Results and formulae needed in examinations

The MEI Students' Handbook is designed for use during the course and contains a large number of results, and other useful information. Not all of this is available in the booklet provided for the examinations, currently designated MF2 and published by OCR.

Consequently there are some results which students need for examinations that they will have to recall or derive. This file contains such results. They are listed by units, rather than by topic as in the Students' Handbook. In a few cases the list would be identical to a page in the Students' Handbook and in that case the page reference is given instead.

Before you go on to look at the lists, there are a number of points we would like to bring to your attention.

- Learning formulae and results is not always the best thing to do. In some cases it is better practice to know how to derive them and to be prepared to do so should the need arise.

- By their nature such lists cannot be completely inclusive. There is almost no limit to the elementary results that a candidate needs. The results listed are those that are given in the Students' Handbook but are not in the examination booklet, MF2.

- The results to be known for any unit include all those that need to be known for earlier units in the same strand. Thus a candidate for C3 needs to recall or derive the results required for C1 and C2.

- The results to be known for an applied unit include some of those needed for equivalent pure units (see the Assumed Knowledge section on the unit's title page).

- There is an underlying assumption that students know all the results needed for Intermediate Tier GCSE.

There is a typesetting error on page 12 of the current printing of the handbook (Fifth edition, July 2004). The integral of $\sin^2 x$ is given at the bottom of the right hand column; the answer should be $\frac{1}{2}\left(x-\frac{1}{2}\sin 2x\right)$ and **not** $\frac{1}{2}\left(x-\frac{1}{2}\sin^2 x\right)$. This will of course be corrected at the next printing.

C1 results that are not given in the examination booklet (page 1)

Quadratic equations

The roots of the equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The discriminant is $\left(b^2 - 4ac\right)$

Discriminant > 0: 2 real roots
Discriminant = 0: 1 repeated root
Discriminant < 0: No real roots

Modulus

$|x|$ means the positive value of x. For $x \geq 0$, $|x| = x$; for $x < 0$, $|x| = -x$.

$|x| < a \Leftrightarrow -a < x < a \quad (a > 0)$

Binomial coefficients

The notations ${}^nC_r$ and $\begin{pmatrix} n \\ r \end{pmatrix}$ are equivalent.

${}^nC_0 = {}^nC_n = 1$

Binomial coefficients may be found using Pascal's triangle:

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

and so on.

Indices

$a^m \times a^n = a^{m+n}$ $\qquad$ $a^m \div a^n = a^{m-n}$ $\qquad$ $a^0 = 1$

$a^{-m} = \dfrac{1}{a^m}$ $\qquad$ $a^{\frac{1}{n}} = \sqrt[n]{a}$ $\qquad$ $\left(a^m\right)^n = a^{mn}$

C1 results that are not given in the examination booklet (page 2)

Straight lines

The line joining (x_1, y_1) to (x_2, y_2) has:

Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

Length $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Equation $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

Mid-point $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Other formulae for the equation of a straight line:

Through $(0, c)$ with gradient *m*: $y = mx + c$

Through (x_1, y_1) with gradient *m*: $y - y_1 = m(x - x_1)$

Through $(a, 0)$ and $(0, b)$: $\frac{x}{a} + \frac{y}{b} = 1$

Perpendicular lines: The product of their gradients $m_1 m_2 = -1$

Circles

The circle with centre (a, b) radius *r* has equation $(x - a)^2 + (y - b)^2 = r^2$

C2 results that are not given in the examination booklet (page 1)

You are also expected to recall or derive C1 results that are not given in the examination booklet.

