

XXXX June 2013 – Morning/Afternoon

A2 GCE MATHEMATICS (MEI)

4798 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4798
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

1 This question concerns curves with parametric equations

$$x = t - k \sin t, \quad y = 1 - \cos t,$$

where k is positive and t takes all real values.

(i) Investigate the curves for values of k where $0 < k < 1$. Describe the common features of these curves and sketch, for $-2\pi \leq t \leq 4\pi$, a typical example. [5]

(ii) Prove that, if $0 < k < 1$, the curve is never parallel to the y -axis. [3]

(iii) Sketch the curve for $k = 1$. Describe the main feature by which this curve differs from the curves where $0 < k < 1$. [3]

(iv) For the case $k = 1$, confirm the behaviour of the curve at the point where $t = 0$ by investigating the gradient as $t \rightarrow 0$. [5]

(v) Sketch the curve for $k = 2$. Show that the width of each loop, measured parallel to the x -axis, is

$$2\sqrt{3} - \frac{2}{3}\pi. \quad [6]$$

(vi) Form an equation to find the value of k where the loops will just touch each other. Find the value of k to 4 decimal places. [3]

2 (i) The function f is defined by $f(z) = z^3 - (3 - 3i)z^2 - 6iz + 2 + i$. Solve the equation $f(z) = 0$ and plot the roots as points on an Argand diagram.

Show that these points form an equilateral triangle. [6]

(ii) Find $f'(z)$ and show that the equation $f'(z) = 0$ has a repeated root.

Plot this root on the Argand diagram drawn in part (i). [4]

The function f has the following properties:

(*) the roots of the equation $f(z) = 0$ form an equilateral triangle when plotted in the Argand diagram;

(**) the equation $f'(z) = 0$ has a single repeated root.

The rest of this question explores the relationship between (*) and (**) for other cubic functions.

(iii) Let $g(z) = z^3 + bz^2 + cz + d$. Find the relationship between b and c when there is a single repeated root of the equation $g'(z) = 0$. [3]

(iv) (A) Obtain another cubic, with distinct roots, for which (**) is true. Show that (*) is also true for your cubic.

(B) Obtain a cubic for which (*) and (**) are both false. [3]

The three points representing $0, z_1, z_2$ form an equilateral triangle on the Argand diagram.

(v) Given that $z_2 = \beta z_1, \beta \in \mathbb{C}$, find the two possible values of β . [3]

(vi) Use one of your values of β to write down a cubic with roots $0, z_1$ and z_2 . Show that (**) is true for this cubic. [5]

- 3** (i) Create a program to find all the positive integer solutions to $x^2 - 3y^2 = 1$ with $x \leq 100$, $y \leq 100$.
Write out your program in full and list the solutions it gives. [10]
- (ii) Show how the other solutions can be derived from the solution with the smallest x -value.
Use each solution to give a rational approximation to $\sqrt{3}$. [5]
- (iii) Edit your program so that it will find solutions to $x^2 - ny^2 = 1$, where n is a positive integer.
Write out the lines of your program that you have changed.
Use the edited program to find a rational approximation to $\sqrt{5}$ that is accurate to within 0.1%. [6]
- (iv) Explain why the edited program will not give any results if n is a square number. [2]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE



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A2 GCE MATHEMATICS (MEI)

4798 Further Pure Mathematics with Technology (FPT)

PRINTED ANSWER BOOK

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- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator
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Duration: Up to 2 hours



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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1 (i)	
1 (ii)	

1 (iii)	
1 (iv)	

1 (v)	
1 (vi)	

2 (i)	

2 (ii)	
2 (iii)	

2(iv)(A)	
2(iv)(B)	
2(v)	

2 (vi)	

3 (iii)	
3 (iv)	

PLEASE DO NOT WRITE ON THIS PAGE



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xxxx June 2013 –

GCE MATHEMATICS (MEI)

4798 Further Pure Mathematics with Technology (FPT)

MARK SCHEME

Duration: Up to 2 hours

MAXIMUM MARK 72

Final Version
Last updated: **xx/xx/xxxx**

This document consists of 17 pages

MARKING INSTRUCTIONS

PREPARATION FOR MARKING SCORIS

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *scoris assessor Online Training*; *OCR Essential Guide to Marking*.
2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal <http://www.rm.com/support/ca>
3. Log-in to scoris and mark 10 practice responses (“scripts”) and 10 standardisation responses

YOU MUST MARK 10 PRACTICE AND 10 STANDARDISATION RESPONSES BEFORE YOU CAN BE APPROVED TO MARK LIVE SCRIPTS.

