4773

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- There is an insert for use in Question 2.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.
- You **may** use a graphical or scientific calculator in this paper.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.

1 A drug therapy involves administering 200 units of a drug to the patient at time $t = 0$. The drug will then be slowly excreted. One day later, at time $t = 1$, a blood sample is taken and sent to the laboratory, so that in another two day's time, at time $t = 3$, it will be known how much drug remained in the patient at time $t = 1$. This is repeated at time $t = 2$ and subsequently.

At time $t = 3$ the results of the first test are used to calculate a top-up dose of the drug. The top-up dose at time $t = n + 2$, if one is required, is the difference between 200 units and the amount of drug in the patient's body at time $t = n$. No top-up dose is given at time $t = n + 2$ if the amount of drug present at time $t = n$ is 200 units or more.

Suppose that during the course of a day patient X excretes 25% of the drug that was in his body at the beginning of the day. Let x_n be the amount of drug in patient X at time $t = n$.

(i) Explain why
$x_{n+2} = 0.75x_{n+1} + \max((200 - x_n), 0)$ for $n = 1, 2, \ldots,$
where $x_1 = 150$ and $x_2 = 112.5$ **[3]**

(ii) Create a spreadsheet in which column A represents time in days, and in which the entries are the numbers 1, 2, …, 20, 21. Column B should contain the amount of drug in patient X at the corresponding time. (You do not need to print out your spreadsheet until you have finished the question.) **[2]**

Patient X's doctor is worried about the fluctuating level of the drug that would be created in such a patient by the therapy. She wonders if it would help to administer a top-up dose which is the difference between 250 units and the amount of drug in the patient's body at time $t = n$ (with no top-up if the amount present at time $t = n$ is 250 units or more).

(iii) Investigate the effects of this revised therapy by calculating revised drug levels in column C of your spreadsheet. Briefly describe the effect of the change. **[3]**

As another alternative the doctor wonders whether it might be worth trying a top-up dose which is a half of the difference between 200 units and the amount of drug in the patient's body at time $t = n$, if that amount is 200 units or more.

(iv) Investigate the effects of this therapy by calculating revised drug levels in column D of your spreadsheet. Describe what would happen if this therapy was used. **[3]**

The laboratory installs new equipment which enables the results to be returned in one day instead of in two days. Thus, the therapy is described by the recurrence relation:

$$x_{n+1} = 0.75x_n + \max((200 - x_n), 0) \text{ for } n = 1, 2, \ldots, \text{ with } x_1 = 150$$

(v) Given that x_n is always less than 200, solve this recurrence relation to find x_n in terms of n, and describe what happens under the therapy now. **[4]**

(vi) Use your spreadsheet to compare your results in part **(v)** to the results obtained by dispensing with the blood tests and administering a constant top-up dose of 40 units, starting at time $t = 1$. Print out your completed spreadsheet. **[3]**

2 *[There are inserts for use in parts (ii) and (iii) of this question. They can be found at the back of this paper.]*

Fig.2.1 represents a directed network of connected pipes together with weights representing their capacities.

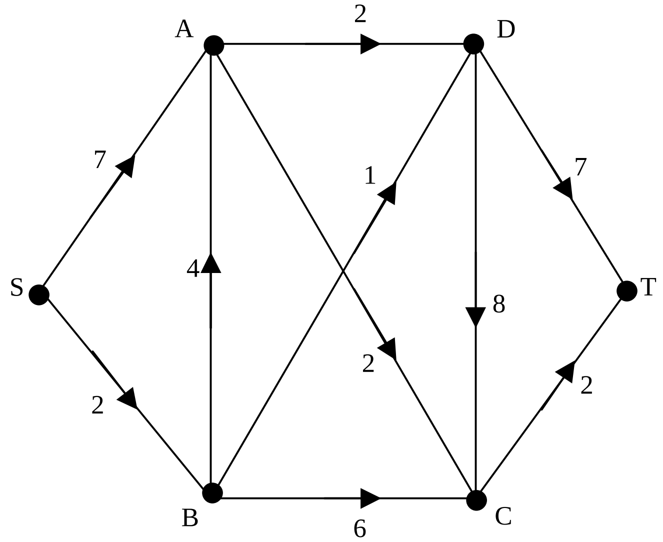


Fig.2.1

(i) The maximum flow from S to T is 5 units. Give a cut with this capacity. **[1]**

A flow of 2 units is established along SBADT and a flow of 2 units along SACT.

(ii) Label these flows, together with potential flows and potential backflows, on Fig.2.2 on the insert. **[2]**

(iii) Give a single flow-augmenting path that will achieve a total flow of 5 units.
Mark the resulting flow along each pipe on Fig.2.3 on the insert. **[2]**

(iv) Construct a linear programming model to find the maximum flow through the network, using variables such as SA to represent the flow along the pipe from vertex S to vertex A.
Use your linear programming package to solve the problem and include a copy of the printout. **[4]**

(v) The network also models another problem in which the weights now represent distances in the indicated directions.
Change your LP formulation so that it finds the shortest distance from S to T.
Run your LP, include a copy of the printout, and interpret the solution. **[5]**

(vi) The arc AD is now changed to be undirected with a weight of 2.
Why does this not affect your answer to part **(iv)**?
Change your LP in part **(v)** and use it to find the new shortest route from S to T. **[4]**

3 An island has three electricity power stations. Their maximum power outputs are 4, 5 and 7 MW (megawatts) respectively. Each can be operated at any power output up to its maximum. Their respective hourly costs are 0.75, 0.70 and 0.73 monetary units per MW.

(i) Explain why the following linear program will find the best way of fulfilling an hourly demand for 6.5 MW.

minimise $3x_1 + 3.5x_2 + 5.11x_3$

subject to Demand = 6.5

$4x_1 + 5x_2 + 7x_3$ – Demand = 0

$0 \le x_1 \le 1$

$0 \le x_2 \le 1$

$0 \le x_3 \le 1$ **[2]**

(ii) Use your linear programming package to solve the problem.
Include a printout of the solution and interpret that solution. **[3]**

(iii) Find the best solution to satisfy an hourly demand of 4.7 MW. **[2]**

(iv) A larger island has 10 power stations, with maximum power outputs and hourly costs per MW as follows.

Station number	1	2	3	4	5	6	7	8	9	10
Max power (MW)	4	5	7	6	4	3	8	4.5	6.2	9
Hourly costs per MW	0.75	0.70	0.73	0.72	0.76	0.69	0.77	0.74	0.76	0.73

Use your linear programming package to find the cheapest way to satisfy an hourly demand for 38.2 MW. **[4]**

(v) A new power station is to be constructed on the larger island. This will cost 4.7 units per hour to run, plus an hourly cost of 0.24 units per MW. Its capacity will be 10 MW. Incorporate this power station into your electricity supply model for the larger island and find the best solution for an hourly demand of 38.2 MW. **[6]**

(vi) By using your LP model or otherwise, find the minimum hourly demand for power for which it is worth using the new station.
You are given that this minimum number of MW is an integer. **[1]**

4 **(i)** Into the first row of the first column of a spreadsheet enter a formula to give a uniformly distributed random number between 0 and 1.
Repeat for the first row of the second column.

In the first row of the third column of your spreadsheet enter a formula to give the sum of the squares of your two random numbers.

In the first row of the fourth column enter a formula to give the result 1 if the sum of the squares is less than 1 and 0 otherwise.

Print out your formulae. **[3]**

(ii) Copy down your four columns for 1000 rows.

Create a cell containing 0.004 times the sum of the entries in the fourth column.

Print out the formula for this cell and its value. **[2]**

(iii) The value which you printed out in part **(ii)** should be a simulated estimate of π .
By regarding your two random numbers as the x- and y- coordinates of a point in the plane, explain why this is. **[3]**

(iv) Repeat your simulation 12 times by using the recalculation facility of your spreadsheet, recording the 12 simulated values of π .