Logarithms and exponentials

$\log_a(xy) = \log_a x + \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a(x^n) = n\log_a x$

$\log_a\left(\sqrt[n]{x}\right) = \frac{1}{n}\log_a x$

$\log_a\left(\frac{1}{x}\right) = -\log_a x$

$\log_a a = 1$

$\log_a 1 = 0$

$y = a^x \Leftrightarrow x = \log_a y$

Trigonometric ratios of some angles

θ	0°	30°	45°	60°	90°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	0

Trigonometric identities

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\sin^2\theta + \cos^2\theta = 1$

Triangles

Area $= \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$

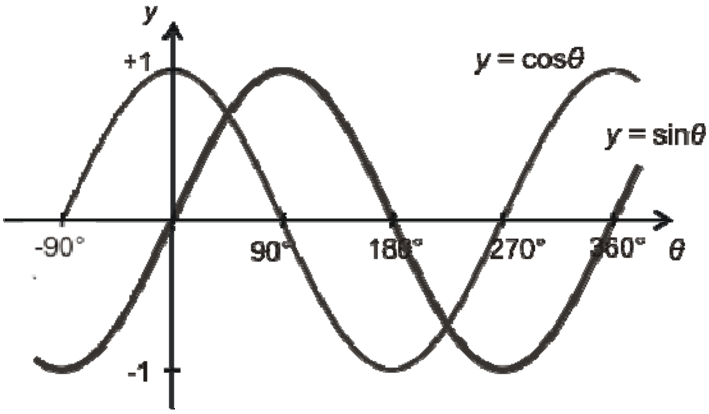
Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

C2 results that are not given in the examination booklet (page 2)

Circular measure

2π radians $= 360^{\circ}$ Arc length $s = r\theta$ Area of sector $A = \frac{1}{2}r^2\theta$

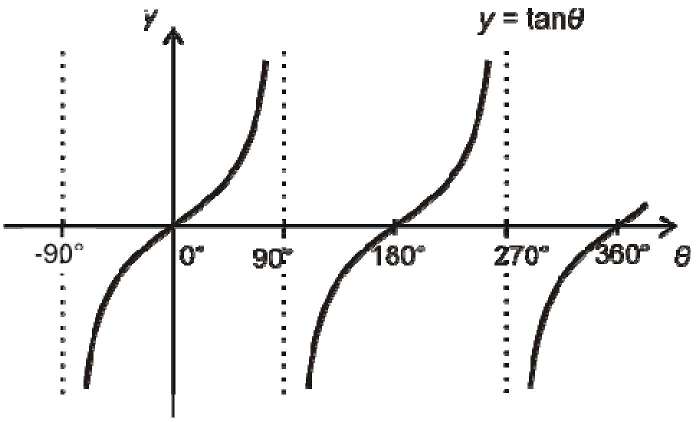
Trigonometrical relationships



$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$

$\sin(\theta + 90°) = \cos\theta$ $\cos(\theta + 90°) = -\sin\theta$

$\sin(\theta + 180°) = -\sin\theta$ $\cos(\theta + 180°) = -\cos\theta$



$\tan(-\theta) = -\tan\theta$

$\tan(\theta + 180°) = \tan\theta$

C2 results that are not given in the examination booklet (page 3)

Differentiation

Differentiation from first principles

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$\frac{dy}{dx}$ is also written as $f'(x)$

For $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$

For $y = f(x) + g(x)$, $\frac{dy}{dx} = f'(x) + g'(x)$

Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

$$\int \left(f'(x) + g'(x)\right) dx = f(x) + g(x) + c$$

Definite integrals

$\int_a^b f(x)dx$ gives the area of the region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$.

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

C3 results that are not given in the examination booklet

You are also expected to recall or derive C1 and C2 results that are not given in the examination booklet.

Exponentials

$y = \mathrm{e}^x \quad \Leftrightarrow \quad x = \log_{\mathrm{e}} x = \ln x$

$\mathrm{e}^{\ln x} = x \qquad \ln\left(\mathrm{e}^x\right) = x$

Differentiation

Standard derivatives

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
e^{kx}	$k\mathrm{e}^{kx}$
$\ln x$	$\dfrac{1}{x}$
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$

Product rule

$$y = uv, \quad \frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$$

Chain rule

y is a function of u and u is a function of x, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u} \times \dfrac{\mathrm{d}u}{\mathrm{d}x}$