TRADITIONAL

Before the Standardisation meeting you must mark at least 10 scripts from several centres. For this preliminary marking you should use **pencil** and follow the **mark scheme**. Bring these **marked scripts** to the meeting.

MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the scoris 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.
4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the scoris messaging system, or by email.
5. Work crossed out:
 - a. where a candidate crosses out an answer and provides an alternative response, the crossed out response is not marked and gains no marks
 - b. if a candidate crosses out an answer to a whole question and makes no second attempt, and if the inclusion of the answer does not cause a rubric infringement, the assessor should attempt to mark the crossed out answer and award marks appropriately.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there then add a tick to confirm that the work has been seen.
7. There is a NR (No Response) option. Award NR (No Response)
- if there is nothing written at all in the answer space
 - OR if there is a comment which does not in anyway relate to the question (e.g. 'can't do', 'don't know')
 - OR if there is a mark (e.g. a dash, a question mark) which isn't an attempt at the question
- Note: Award 0 marks - for an attempt that earns no credit (including copying out the question)
8. The scoris **comments box** is used by your team leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.** If you have any questions or comments for your team leader, use the phone, the scoris messaging system, or e-mail.
9. Assistant Examiners will send a brief report on the performance of candidates to your Team Leader (Supervisor) by the end of the marking period. The Assistant Examiner's Report Form (AERF) can be found on the RM Cambridge Assessment Support Portal (and for traditional marking it is in the *Instructions for Examiners*). Your report should contain notes on particular strength displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.
10. [DOES NOT APPLY TO GCE MATHEMATICS] For answers marked by levels of response:
- a. **To determine the level** – start at the highest level and work down until you reach the level that matches the answer
 - b. **To determine the mark within the level**, consider the following:

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

11. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

12. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

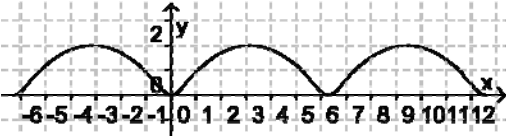
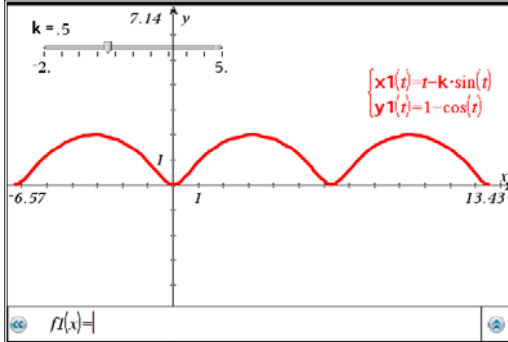
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

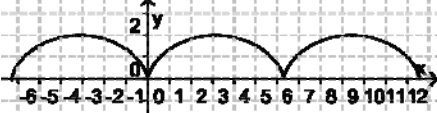
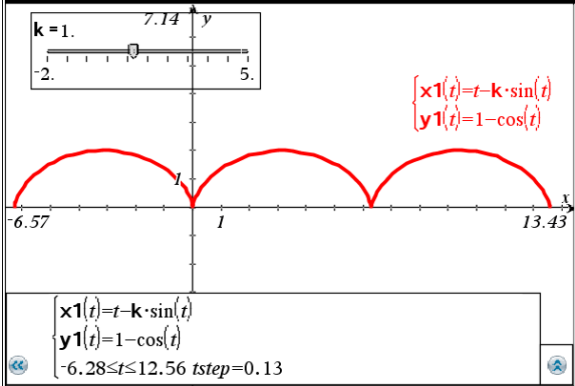
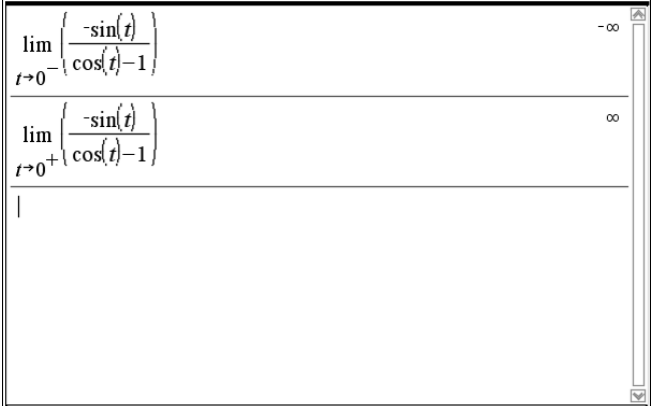
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