Find the mean and standard deviation of your simulated values. **[3]**

(v) Use the standard deviation which you computed in part **(iv)** to compute an estimate of the number of times which you need to repeat the simulation so that you can be confident that the mean value of your simulated values of π is correct to within 0.0005. **[4]**

(vi) An alternative method to simulate a value of π uses three uniformly distributed random variables between 0 and 1.
The sum of the squares of these are added, and the result is compared to 1.
This is repeated 1000 times, and the number of results less than 1 is multiplied by 0.006.

Build a spreadsheet to simulate an estimate of π using this method.

Use it to produce 12 estimates of π .

Investigate whether or not the method seems to improve on the earlier method which used only 2 random variables. **[3]**

Fig. 2.2 [Insert for question 2(ii)]

Key:

capacity	Forward potential
flow	Backward potential

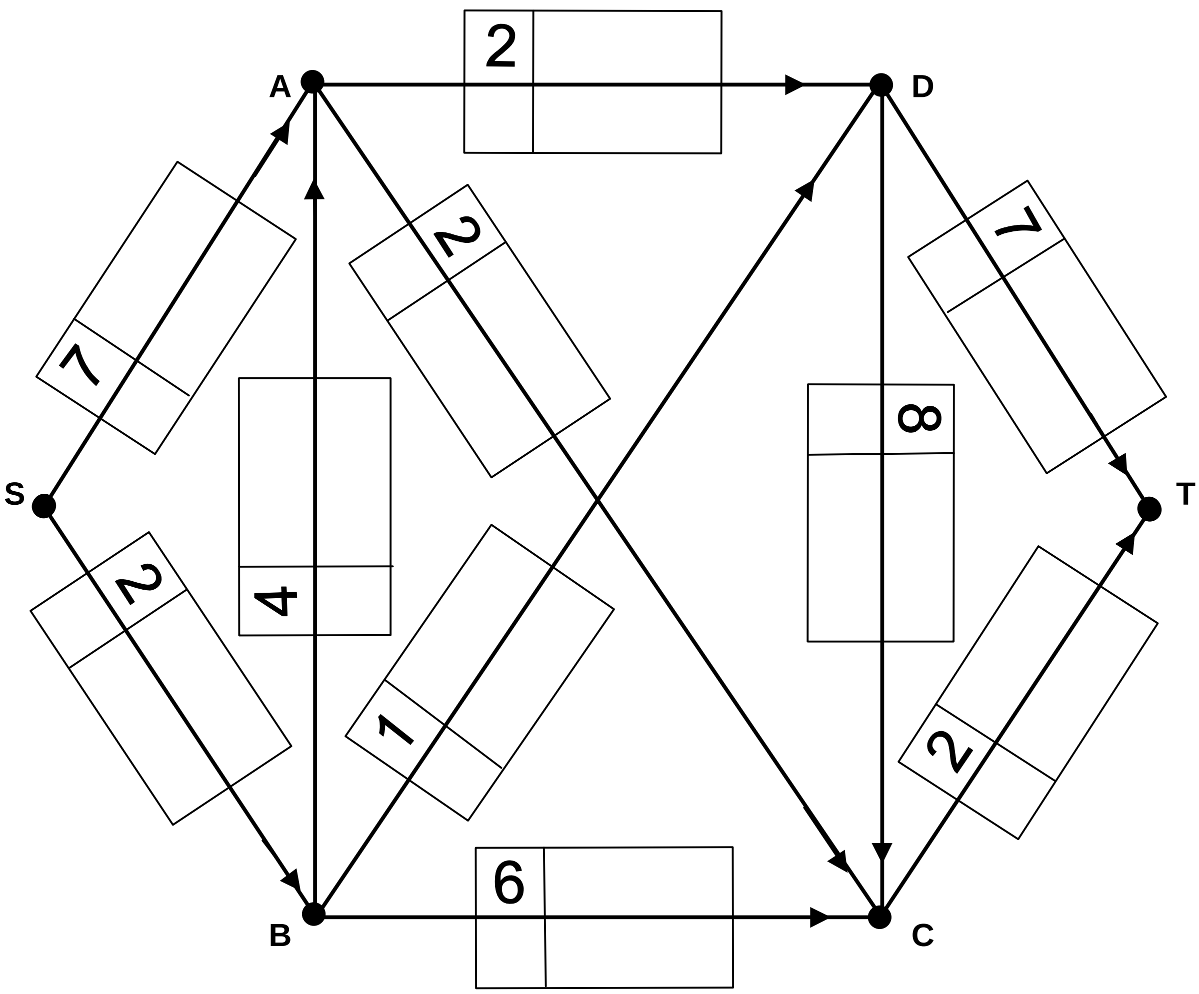
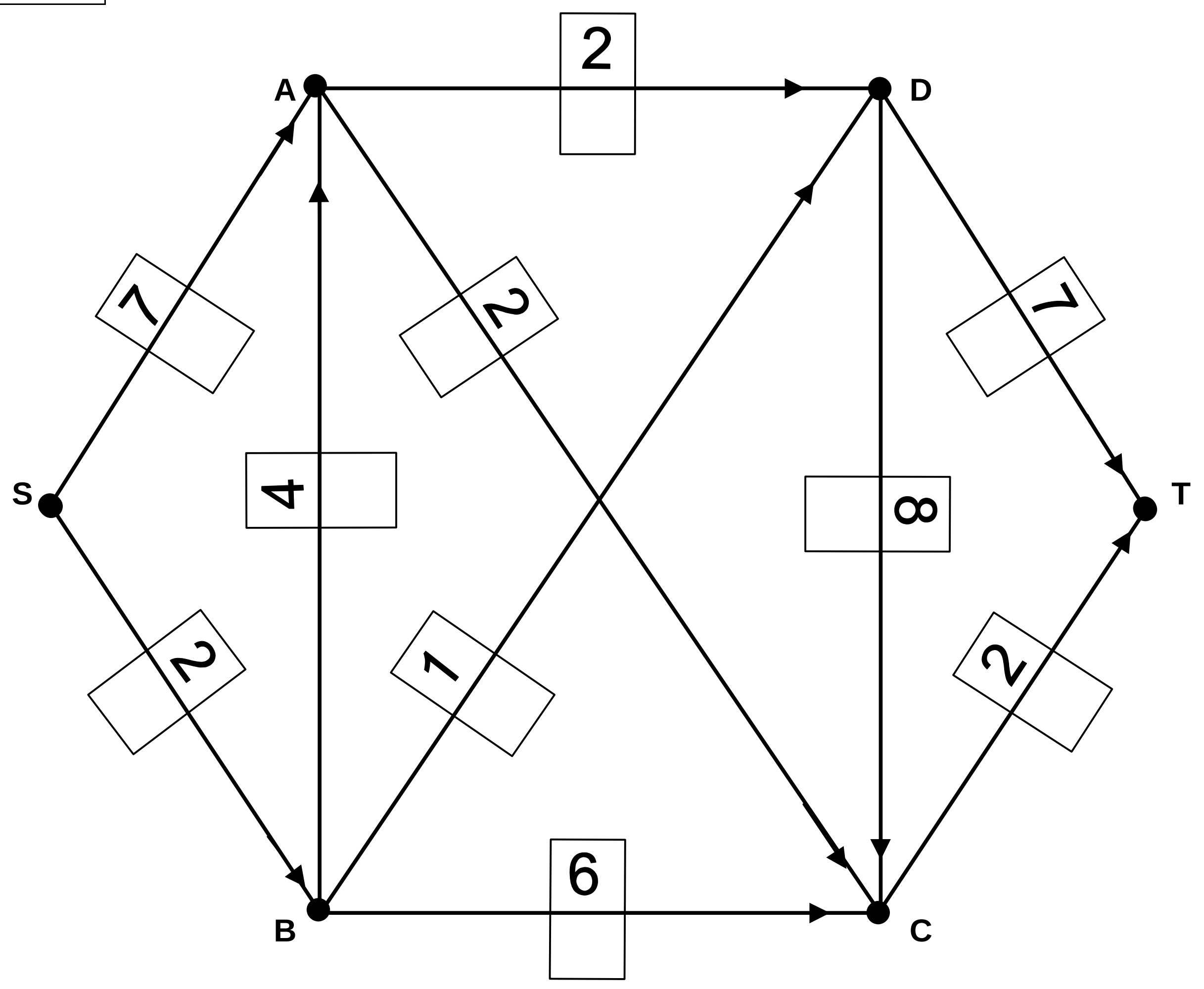