C3 results that are not given in the examination booklet (page 2)

Integration

Standard integrals

$f(x)$	$\int f(x)\,dx$ (+ a constant)
e^{kx}	$\frac{1}{k}e^{kx}$
$\frac{1}{x}$	$\ln\lvert x\rvert$
$\sin kx$	$-\frac{1}{k}\cos kx$
$\cos kx$	$\frac{1}{k}\sin kx$

Integration by substitution

$$y = f(u),\ u = g(x), \quad \int f'[g(x)]g'(x)\,dx = f[g(x)] + c$$

C4 results that are not given in the examination booklet

You are also expected to recall or derive C1, C2 and C3 results that are not given in the examination booklet.

Trigonometry

$\sec^2\theta \equiv 1 + \tan^2\theta$
$\text{cosec}^2\theta \equiv 1 + \cot^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Vectors

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} . \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = u_1v_1 + u_2v_2 + u_3v_3$$

FP1 results that are not given in the examination booklet

You are also expected to recall or derive relevant C1 and C2 results that are not given in the examination booklet.

Finite series

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

Complex numbers

Real-imaginary form

For $z = x + \mathrm{j}y$, $\mathrm{Re}(z) = x$ and $\mathrm{Im}(z) = y$

$\mathrm{mod}(z) = |z| = \sqrt{x^2 + y^2}$

$\arg(z) = \theta$ where $\cos\theta = \dfrac{x}{|z|}$ and $\sin\theta = \dfrac{y}{|z|}$ and $-\pi < \theta \le \pi$ ($\arg(0)$ is undefined)

The conjugate of z is z^* where $z^* = x - \mathrm{j}y$

$zz^* = |z|^2$, $\dfrac{1}{z} = \dfrac{x - \mathrm{j}y}{x^2 + y^2} = \dfrac{z^*}{|z|^2}$

Modulus-argument (polar) form

If $|z| = r$ and $\arg(z) = \theta$, then $z = [r,\ \theta] = r(\cos\theta + \mathrm{j}\sin\theta)$

$z^* = r(\cos\theta - \mathrm{j}\sin\theta)$, $\dfrac{1}{z} = \dfrac{1}{r}(\cos\theta - \mathrm{j}\sin\theta)$

Mechanics 1 results that are not given in the examination booklet

No results for Mechanics 1 are given in the examination booklet.

You are expected to be able to recall those on page 14 of the Students' Handbook.

You are also expected to recall or derive relevant C1 and C2 results that are not given in the examination booklet.

Particular attention should be given to the **constant acceleration formulae.**

Written in scalars

$$s = ut + \tfrac{1}{2}at^2 \qquad s = \tfrac{1}{2}(u+v) \qquad s = vt - \tfrac{1}{2}at^2$$

$$v = u + at \qquad v^2 - u^2 = 2as$$

Written in vectors

$$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2 \qquad \mathbf{s} = \tfrac{1}{2}(\mathbf{u}+\mathbf{v}) \qquad \mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$$

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

(There is no simple vector equivalent of $v^2 - u^2 = 2as$.)

Attention should also be given to the equations for **general motion** where the acceleration need not be constant.

Written in scalars

$$v = \frac{ds}{dt} = \dot{s} \qquad s = \int v\,dt$$

$$a = \frac{dv}{dt} = \dot{v} = \frac{d^2s}{dt^2} = \ddot{s} \qquad v = \int a\,dt$$

These results can also be written using vectors; in that case the symbol **r** is often used instead of **s** for displacement.

Mechanics 2 results that are not given in the examination booklet

You are also expected to recall or derive Mechanics 1 results that are not given in the examination booklet.