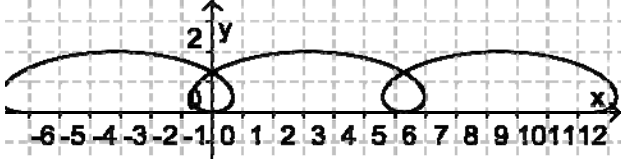
NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

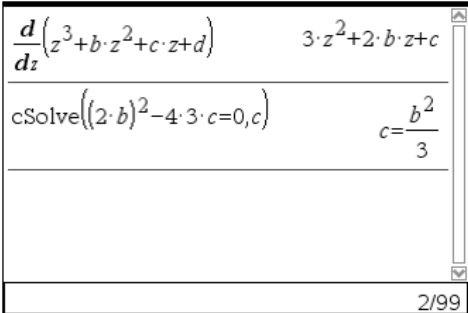
Question	Answer	Marks	Guidance
<p>1 (i)</p>	<p>Periodic with period 2π Maximum 2, minimum 0</p> <p>Comment about symmetry</p> 	<p>B1 B1 B1 G2 [5]</p>	<p>Both required for mark. Accept comment about coordinates of maxima/minima.</p> <p>Minima (zero gradient) clearly shown. Give B1 for general shape correct (three cycles).</p> 
<p>1 (ii)</p>	<p>$\frac{dy}{dx} = \frac{\sin t}{1 - k \cos t}$ Denominator > 0 for all t.</p> <p>OR</p> <p>$\frac{dx}{dt} = 1 - k \cos t$ $\frac{dx}{dt} > 0$ for all t.</p>	<p>M1 A1 E1 M1 A1 E1 [3]</p>	<p>Evidence of attempt to find $\frac{dy}{dx}$. Accept “denominator is never 0”.</p> <p>Accept “is never 0”.</p>

<p>1 (iii)</p>	 <p>The curve has cusps.</p>	<p>G2 B1 [3]</p>	<p>Periodic with three complete cycles. Cusps clearly shown.</p> 
<p>1 (iv)</p>	$\lim_{t \rightarrow 0^+} \left(\frac{\sin t}{1 - k \cos t} \right) \rightarrow +\infty$ $\lim_{t \rightarrow 0^-} \left(\frac{\sin t}{1 - k \cos t} \right) \rightarrow -\infty$ <p>The point is defined at $t = 0$: $x = 0$, $y = 0$.</p> <p>Therefore the curve has a cusp at $(0,0)$</p>	<p>M1 M1 A1 E1 E1 [5]</p>	<p>Limit from one direction only scores M1M0A0</p> 

<p>1 (v)</p>	 <p> $\frac{dy}{dx}$ is infinite when $1 - 2 \cos t = 0$ $t = \frac{\pi}{3}$ $x = \frac{\pi}{3} - 2 \sin \frac{\pi}{3}$ $= -\left(\sqrt{3} - \frac{\pi}{3}\right)$ Hence width of loop is $2\left(\sqrt{3} - \frac{\pi}{3}\right)$ $= 2\sqrt{3} - \frac{2\pi}{3}$ </p>	<p>G2</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>[6]</p>	<p>Any correct value of t.</p> <p>Substituting $\frac{\pi}{3}$ to find width of half the loop.</p> <p>Taking positive value and doubling.</p>
<p>1 (vi)</p>	<p> $\arccos\left(\frac{1}{k}\right) - k \sin\left(\arccos\left(\frac{1}{k}\right)\right) = -\pi$ $k = 4.6033$ </p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>For lhs correct but rhs = $+\pi$ score M1A0A0</p> <div data-bbox="1348 951 1989 1383" style="border: 1px solid black; padding: 5px;"> <p>solve $\left(\cos^{-1}\left(\frac{1}{k}\right) - k \cdot \sin\left(\cos^{-1}\left(\frac{1}{k}\right)\right) = -\pi, k\right)$ $k=4.60334$</p> <hr/> <p> </p> <p><small>More solutions may exist. Try specifying appropriate...</small></p> </div>