Fig. 2.3 [Insert for question 2(iii)]

Key:

capacity
flow



Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

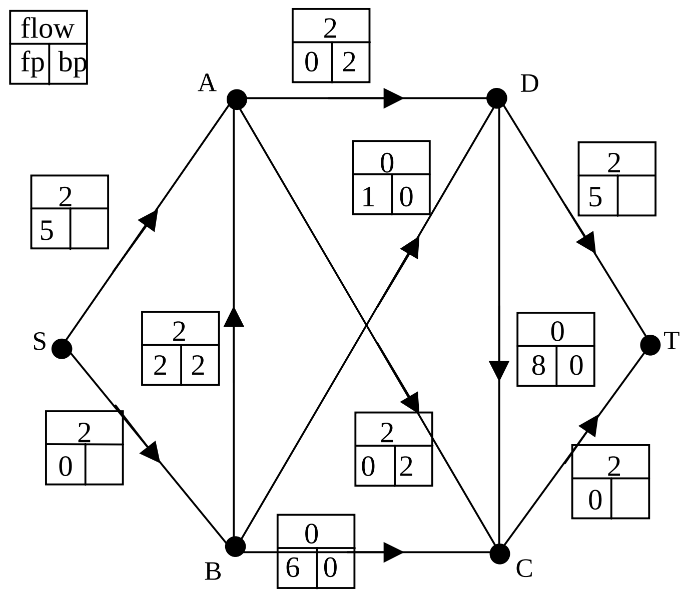
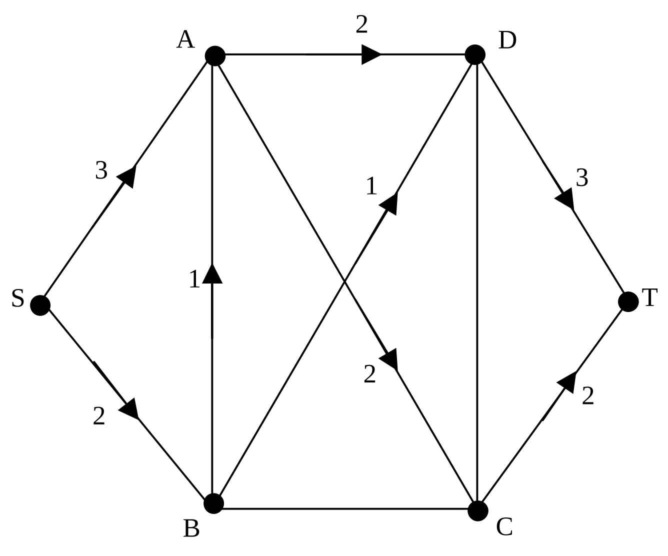
DECISION MATHEMATICS COMPUTATION, DC

4773

MARK SCHEME

Qu	Answer	Mark	Comment
1(i)	0.75= remainder after excretion $200 - x_n =$ top-up dose $150 = 0.75 \times 200$ and $112.5 = 0.75 \times 150$	B1 B1 B1 **[3]**	
1(ii)	*See below*	M1 A1 **[2]**	
1(iii)	*See below for s/sheet* Doesn't help. Fluctuates at higher level.	M1 A1 B1 **[3]**	Column C Comment
1(iv)	*See below for s/sheet* Converges to 133.33...	M1 A1 B1 **[3]**	Column D Comment
1(v)	$x_n = 160(1-(-\frac{1}{4})^{n+1}$ Quickly convergent oscillation to 160	M1 A2 B1 **[4]**	 Comment
1(vi)	Also converges to 160 – less quickly, but no tests needed	M1 A1 B1 **[3]**	Column F Comment

time	drug(ii)	drug(iii)	drug(iv)	drug(v)	drug(vi)
1	150.0	150.0	150.0	150.0	190.00
2	112.5	112.5	112.5	162.5	182.50
3	134.4	184.4	109.4	159.4	176.88
4	188.3	275.8	125.8	160.2	172.66
5	206.8	272.5	139.6	160.0	169.49
6	166.8	204.3	141.8	160.0	167.12
7	125.1	153.3	136.6	160.0	165.34
8	127.0	160.6	131.5	160.0	164.00
9	170.1	217.2	130.3	160.0	163.00
10	200.6	252.3	132.0	160.0	162.25
11	180.3	222.0	133.8	160.0	161.69
12	135.2	166.5	134.4	160.0	161.27
13	121.1	152.9	133.9	160.0	160.95
14	155.6	198.1	133.2	160.0	160.71
15	195.6	245.7	133.0	160.0	160.53
16	191.1	236.2	133.1	160.0	160.40
17	147.7	181.4	133.4	160.0	160.30
18	119.7	149.9	133.5	160.0	160.23
19	142.0	181.0	133.4	160.0	160.17
20	186.8	235.9	133.3	160.0	160.13
21	198.1	245.9	133.3	160.0	160.10

Qu	Answer	Mark	Comment
2(i)	SABC|DT	B1 **[1]**	
2(ii)		M1 A1 **[2]**	
2(iii)	SABDT	B1	
		B1 **[2]**	
2(iv)	max SA+SB		
	st SA+BA-AD-AC=0	B1	Objective
	SB-BA-BC-BD=0		
	BC+AC+DC-CT=0		
	AD+BD-DT-DC=0	B1	Vertices
	SA<=7 AD<=2 BC<=6 CT<=2		
	SB<=2 AC<=2 DC<=8		
	BA<=4 BD<=1 DT<=7	B1	Pipes

Qu	Answer	Mark	Comment
2(iv) cont'd	e.g. LP OPTIMUM FOUND AT STEP 4 OBJECTIVE FUNCTION VALUE 1) 5.000000 VARIABLE VALUE REDUCED COST SA 4.000000 0.000000 SB 1.000000 0.000000 BA 0.000000 0.000000 AD 2.000000 0.000000 AC 2.000000 0.000000 BC 0.000000 0.000000 BD 1.000000 0.000000 DC 0.000000 1.000000 CT 2.000000 0.000000 DT 3.000000 0.000000	B1 **[4]**	
2(v)	min 7SA+2SB+4BA+2AD+2AC +BD+6BC+8DC+7DT+2CT st SA+BA-AD-AC=0 SB-BA-BC-BD=0 BC+AC+DC-CT=0 AD+BD-DT-DC=0 SA+SB=1 DT+CT=1 end	B1 B1 B1	
	e.g. LP OPTIMUM FOUND AT STEP 5 OBJECTIVE FUNCTION VALUE 1) 10.000000 VARIABLE VALUE REDUCED COST SA 0.000000 1.000000 SB 1.000000 0.000000 BA 0.000000 0.000000 AD 0.000000 5.000000 AC 0.000000 0.000000 BD 2.000000 0.000000 BC 1.000000 0.000000 DC 0.000000 3.000000 DT 0.000000 0.000000 CT 1.000000 0.000000	B1	There are other possible solutions which may be seen.
	Shortest path=SBCT, of length 10	B1 **[5]**	