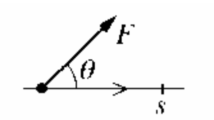
Friction

$F = \mu R$ when sliding takes place
$F \leq \mu R$ when no sliding takes place

Centre of mass

$\left(\sum m\right)\bar{x} = \sum(mx)$ $\left(\sum m\right)\bar{\mathbf{r}} = \sum(m\mathbf{r})$

Work, energy and power



Work done by a constant force, $W = F(s\cos\theta) = (F\cos\theta)s$

Kinetic energy = $\frac{1}{2}mv^2$

Work done against gravity = mgh

Power $= \dfrac{\mathrm{d}W}{\mathrm{d}t} = Fv\cos\theta$

The work energy principle

The total work done by the forces acting on a body is equal to the increase in the kinetic energy of the body.

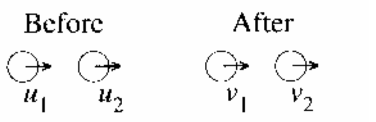
Mechanics 2 results that are not given in the examination booklet (page 2)

Impulse

Impulse $= \mathbf{F}t$ for constant $\mathbf{F}$
Impulse = change in momentum $= m\mathbf{v} - m\mathbf{u}$

Impacts

Newton's Experimental Law



$$v_2 - v_1 = -e\left(u_2 - u_1\right)$$

where e is the coefficient of restitution

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

Statistics 1 results that are not given in the examination booklet (page 1)

Samples

There are two notations, according to whether the data are grouped or not.

Ungrouped data A sample has n observations of x, $x_1, x_2, \ldots x_n$.

Sample mean: $$\bar{x} = \frac{\sum x_i}{n}$$

Sum of squares of deviations: $$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Grouped data A sample has n observations of x, with f_i observations of x_i. $\sum f_i = n$

Sample mean: $$\bar{x} = \frac{\sum x_i f_i}{n}$$

Sum of squares of deviations: $$S_{xx} = \sum (x_i - \bar{x})^2 f_i$$
$$= \sum x_i^2 f_i - \frac{\left(\sum x_i f_i\right)^2}{n} = \sum x_i^2 f_i - n\bar{x}^2$$

For both notations

Mean square deviation: $$msd = \frac{S_{xx}}{n}$$

Root mean square deviation: $$rmsd = \sqrt{msd}$$

Sample variance: $$s^2 = \frac{S_{xx}}{n-1}$$

Sample standard deviation: $$s = \sqrt{s^2}$$

Some calculators give both *rmsd* and *s*; make sure you understand the output from your calculator.

Coding

If $y = a + bx$ then $\bar{y} = a + b\bar{x}$ and $s_y^2 = b^2 s_x^2$

Statistics 1 results that are not given in the examination booklet (page 2)

Selections and arrangements

Number of ways of

* arranging n unlike objects in line $= n!$

* selecting r objects from n unlike objects when the order does not matter $= {}^nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

* selecting r objects from n unlike objects when order does matter $= {}^n\mathrm{P}_r = \dfrac{n!}{(n-r)!}$

Conditional probability

$$\mathrm{P}(B|A) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)}$$

Discrete random variables

[The formulae for finding the mean or expectation, $\mathrm{E}(X)$, and the variance, $\mathrm{Var}(X)$, of X are given in MF2.]

$\mathrm{E}(a+bX) = a + b\mathrm{E}(X)$, $\mathrm{Var}(a+bX) = b^2\mathrm{Var}(X)$

The binomial distribution

For the binomial distribution, $\mathrm{B}(n, p)$, the random variable, X, is the number of successes from n independent trials of a process for which $\mathrm{P(success)} = p$.

$\mathrm{P}(X = r) = {}^nC_r\, p^r q^{n-r}$ for $r = 0, 1, 2, \ldots, n$ where $q = 1 - p$.

Statistics 1 results that are not given in the examination booklet (page 3)

The binomial hypothesis test

(A description of the process of hypothesis testing is given on page 18 of the Students' Handbook.)

Null hypothesis	H_0:	The probability of the underlying population, p, has a given value
Alternative hypotheses	H_1:	$p \neq$ the given value (2-tail test)
	or	$p >$ the given value (1-tail test)
	or	$p <$ the given value (1-tail test)
Test statistic		Observed number of successes in a sample of size n trials
Critical values		Can be calculated, or derived from cumulative frequency tables

Statistics 2 results that are not given in the examination booklet

You are also expected to recall or derive Statistics 1 results that are not given in the examination booklet.