2 (i)	$z = \frac{\sqrt{3}+2}{2} - \frac{1}{2}i, \frac{-\sqrt{3}-2}{2} - \frac{1}{2}i, 1-2i$ $\left \frac{\sqrt{3}+2}{2} - \frac{1}{2}i - \left(\frac{-\sqrt{3}-2}{2} - \frac{1}{2}i \right) \right = \left \frac{\sqrt{3}+2}{2} - \frac{1}{2}i - (1-2i) \right =$ $= \left \frac{-\sqrt{3}-2}{2} - \frac{1}{2}i - (1-2i) \right = \sqrt{3}$	<p>B2</p> <p>B2</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>B1 for 1 or 2 correct only.</p> <p>Points marked correctly. B1 for two correct.</p> <p>Evaluation of the distance between the points. All shown to be equal.</p> <p>Any other acceptable method should be awarded full marks.</p> <div data-bbox="1346 916 2004 1362" style="border: 1px solid black; padding: 5px;"> <pre> cSolve(z^3-(3-3*i)*z^2-6*i*z+2+i=0,z) z = (sqrt(3)+2)/2 - 1/2*i or z = (-sqrt(3)-2)/2 - 1/2*i or z = 1-2*i (sqrt(3)+2)/2 - 1/2*i - (-sqrt(3)-2)/2 - 1/2*i sqrt(3) (sqrt(3)+2)/2 - 1/2*i - (1-2*i) sqrt(3) (-sqrt(3)-2)/2 - 1/2*i - (1-2*i) sqrt(3) </pre> </div>
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<p>2 (ii)</p>	<p>$f'(z) = 3z^2 - (6 + 6i)z - 6i$</p> <p>$f'(z) = 3(z - (1 - i))^2$. Repeated root at $z = 1 - i$.</p> <p>or</p> <p>Discriminant $= -(6 + 6i)^2 - 4 \times 3 \times (-6i)$ $= 0$</p> <p>or</p> <p>$\text{csolve}(3z^2 - (6 + 6i)z - 6i = 0, z) = 1 - i$. This root must be repeated as a quadratic will have two roots over the complex numbers.</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>Factorising $f'(z) = 0$.</p> <p>Must show that the root is repeated for these marks by either factorising, finding the discriminant or stating that it must have two solutions over the complex numbers.</p> <p>“$\text{csolve}(3z^2 - (6 + 6i)z - 6i = 0, z) = 1 - i$” without further explanation scores M0A0.</p> <p>Point marked correctly.</p>
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<p>2 (iii)</p>	<p>$g'(z) = 3z^2 + bz + c$ Discriminant: $b^2 - 12c$ Discriminant = 0 $\Rightarrow c = \frac{b^2}{3}$</p>	<p>M1 M1 A1 [3]</p>	<p>Accept $b = \pm\sqrt{3c}$ Award full marks for integrating $k(z - \alpha)^2$ and finding the same result by comparing coefficients.</p>  <p>2/99</p>
<p>2 (iv)</p>	<p>Function that has distinct roots, repeated root to $f'(z) = 0$. Showing that the roots are in an equilateral triangle. Function that has roots that are not in an equilateral triangle and distinct roots to $f'(z) = 0$.</p>	<p>B1 B1 B1 [3]</p>	

2	(v)	$ \beta =1 \text{ and } \arg(\beta) = \pm \frac{\pi}{3}$ $\beta = \left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right) \text{ or } \beta = e^{\pm i\frac{\pi}{3}}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>Evidence of attempt to find modulus and argument or suitable diagram.</p> <p>Either form acceptable.</p>
2	(vi)	$h(z) = z(z - z_1) \left(z - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) z_1 \right)$ $h'(z) = 3z^2 - 3z_1z + \frac{z_1^2}{2} - \frac{z_1\sqrt{3}(2z - z_1)}{2}i$ $= \frac{(6z - z_1(3 + \sqrt{3}i))^2}{12}$ <p>So $h'(z) = 0$ has a single repeated solution.</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>Or equivalent $e^{i\frac{\pi}{3}}$ form</p> <p>Accept the alternative method of using the relationship between b and c from (iii) acceptable.</p> <div data-bbox="1348 646 2000 1093" style="border: 1px solid black; padding: 5px;"> <p>expand $\left(z(z-z_1) \left(z - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) z_1 \right) \right)$</p> $\frac{z(z-z_1)(2z-z_1)}{2} - \frac{z_1\sqrt{3} \cdot z(z-z_1)}{2} \cdot i$ <hr/> $\frac{d}{dz} \left(\frac{z(z-z_1)(2z-z_1)}{2} - \frac{z_1\sqrt{3} \cdot z(z-z_1)}{2} \cdot i \right)$ $3z^2 - 3z_1z + \frac{z_1^2}{2} - \frac{z_1\sqrt{3} \cdot (2z-z_1)}{2} \cdot i$ <hr/> <p>factor $\left(3z^2 - 3z_1z + \frac{z_1^2}{2} - \frac{z_1\sqrt{3} \cdot (2z-z_1)}{2} \cdot i \right) \quad \frac{(6z - z_1(3 + \sqrt{3}i))^2}{12}$</p> </div> <p style="text-align: right;">16/99</p>