Qu	Answer	Mark	Comment
2(vi)	Min cut as before min 7SA+2SB+4BA+2AD+2DA+2AC +BD+6BC+8DC+7DT+2CT st SA+BA+DA-AD-AC=0 SB-BA-BC-BD=0 BC+AC+DC-CT=0 AD+BD-DA-DT-DC=0 SA+SB=1 DT+CT=1 end e.g. LP OPTIMUM FOUND AT STEP 4 OBJECTIVE FUNCTION VALUE 1) 9.000000 VARIABLE VALUE REDUCED COST SA 0.000000 2.000000 SB 1.000000 0.000000 BA 0.000000 1.000000 AD 0.000000 4.000000 DA 1.000000 0.000000 AC 1.000000 0.000000 BD 1.000000 0.000000 BC 0.000000 1.000000 DC 0.000000 4.000000 DT 0.000000 1.000000 CT 1.000000 0.000000 Shortest route=SBDACT, with length 9	B1 B1 B1 B1 **[4]**	
3(i)	Costings: $4 \times 0.75 = 3$, etc. *x*s represent proportion of max output used from each station	B1 B1 **[2]**	
3(ii)	LP OPTIMUM FOUND AT STEP 2 OBJECTIVE FUNCTION VALUE 1) 4.595 VARIABLE VALUE REDUCED COST X1 0.000000 0.080000 X2 1.000000 0.000000 X3 0.214286 0.000000 DEMAND 6.500000 0.000000 Run station 2 at max and station 3 at 3/14 of max. Cost=4.595 per hour	 B1 B1 B1 **[3]**	

Qu	Answer	Mark	Comment
3(iii)	LP OPTIMUM FOUND AT STEP 1 OBJECTIVE FUNCTION VALUE 1) 3.290000 VARIABLE VALUE REDUCED COST X1 0.000000 0.200000 X2 0.940000 0.000000 X3 0.000000 0.200000 DEMAND 4.700000 0.000000	B1	
	Run station 2 only, at 94% of capacity Cost=3.29 per hour	B1 **[2]**	
3(iv)	min 3X1+3.5X2+5.11X3+4.32X4+3.04X5+2.07X6 +6.16X7+3.33X8+4.712X9+6.57X10	B1	
	st Demand=38.2	B1	
	4X1+5X2+7X3+6X4+4X5+3X6+8X7 +4.5X8+6.2X9+9X10-Demand=0 X1>=0 X1<=1 etc.	B1	
	LP OPTIMUM FOUND AT STEP 7 OBJECTIVE FUNCTION VALUE 1) 27.67500 VARIABLE VALUE REDUCED COST X1 0.925000 0.000000 X2 1.000000 0.000000 X3 1.000000 0.000000 X4 1.000000 0.000000 X5 0.000000 0.040000 X6 1.000000 0.000000 X7 0.000000 0.160000 X8 1.000000 0.000000 X9 0.000000 0.062000 X10 1.000000 0.000000 DEMAND 38.200001 0.000000	B1 **[4]**	

Qu	Answer	Mark	Comment
3(v)	Objective: ...+2.4X11+4.7I	B1	
	Constraints: ...+10X11	B1	
	$X11 \geq 0$		
	$X11 \leq 1$	B1	
	$I - X11 \geq 0$	B1	
	Int I	B1	
	OBJECTIVE FUNCTION VALUE		
	1) 27.35600		
	e.g. VARIABLE / VALUE / REDUCED COST		
	X1 1.000000 -0.200000		
	X2 0.000000 0.080000		
	X3 1.000000 0.000000		
	X4 1.000000 0.000000	B1	
	X5 0.000000 0.120000		
	X6 1.000000 0.000000		
	X7 0.000000 0.320000		
	X8 0.000000 0.045000		
	X9 0.000000 0.186000		
	X10 0.800000 0.000000		
	X11 1.000000 0.000000		
	DEMAND 38.200001 0.000000	**[6]**	
3(vi)	13 units, either by iteration or by:	B1	
	$3*0.69+5*0.7+5*0.72=4.7+2.4+3*0.69$	**[1]**	
4(i)	=RAND()	B1	
	=A1^2+B1^2	B1	
	IF(C1<1,1,0)	B1	
		[3]	
4(ii)	=0.004*SUM(D1:D1000)	B1	
	e.g. 3.084	B1	
		[2]	
4(iii)	All points in unit square.	B1	
	Points for which test<1 lie within quadrant of unit circle.	B1	
	So $\frac{sum}{1000} \cong \frac{\pi 1^2 / 4}{1^2}$ giving $\pi \cong 0.004 * \text{sum}$	B1	
		[3]	
4(iv)	e.g.: 3.084 3.152 3.096 3.240 3.124 3.116	B1	
	3.128 3.120 3.188 3.164 3.144 3.140		
	3.14133 0.04207	B1,B1	
		[3]	

Qu	Answer	Mark	Comment
4(v)	e.g. require n s.t. $2\times\dfrac{0.042}{\sqrt{n}}=0.0005$, i.e. $n\cong 30000$	M1,A1 B1 B1 **[4]**	$s/\sqrt{n}$ $2\times$ Solving
4(vi)	e.g.: 3.216 3.048 3.186 3.078 3.138 3.168 3.246 3.096 3.168 3.114 3.276 3.282 3.168 0.07679 Comparing s.d.s, and hence standard errors – seems to be worse	M1 A1 B1 **[3]**	
			Total: 72

AO	Range	Total	Question Number			
			1	2	3	4
1	14-22	15	3	2	3	7
2	14-22	16	5	2	4	5
3	18-26	19	6	8	5	-
4	3-11	7	1	1	1	4
5	7-22	15	3	5	5	2
	Totals	**72**	18	18	18	18

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
NUMERICAL METHODS, NM

4776

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 **(i)** Show that the equation $x^4 - 5x + 1 = 0$ has a root between $x = 1$ and $x = 2$. **[2]**

(ii) Use the bisection method to find this root with a maximum possible error less than or equal to 0.1. **[5]**

2 A rough approximation to $\sqrt{x}$ where $0.25 \le x \le 1$ is given by s where $s = \frac{2}{3}x + 0.36$.

(i) Find the absolute and relative errors when the approximation is used for $x = 0.25$ and $x = 0.64$. **[4]**

Once s has been found, an improved approximation to $\sqrt{x}$ is given by $\frac{(s^2 + x)}{2x}$.

(ii) Find the relative error in the improved approximation when $x = 0.25$. **[3]**

3 A function $\mathrm{f}(x)$ has the values shown in the table.
The values of x are exact; the values of $\mathrm{f}(x)$ are correct to 5 decimal places.

$\boldsymbol{x}$	2	2.1	2.2	2.4
f($\boldsymbol{x}$)	0.80711	0.81934	0.83135	0.85471

(i) Obtain **three** estimates of $\mathrm{f}'(2)$ using the forward difference method with h taking values 0.4, 0.2, 0.1. **[3]**

(ii) Show that, as h is halved, the differences between the estimates are approximately halved. **[2]**

(iii) Hence obtain the best estimate you can of $\mathrm{f}'(2)$. **[2]**

4 **(i)** Given that $I = \int_1^2 \sqrt{1 + x^4}\,\mathrm{d}x$, find the estimates of I given by single applications of the trapezium rule and the mid point rule (i.e. take $h = 1$ in each case). **[5]**

(ii) Show how these two estimates may be used to find a better estimate of I. **[2]**

5 The function $\mathrm{f}(x)$ has three known values as given in the table.

x	1	2	4
$\mathbf{f}(x)$	–2	6	–1

(i) State the lowest possible degree of a polynomial that will pass through the three data points. Explain how you can tell, without doing any calculations, that no polynomial of lower degree will fit the data. **[3]**

(ii) Use Lagrange's method to obtain an estimate of $\mathrm{f}(3)$. **[5]**

Section B (36 marks)

6 In the difference table below, the values of x are exact but the values of $\mathrm{f}(x)$ may be subject to error.

x	$\mathrm{f}(x)$	$\Delta \mathrm{f}(x)$	$\Delta^2 \mathrm{f}(x)$	$\Delta^3 \mathrm{f}(x)$	$\Delta^4 \mathrm{f}(x)$
0	1.77				
		1.87			
1	3.64		0.26		
		2.13		0.23	
2	5.77		0.49		0.18
		2.62		0.41	
3	8.39		0.90		
		3.52			
4	11.91				

(i) **(*A*)** Use Newton's forward difference method to obtain a sequence of **four** estimates, linear, quadratic, cubic and quartic, for $\mathrm{f}(0.8)$. **[8]**

(*B*) Assuming that the values of $\mathrm{f}(x)$ are exact, give an estimated value for $\mathrm{f}(0.8)$ to the accuracy that appears justified, explaining your reasoning. **[2]**

(ii) Now assume that the values of $\mathrm{f}(x)$ are rounded to 2 decimal places.