The χ^2 test on a contingency table

The test statistic is $X^2 = \sum \frac{(f_o - f_e)^2}{f_e}$.

For a contingency table with r rows and c columns there are $(r-1)(c-1)$ degrees of freedom.

Product moment correlation and regression

The formulae are given in the examination booklet but you are advised to work with the forms using sums of squares and sums of products, as given below.

Correlation coefficient: $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

Least squares regression line: $y - \bar{y} = b(x - \bar{x})$ where $b = \frac{S_{xy}}{S_{xx}}$

Decision Mathematics 1 results that are not given in the examination booklet

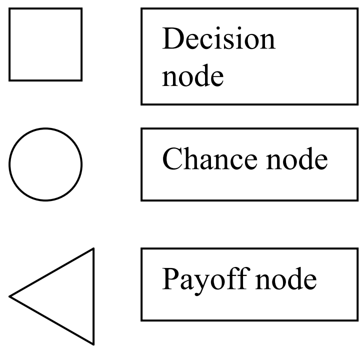
No results are given in the examination booklet.

You are expected to be able to recall or derive those on page 26 of the Students' Handbook, and those at the top of page 28.

Decision Mathematics 2 results that are not given in the examination booklet

You are also expected to recall or derive Decision Mathematics 1 results that are not given in the examination booklet.

Decision trees



EMV   Expected monetary value

Propositional connectives

$\vee$	or	$\wedge$	and
$\Leftrightarrow$	equivalence	$\Rightarrow$	implies
$\sim$	not		

Distributive laws

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$
$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

de Morgan's Laws

$$\sim (p \wedge q) \;\Leftrightarrow\; \sim p \vee \sim q$$
$$\sim (p \vee q) \;\Leftrightarrow\; \sim p \wedge \sim q$$

Decision Mathematics 2 results that are not given in the examination booklet (page 2)

Algorithms and vocabulary

See page 28 of the Students' Handbook for step-by-step descriptions of various algorithms and also for explanations of standard terms used in linear programming.

Decision Computation results that are not given in the examination booklet

You are also expected to recall or derive Decision Mathematics 1 results that are not given in the examination booklet.

Recurrence relations

First order

For $u_{n+1} = au_n + \mathrm{f}(n) \quad (a \neq 1)$,

if $\mathrm{f}(n) = 0$, $u_n = a^n u_0$
otherwise, $u_n = Aa^n +$ particular solution

For $u_{n+1} = u_n + \mathrm{f}(n)$ (i.e. $a = 1$ in the above relation),

$$u_{n+1} = u_0 + \sum_{i=0}^{n} \mathrm{f}(i)$$

Second order

For $u_{n+2} + au_{n+1} + bu_n = \mathrm{f}(n)$

The auxiliary equation is $\lambda^2 + a\lambda + b = 0$ with roots λ_1 and λ_2

If $\mathrm{f}(n) = 0$, $u_n = A\lambda_1^n + B\lambda_2^n \quad (\lambda_1 \neq \lambda_2)$

$u_n = (An + B)\lambda^n \quad (\lambda_1 = \lambda_2 = \lambda)$

If $\mathrm{f}(n) \neq 0$, a particular solution is added to the corresponding expression for u_n where $\mathrm{f}(n) = 0$.

Algorithms and vocabulary

See page 28 of the Students' Handbook for step-by-step descriptions of various algorithms and also for explanations of standard terms used in linear programming.

Numerical Methods results that are not given in the examination booklet

You are also expected to recall or derive C1 and C2 results that are not given in the examination booklet.

You will be given all the results except those below for numerical differentiation.

Numerical differentiation

$$\mathrm{f}'(x) \approx \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}, \quad \mathrm{f}'(x) \approx \frac{\mathrm{f}(x+h)-\mathrm{f}(x-h)}{2h}$$