3	(i)	<p>Example program: Define program1(m)= Prgm Local i,j For i,1,m For j,1,m If $i^2-3*j^2=1$ Then Disp i,j EndIf EndFor EndFor EndPrgm</p> <p>$x = 2, y = 1; x = 7, y = 4; x = 26, y = 15, x = 97, y = 56$</p>	<p>M6</p> <p>A4</p> <p>[10]</p>	<p>If some (or all) of the answers are incorrect allocate method marks as follows:</p> <p>M1 Appropriate structure program: nested loops or equivalent M1 Appropriate use of variables M1 Maximum value or 100 used. M1 Loop for x or equivalent M1 Loop for y or equivalent M1 Check (If) statement</p> <p>More efficient programs are possible.</p> <p>A1 for each correct x, y pair Subtract (maximum) A1 if any additional value(s) of x and/or y solutions are given (only penalise once).</p>
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3 (ii)		Using $x = 2, y = 1$.	M1	Evidence of using $x = 2, y = 1$ to find at least 1 other solution. At least one shown correctly.
		$x_2 + y_2\sqrt{3} = (2 + \sqrt{3})^2$	A1	
		$\quad = 7 + 4\sqrt{3}$		
		So $x = 7, y = 4$ is a solution.		
		$x_3 + y_3\sqrt{3} = (2 + \sqrt{3})^3$	A1	Other two shown to be solutions based on $x = 2, y = 1$.
		$\quad = 26 + 15\sqrt{3}$		
		So $x = 26, y = 15$ is a solution,		
		$x_3 + y_3\sqrt{3} = (2 + \sqrt{3})^4$		
		$\quad = 97 + 56\sqrt{3}$		
		So $x = 97, y = 56$ is a solution,		
		If $x^2 - 3y^2 = 1$ then $x^2 \approx 3y^2 \therefore \frac{x}{y} \approx \sqrt{3}$	M1	Rearranging $x^2 - 3y^2 = 1$.
		$2, \frac{7}{4}, \frac{26}{15}, \frac{97}{56}$	A1	Must be based on appropriate rearrangement.
			[5]	

3	(iii)	<p>Change input to Define program1(m,n)=</p> <p>Change If $i^2-3j^2=1$ Then to If $i^2-nj^2=1$ Then</p> <p>Smallest solution is $x = 9, y = 4$</p> <p>$\frac{9}{4}$ is not accurate enough.</p> <p>Finding another solution (program or squares...) $x = 161, y = 72$</p> <p>$\frac{161}{72}$ is accurate to within 0.1%</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>Accept method based on changing to $i^2-5j^2=1$ if clear.</p> <p>Seen or implied</p>
3	(iv)	<p>Writing $n = d^2$ would give any solution as $x^2 = (dy)^2 + 1$ But there are no square numbers that are 1 apart.</p>	<p>E2</p> <p>[2]</p>	

ASSESSMENT GRID

Qualification type	GCE	Specification name		Mathematics (MEI)			Unit name		<i>Further Pure Mathematics with Technology</i>
Spec. No.		Year	20XX	Status	choose			Name Setting Task	
Unit No.	4798	Series	Specimen				Max. no. of marks	72	

Q. No. (and part)	Spec. Ref	Assessment Objectives (Defined below or on a separate tab)					Demand of question (e.g. high, medium, low, or target)			Total (mark for question or part)	Marks for each spec area (eg Calculus, c, Algorithms, A, ...)							relevant additional attributes (e.g. synoptic content, extended writing, objective, SPAG, etc.)
		AO1	AO2	AO3	AO4	AO5	A	C	E		C	j	T					
1 (i)		3				2			5	5	5							
1 (ii)		1	2						2	3	3							
1 (iii)		1				2			2	3	3							
1 (iv)		2	2			1	3		2	5	5							
1 (v)		1	3			2			3	6	6							
1 (vi)		1	2				3			3	3							
2 (i)		2	2			2			2	6		6						
2 (ii)		2	2						3	4		4		6				
2 (iii)		1	2						3	3		3		4				
2 (iv)			3						3	3		3		3				
2 (v)		3					3			3		3		3				
2 (vi)			5				5			5		5		5				
3 (i)		2				8			10	10				10				
3 (ii)		3	2				2		3	5				5				
3 (iii)		3				3			2	6				6				
3 (iv)			2				2			2				2				
Auto totals		25	27	0	0	20	18	20	34	72	25	24	23	0	0	0	0	
Targets →		22-28	22-28	0	0	18-28	18	18	36		24	24	24					