(*A*) State the maximum possible error in each value. **[1]**

(*B*) Calculate, for the linear estimate, the maximum possible error due to rounding. **[3]**

(*C*) Explain what this implies for the higher order estimates. **[2]**

(*D*) Explain briefly whether, in these circumstances, you would revise your final answer in part **(i)(*B*)**. **[2]**

7 **(i)** The iterative formula $x_{r+1} = 0.8(1 - x_r{}^3)$ is used with starting values given below.
Describe in each case how the sequence of iterates behaves.

(***A***) $x_0 = 1.3$, **[3]**

(***B***) $x_0 = 0.6$. **[2]**

(ii) (***A***) Show graphically, or otherwise, that the equation $x = 0.8(1 - x^3)$ has only one real root, α. **[3]**

(***B***) Use the Newton-Raphson method, with the equation in the form $x - 0.8(1 - x^3) = 0$, to determine α, correct to 5 significant figures. **[5]**

(iii) (***A***) Differentiate $0.8(1 - x^3)$ and evaluate the derivative at $x = \alpha$. **[3]**

(***B***) Explain how this value relates to the behaviour of the iteration in part **(i)(*B*)**. **[2]**

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

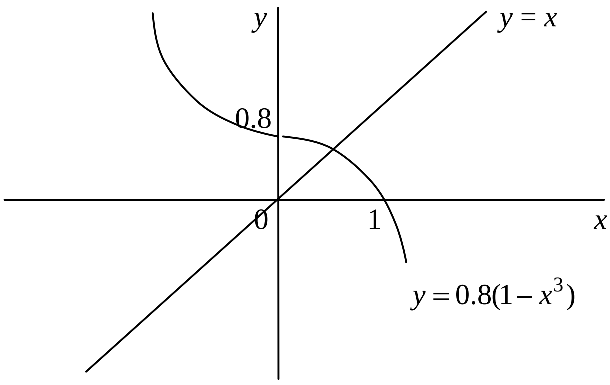
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4776**
NUMERICAL METHODS, NM

MARK SCHEME

Qu	Answer	Mark
Section A		
1(i)	$f(x) = x^4 - 5x + 1,\ f(1) = -3,\ f(2) = 7$ hence root	M1,A1 **[2]**
1(ii)	A B x $f(x)$ mpe 1 2 1.5 -1.4375 0.5 1.5 2 1.75 1.628906 0.25 1.5 1.75 1.625 -0.1521 0.125 1.625 1.75 **1.6875** 0.0625	M1 A1 A1 A1 A1 **[5]**
2(i)	x = 0.25 s = 0.5267 Error: 0.0267 rel error: 0.053 x = 0.64 s = 0.7867 Error: –0.0133 rel error: –0.017	M1,A1,A1 A1 **[4]**
2(ii)	x = 0.25 gives s = 0.5267 which leads to the improved estimate 0.500675 with relative error 0.00135	M1,A1 A1 **[3]**
3(i)	h 0.4 0.2 0.1 est $f'(2)$ 0.119 0.1212 0.1223	M1,A1,A1 **[3]**
3(ii)	diffs 0.0022 0.0011 halved (to 4 dp)	M1,A1 **[2]**
3(iii)	Best estimate: $0.1223 + 0.0011(0.5 + 0.25 + ...) = 0.1234$	M1,A1 **[2]**
4(i)	x 1 1.5 2 Sqrt $(1 + x^4)$ 1.414214 2.462214 4.123106 $T = \dfrac{(f(1) + f(2))}{2} = 2.76866$ $M = f(1.5) = 2.462214$	A1 M1,A1 M1,A1 **[5]**
4(ii)	$S = \dfrac{(T + 2M)}{3} = 2.564363$ is an improved estimate	M1,A1 **[2]**

Qu	Answer	Mark
Section A (continued)		
5(i)	Degree 2 Clearly a turning point (a maximum) so not linear (and not constant)	B1 E1,E1 **[3]**
5(ii)	$f(3) = \frac{-2(3-2)(3-4)}{(1-2)(1-4)} + \frac{6(3-1)(3-4)}{(2-1)(2-4)} - \frac{(3-1)(3-2)}{(4-1)(4-2)}$ $= \frac{2}{3} + 6 - \frac{1}{3}$ $= 6\frac{1}{3}$	M1,A1 A1,A1 A1 **[5]**
	Section A Total: 36	
Section B		
6(i)(*A*)	$f(0.8) = 1.77 + 0.8 \times 1.87$ — 3.266 — linear $f(0.8) = \text{linear } + \frac{0.8(-0.2)0.26}{2}$ — 3.2452 — quadratic $f(0.8) = \text{quadratic } + \frac{0.8(-0.2)(-1.2)0.23}{6}$ — 3.25256 — cubic $f(0.8) = \text{cubic } + \frac{0.8(-0.2)(-1.2)(-2.2)0.18}{24}$ — 3.249392 — quartic	M1,A1 M1,A1 M1,A1 M1,A1 **[8]**
6(i)(*B*)	3.25 seems reliable because the last 3 estimates agree to 2 d.p. (but not to 3 d.p.)	A1,E1 **[2]**
6(ii)(*A*)	mpe in each f value is 0.005	A1 **[1]**
6(ii)(*B*)	greatest possible is $1.775 + 0.8(3.645 - 1.775) = 3.266 + 0.005$ (OR equivalent least possible OR other reasoning) so mpe is 0.005	M1,A1 A1 **[3]**
6(ii)(*C*)	Higher order estimates will all have at least this much mpe as they all contain the linear estimate	E1 E1 **[2]**
6(ii)(*D*)	Error of 0.005 can affect 2nd d.p., hence 2nd d.p. is unreliable. This therefore casts doubt on the accuracy of 3.25	E1 E1 **[2]**

Qu	Answer	Mark
Section B (continued)		
7(i)(*A*)	r 0 1 2 3 4 5 6 x_r 1.3 -0.9576 1.502494 -1.91349 6.404892 -209.396 7345109 Diverges	 M1,A1 E1 **[3]**
7(i)(*B*)	r 0 1 2 3 4 5 6 x_r 0.6 0.6272 0.602618 0.624928 0.604755 0.623059 0.606501 Converges but slowly	 A1 E1 **[5]**
7(ii)(*A*)	 (OR Let $y = x - 0.8(1 - x^3)$ so $y' = 1 + 2.4x^2 > 0$ Curve has no t.p.s., $y(0) < 0$ and $y(1) > 0$, hence a single root	 G3 **[3]**
7(ii)(*B*)	Newton-Raphson: $x_{r+1} = x_r - \dfrac{(x_r - 0.8(1 - x_r{}^3))}{(1 + 2.4x_r{}^2)}$ r 0 1 2 3 4 5 6 x_r 0.6 0.614592 0.61443 0.61443 0.61443 0.61443 0.61443	M1,A1 A1,A1,A1 **[5]**
7(iii)(*A*)	Derivative is $-2.4x^2$ Evaluates to – 0.90606	M1,A1 A1 **[3]**
7(iii)(*B*)	The negative sign indicates oscillation The magnitude, just less that 1, indicates slow convergence	E1 E1 **[2]**
	Section B Total: 36	
	Total: 72	

AO	Range	Total	Question Number							CWk
			1	**2**	**3**	**4**	**5**	**6**	**7**	
1	27-36	32	2	4	3	3	3	6	7	4
2	27-36	31	3	3	3	2	4	5	7	4
3	0-9	0	-	-	-	-	-	-	-	-
4	0-9	6	-	-	-	1	1	3	-	1
5	18-27	21	-	-	1	1	-	4	4	9
	Totals	**90**	7	7	7	7	8	18	18	18

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
NUMERICAL COMPUTATION, NC

4777

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 2 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer any **three** the questions.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions, you are required to write spreadsheet routines to carry out various numerical analysis processes. You should note the following points.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arcos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the *formulae* in the cells as well as the *values* in the cells. You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.

1 **(i)** The iteration $x_0, x_1, x_2, \ldots$ where $x_{r+1} = \mathrm{g}(x_r)$ has a fixed point α, so that $\alpha = \mathrm{g}(\alpha)$.
You are given that $x_{r+1} - \alpha \approx k(x_r - \alpha)$, where k is a constant.

(***A***) Show that k may be estimated as $\dfrac{x_2 - x_1}{x_1 - x_0}$. **[2]**

(***B***) Use the iteration $x_{r+1} = 0.1\mathrm{e}^{x_r}$ with $x_0 = 0.3$ to show how an estimate of k can be used to find an estimate of α using Richardson's extrapolation. **[5]**

(ii) A solution to the simultaneous equations

$$1.1x + \sin y = 0.1\,,$$
$$\sin x + 1.1y = 0.2\,,$$

can be found using the iterations

$$x_{r+1} = \frac{1}{1.1}(0.1 - \sin y_r),$$
$$y_{r+1} = \frac{1}{1.1}(0.2 - \sin x_{r+1}).$$

(***A***) Starting with $y_0 = 0.2$, show on a spreadsheet that the iterations converge slowly. **[3]**

(***B***) Use the values of x_r and y_r up to $r = 3$ and Richardson's extrapolation to estimate the solution. **[6]**

(***C***) Using the y estimate as a new starting point, repeat the process as necessary to obtain the solution for x and y correct to 6 decimal places. **[8]**

2 **(i)** Derive the Gaussian three point integration formula

$$\int_{-h}^{h} \mathrm{f}(x)\mathrm{d}x \approx \frac{h}{9}\left(5\mathrm{f}\left(-\sqrt{\frac{3}{5}}h\right) + 8\mathrm{f}(0) + 5\mathrm{f}\left(\sqrt{\frac{3}{5}}h\right)\right).$$

In your derivation, you should show that this formula is exact up to $\mathrm{f}(x) = x^5$, but not exact for $\mathrm{f}(x) = x^6$. **[8]**

(ii) **(*A*)** On a spreadsheet, obtain a value for:

$$\int_{0}^{1} \exp\left(-\frac{1}{2}x^2\right)\mathrm{d}x$$

using a single application of the Gaussian three point rule with $h = 0.5$. **[4]**

(*B*) Determine the percentage error in this result by comparing it with **two** applications of the Gaussian three point rule each with $h = 0.25$. **[5]**

(iii) The value of

$$\int_{0}^{z} \exp\left(-\frac{1}{2}x^2\right)\mathrm{d}x, \quad z > 0$$

is to be estimated from a single application of the Gaussian three point rule.
Use the routines you developed in part **(ii)** to determine, by trial and error and correct to one decimal place, the range of values of *z* for which the estimate will be accurate to within 0.001%. **[7]**

3 **(i)** **(*A*)** Use a spreadsheet to show that the Runge-Kutta method of order 4 gives an exact solution to the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = x^3$ where $y = 0$ when $x = 0$, for $0 \le x \le 1$ and a step length of 0.2. **[5]**

(*B*) Show that the method does not give an exact solution when the right hand side of the differential equation is x^4. **[2]**

(*C*) Explain how these results relate to an important property of Simpson's rule. **[2]**

(ii) **(*A*)** Use the Runge-Kutta method of order 4 on a spreadsheet to obtain a solution to the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x + \cos y$ where $y = 0$ when $x = 0$, for $0 \le x \le 3\pi$. **[9]**

(*B*) By reducing the step length as necessary, determine the solution correct to 4 decimal places. **[3]**

(*C*) Use your spreadsheet to produce a graph of the solution. **[3]**

4 **(i)** For a given 3×3 matrix, **M**, with non-zero determinant, let

$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be the solution of the equation $\mathbf{Mx} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

(*A*) State how $\mathbf{x}$ relates to the inverse matrix $\mathbf{M}^{-1}$. **[1]**

(*B*) State in similar terms how to find the complete inverse matrix $\mathbf{M}^{-1}$. **[2]**

(*C*) Explain how, in using the process of Gaussian elimination, the determinant of a matrix may be found. **[4]**

(ii) Use Gaussian elimination and the technique of part **(i)** to find the determinant and the inverse of the following matrix.

$$\begin{pmatrix} 3 & -1 & 4 & 7 \\ 2 & 2 & 0 & -1 \\ -4 & -2 & 3 & 0 \\ 0 & 1 & -1 & 3 \end{pmatrix}$$

[17]

RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

NUMERICAL COMPUTATION, NC

4777

MARK SCHEME

Qu	Answer (Note: Presented in Printout Format)	Mark	Comment
1(i)(*A*)	$x_1-\alpha=k(x_0-\alpha)$ subtract to give $x_2-x_1=k(x_1-x_0)$ $x_2-\alpha=k(x_1-\alpha)$ hence $k=\dfrac{(x_2-x_1)}{(x_1-x_0)}$	M1 M1 **[2]**	
1(i)(*B*)	r 0 1 2 x_r 0.3 0.134986 0.114452	A1	
	$k=\dfrac{(0.114452-0.134986)}{(0.134986-0.3)}$ $=0.124437$	A1	
	hence $\alpha=0.114452+(0.114452-0.134986)(0.124437+0.124437^2+...)$ $=0.111534$	M2 A1 **[5]**	
1(ii)(*A*)	r 0 1 2 3 4 5 6 7 x_r -0.0897 -0.14566 -0.18968 -0.22356 -0.24913 -0.2681 -0.28197 y_r 0.2 0.263254 0.313767 0.353219 0.383369 0.405968 0.422634 0.434769		
	Clearly very slow convergence	M1,A2 **[3]**	
1(ii)(*B*)	r 0 1 2 3 x_r -0.0897 -0.14566 -0.18968 y_r 0.2 0.263254 0.313767 0.353219	M3	For correctly setting up spreadsheet
	$kx=0.786622,\ ky=0.781032$ $\alpha=-0.351950,\ \beta=0.49394$	M1 A2 **[6]**	
1(ii)(*C*)	r 0 1 2 3 x_r -0.34009 -0.33297 -0.32805 y_r 0.49394 0.485065 0.478955 0.474723	M2	For correctly re-iterating
	$kx=0.691206,\ ky=0.69265$ $\alpha=-0.317030,\ \beta=0.465185$	A2	
	r 0 1 2 3 x_r -0.3169 -0.31684 -0.31679 y_r 0.465185 0.46511 0.465057 0.46502		
	$kx=0.701891,\ ky=0.701903$ $\alpha=-0.316693,\ \beta=0.464933$	A1	
	r 0 1 2 3 x_r -0.31669 -0.31669 -0.31669 y_r 0.464933 0.464933 0.464933 0.464933	M1	For stopping at the right point
	$kx=0.701984,\ ky=0.701984$ $\alpha=-0.316693,\ \beta=0.464933$	A2 **[8]**	

<table>
<tr><th>Qu</th><th>Answer (Note: Presented in Printout Format)</th><th>Mark</th><th>Comment</th></tr>
<tr><td>2(i)</td><td>Set up formula as $I = af(-\alpha) + bf(0) + af(\alpha)$
$f(x) = 1: \quad 2h = 2a + b \quad (1)$
$f(x) = x: \quad 0 = 0$
$f(x) = x^2: \quad \frac{2h^3}{3} = 2a\alpha^2 \quad (2)$
$f(x) = x^3: \quad 0 = 0$
$f(x) = x^4: \quad \frac{2h^5}{5} = 2a\alpha^4 \quad (3)$
$f(x) = x^5: \quad 0 = 0$
$f(x) = x^6: \quad \frac{2h^7}{7} = 2a\alpha^6 \quad (4)$</td><td>M1

M3</td><td>

Setting up equations</td></tr>
<tr><td></td><td>From (2) and (3), $\alpha^2 = \frac{3h^2}{5}$ hence from (2) $a = \frac{5h}{9}$
and from (1) $b = \frac{8h}{9}$.</td><td>M3</td><td>Convincing algebra</td></tr>
<tr><td></td><td>(4) gives $\frac{2}{7} = \frac{6}{25}$ which is incorrect</td><td>E1
[8]</td><td>Analysis of (4)</td></tr>
<tr><td>2(ii)(A)</td><td><table>
<tr><td>m</td><td>h</td><td>m–a</td><td>m</td><td>m+a</td></tr>
<tr><td>0.5</td><td>0.5</td><td>0.112702</td><td>0.5</td><td>0.887298</td></tr>
<tr><td></td><td>ordinates:</td><td>0.993669</td><td>0.882497</td><td>0.674591</td></tr>
<tr><td></td><td>weights:</td><td>0.555556</td><td>0.888889</td><td>0.555556</td></tr>
<tr><td></td><td>integral:</td><td>0.8556264</td><td></td><td></td></tr>
</table></td><td>M3

A1
[4]</td><td>Spreadsheet</td></tr>
<tr><td>2(ii)(B)</td><td><table>
<tr><td>m</td><td>h</td><td>m–a</td><td>m</td><td>m+a</td></tr>
<tr><td>0.25</td><td>0.25</td><td>0.056351</td><td>0.25</td><td>0.443649</td></tr>
<tr><td></td><td>ordinates:</td><td>0.998414</td><td>0.969233</td><td>0.906275</td></tr>
<tr><td></td><td>weights:</td><td>0.555556</td><td>0.888889</td><td>0.555556</td></tr>
<tr><td></td><td>integral:</td><td>0.479925 (1)</td><td></td><td></td></tr>
</table></td><td>M1

A1</td><td>Modification</td></tr>
<tr><td></td><td><table>
<tr><td>m</td><td>h</td><td>m–a</td><td>m</td><td>m+a</td></tr>
<tr><td>0.75</td><td>0.25</td><td>0.556351</td><td>0.75</td><td>0.943649</td></tr>
<tr><td></td><td>ordinates:</td><td>0.856618</td><td>0.75484</td><td>0.640672</td></tr>
<tr><td></td><td>weights:</td><td>0.555556</td><td>0.888889</td><td>0.555556</td></tr>
<tr><td></td><td>integral:</td><td>0.375699 (2)</td><td></td><td></td></tr>
</table></td><td>A1</td><td></td></tr>
<tr><td></td><td>Sum of (1) and (2) $= 0.8556244$
Percentage error $= 0.000231$</td><td>M1
A1</td><td></td></tr>
<tr><td></td><td>(Given the rate of convergence of G3 (h^6), 0.8556244 is very nearly correct, so the single application has an error of about 20/8556244, or 0.00023%.)</td><td>[5]</td><td></td></tr>
</table>

<table>
<tr><th>Qu</th><th>Answer (Note: Presented in Printout Format)</th><th>Mark</th><th>Comment</th></tr>
<tr><td>2(iii)</td><td>
<table>
<tr><td>m</td><td>h</td><td>m–a</td><td>m</td><td>m+a</td></tr>
<tr><td>0.695</td><td>0.695</td><td>0.156655</td><td>0.695</td><td>1.233345</td></tr>
<tr><td></td><td>ordinates:</td><td>0.987805</td><td>0.785439</td><td>0.4674</td></tr>
<tr><td></td><td>weights:</td><td>0.555556</td><td>0.888889</td><td>0.555556</td></tr>
<tr><td></td><td>integral:</td><td>1.0470974</td><td></td><td></td></tr>
</table>
<table>
<tr><td>m</td><td>h</td><td>m–a</td><td>m</td><td>m+a</td></tr>
<tr><td>0.3475</td><td>0.3475</td><td>0.078328</td><td>0.3475</td><td>0.616672</td></tr>
<tr><td></td><td>ordinates:</td><td>0.996937</td><td>0.941408</td><td>0.826841</td></tr>
<tr><td></td><td>weights:</td><td>0.555556</td><td>0.888889</td><td>0.555556</td></tr>
<tr><td></td><td>integral:</td><td>0.642881 (1)</td><td></td><td></td></tr>
</table>
<table>
<tr><td>m</td><td>h</td><td>m–a</td><td>m</td><td>m+a</td></tr>
<tr><td>1.0425</td><td>0.3475</td><td>0.773328</td><td>1.0425</td><td>1.311672</td></tr>
<tr><td></td><td>ordinates:</td><td>0.741546</td><td>0.580768</td><td>0.42306</td></tr>
<tr><td></td><td>weights:</td><td>0.555556</td><td>0.888889</td><td>0.555556</td></tr>
<tr><td></td><td>integral:</td><td>0.404227 (2)</td><td></td><td></td></tr>
</table>
Sum of (1) and (2) $=1.0471077$
Percentage error $=-0.00098$
</td><td>M5

A2
[7]</td><td>By trial and error on this spreadsheet the maximum value of m is 0.695 (to 3 d.p.) for the percentage error to remain below 0.001 in magnitude, hence the integral goes 0 to 1.39

Evidence of trial and error</td></tr>
<tr><td>3(i)(A)</td><td>
<table>
<tr><td>h</td><td>x</td><td>y</td><td>k_1</td><td>k_2</td><td>k_3</td><td>k_4</td><td>new_y</td></tr>
<tr><td>0.2</td><td>0</td><td>0</td><td>0</td><td>0.0002</td><td>0.0002</td><td>0.0016</td><td>0.0004</td></tr>
<tr><td></td><td>0.2</td><td>0.0004</td><td>0.0016</td><td>0.0054</td><td>0.0054</td><td>0.0128</td><td>0.0064</td></tr>
<tr><td>RHSx^3</td><td>0.4</td><td>0.0064</td><td>0.0128</td><td>0.025</td><td>0.025</td><td>0.0432</td><td>0.0324</td></tr>
<tr><td></td><td>0.6</td><td>0.0324</td><td>0.0432</td><td>0.0686</td><td>0.0686</td><td>0.1024</td><td>0.1024</td></tr>
<tr><td></td><td>0.8</td><td>0.1024</td><td>0.1024</td><td>0.1458</td><td>0.1458</td><td>0.2</td><td>0.25</td></tr>
<tr><td></td><td>1</td><td>0.25</td><td>0.2</td><td>0.2662</td><td>0.2662</td><td>0.3456</td><td>0.5184</td></tr>
</table>
by inspection $y = x^4/4$, as required for an exact solution
</td><td>M4

A1
[5]</td><td>Setup</td></tr>
<tr><td>3(i)(B)</td><td>
<table>
<tr><td>h</td><td>x</td><td>y</td><td>k_1</td><td>k_2</td><td>k_3</td><td>k_4</td><td>new_y</td></tr>
<tr><td>0.2</td><td>0</td><td>0</td><td>0</td><td>0.00002</td><td>0.00002</td><td>0.00032</td><td>6.67E-05</td></tr>
<tr><td></td><td>0.2</td><td>6.67E-05</td><td>0.00032</td><td>0.00162</td><td>0.00162</td><td>0.00512</td><td>0.002053</td></tr>
<tr><td>RHSx^4</td><td>0.4</td><td>0.002053</td><td>0.00512</td><td>0.0125</td><td>0.0125</td><td>0.02592</td><td>0.01556</td></tr>
<tr><td></td><td>0.6</td><td>0.01556</td><td>0.02592</td><td>0.04802</td><td>0.04802</td><td>0.08192</td><td>0.065547</td></tr>
<tr><td></td><td>0.8</td><td>0.065547</td><td>0.08192</td><td>0.13122</td><td>0.13122</td><td>0.2</td><td>0.200013</td></tr>
<tr><td></td><td>1</td><td>0.200013</td><td>0.2</td><td>0.29282</td><td>0.29282</td><td>0.41472</td><td>0.49768</td></tr>
</table>
by inspection y is not equal to $x^5/5$, so not an exact solution
</td><td>M1

A1
[2]</td><td>Mod'n</td></tr>
<tr><td>3(i)(C)</td><td>In the case where f(x,y) does not depend on y, RK4 is equivalent to Simpson's rule. Simpson's rule is exact up to cubics.</td><td>E1
E1
[2]</td><td></td></tr>
</table>

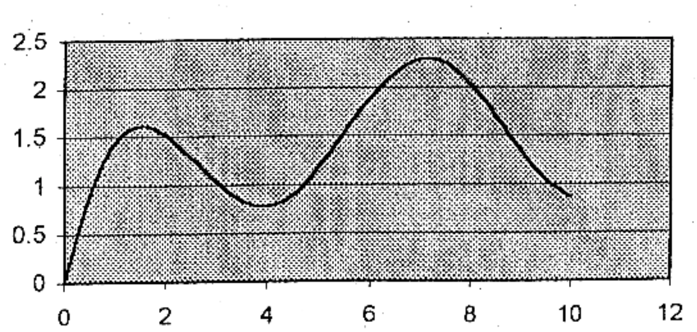
Qu	Answer (Note: Presented in Printout Format)	Mark	Comment
3(ii)(*A*)		M4	Mod'n

h_1	x	y	k_1	k_2	k_3	k_4	new_y
0.5	0	0	1	0.923247	0.932121	0.736857	0.907933
	0.5	0.907933	0.74648	0.50864	0.564481	0.319263	1.443263
	1	1.443263	0.333745	0.137996	0.186912	0.005697	1.60814
	1.5	1.60814	0.016701	-0.11196	-0.0798	-0.18685	1.51586
	2	1.51586	-0.18062	-0.24172	-0.22664	-0.26164	1.286031
	2.5	1.286031	-0.26011	-0.26064	-0.26052	-0.23567	1.029683
	3	1.029683	-0.23745	-0.19057	-0.19991	-0.13071	0.838162
	3.5	0.838162	-0.13381	-0.05176	-0.06636	0.031505	0.781738
	4	0.781738	0.028023	0.126831	0.108765	0.209113	0.899792
	4.5	0.899792	0.205488	0.287884	0.270299	0.336865	1.176246
	5	1.176246	0.334028	0.368832	0.360339	0.371438	1.53688
	5.5	1.53688	0.37129	0.355023	0.359045	0.32037	1.890179
	6	1.890179	0.323095	0.268423	0.280625	0.205969	2.161373
	6.5	2.161373	0.209874	0.126113	0.142466	0.042384	2.292942
	7	2.292942	0.046453	-0.05516	-0.03606	-0.14344	2.24637
	7.5	2.24637	-0.13935	-0.23285	-0.21333	-0.29573	2.025127
	8	2.025127	-0.29218	-0.34457	-0.33204	-0.362	1.690563
	8.5	1.690563	-0.36075	-0.36014	-0.36029	-0.33646	1.334219
	9	1.334219	-0.33838	-0.29502	-0.30503	-0.24083	1.037669
	9.5	1.037669	-0.24447	-0.16907	-0.18423	-0.09084	0.86402
	10	0.86402	-0.09484	0.003155	-0.0151	0.09263	0.859668

Qu	Answer	Mark	Comment
		A5	For several runs with decreasing h
		[9]	
3(ii)(*B*)		A3	For establishing accuracy

h	y(10)
0.5	0.86402
0.25	0.863967
0.125	0.863962

Qu	Answer	Mark	Comment
		[3]	
3(ii)(*C*)			



Qu	Answer	Mark	Comment
		G3 [3]	
4(i)(*A*)	**x** is the first column of $\mathbf{M}^{-1}$	E1 [1]	
4(i)(*B*)	The second column is the solution of $\mathbf{Mx} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^{\mathrm{T}}$	E1	
	The third column is the solution of $\mathbf{Mx} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}$	E1 [2]	
4(i)(*C*)	When the matrix is in triangular form	E1	
	the determinant is the product of the terms on the leading diagonal	E1	
	provided:		
	– no division across rows has occurred	E1	
	– due account has been taken of any row interchanges.	E1 [4]	

Qu	Answer (Note: Presented in Printout Format)								Mark	Comment
4(ii)	$\boxed{3}$	-1	4	7	1	0	0	0		
	2	2	0	-1	0	1	0	0		
	-4	-2	3	0	0	0	1	0		
	0	1	-1	3	0	0	0	1		
	3	-1	4	7	1	0	0	0		
	0	$\boxed{2.666667}$	-2.66667	-5.66667	-0.66667	1	0	0		
	0	-3.33333	8.333333	9.333333	1.333333	0	1	0		
	0	1	-1	3	0	0	0	1	M1,A1,A1	
	3	-1	4	7	1	0	0	0		
	0	2.666667	-2.66667	-5.66667	-0.66667	1	0	0		
	0	0	5	2.25	0.5	1.25	1	0		
	0	0	0	$\boxed{5.125}$	0.25	-0.375	0	1	M1,A1,A1	
	determinant is $3 \times 2.666667 \times 5 \times 5.125 = 205$								A1	
	3	-1	4	0	0.658537	0.512195	0	-1.36585		
	0	2.666667	-2.66667	0	-0.39024	0.585366	0	1.105691		
	0	0	$\boxed{5}$	0	0.390244	1.414634	1	-0.43902		
	0	0	0	5.125	0.25	-0.375	0	1	M1,A1,A1	
	3	-1	0	0	0.346341	-0.61951	-0.8	-1.01463		
	0	$\boxed{2.666667}$	0	0	-0.18211	1.339837	0.533333	0.871545		
	0	0	5	0	0.390244	1.414634	1	-0.43902		
	0	0	0	5.125	0.25	-0.375	0	1	M1,A1,A1	
	3	0	0	0	0.278049	-0.11707	-0.6	-0.6878		
	0	2.666667	0	0	-0.18211	1.339837	0.533333	0.871545		
	0	0	5	0	0.390244	1.414634	1	-0.43902		
	0	0	0	5.125	0.25	-0.375	0	1	M1A1	
	1	0	0	0	0.092683	-0.03902	-0.2	-0.22927		
	0	1	0	0	-0.06829	0.502439	0.2	0.326829		
	0	0	1	0	0.078049	0.282927	0.2	-0.0878		
	0	0	0	1	0.04878	-0.07317	0	0.195122	M1A1	
									[17]	
									Total: 72	

AO	Range	Total	Question Number			
			1	2	3	4
1	24-34	26	5	8	5	8
2	24-34	28	6	8	6	8
3	0-10	0	-	-	-	-
4	9-20	14	7	2	2	3
5	19-29	28	6	6	11	5
	Totals	**96**	24	24	24	24