OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4754/01A

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Tuesday 16 June 2015 – Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4754/01A
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (NH/FD) 5312830951 109566/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

This paper will be followed by PAPER B: COMPREHENSION.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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SECTION A (36 marks)

1 Solve the equation $\frac{5x}{2x+1} - \frac{3}{x+1} = 1$. [5]

2 Express $6\cos 2\theta + \sin\theta$ in terms of $\sin\theta$.

Hence solve the equation $6\cos 2\theta + \sin\theta = 0$, for $0° \leqslant \theta \leqslant 360°$. [7]

3 (i) Find the first three terms of the binomial expansion of $\frac{1}{\sqrt[3]{1-2x}}$. State the set of values of x for which the expansion is valid. [5]

(ii) Hence find a and b such that

$$\frac{1-3x}{\sqrt[3]{1-2x}} = 1 + ax + bx^2 + \ldots .$$ [3]

4 You are given that $\mathrm{f}(x) = \cos x + \lambda \sin x$ where λ is a positive constant.

(i) Express $\mathrm{f}(x)$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving R and α in terms of λ. [4]

(ii) Given that the maximum value (as x varies) of $\mathrm{f}(x)$ is 2, find R, λ and α, giving your answers in exact form. [4]

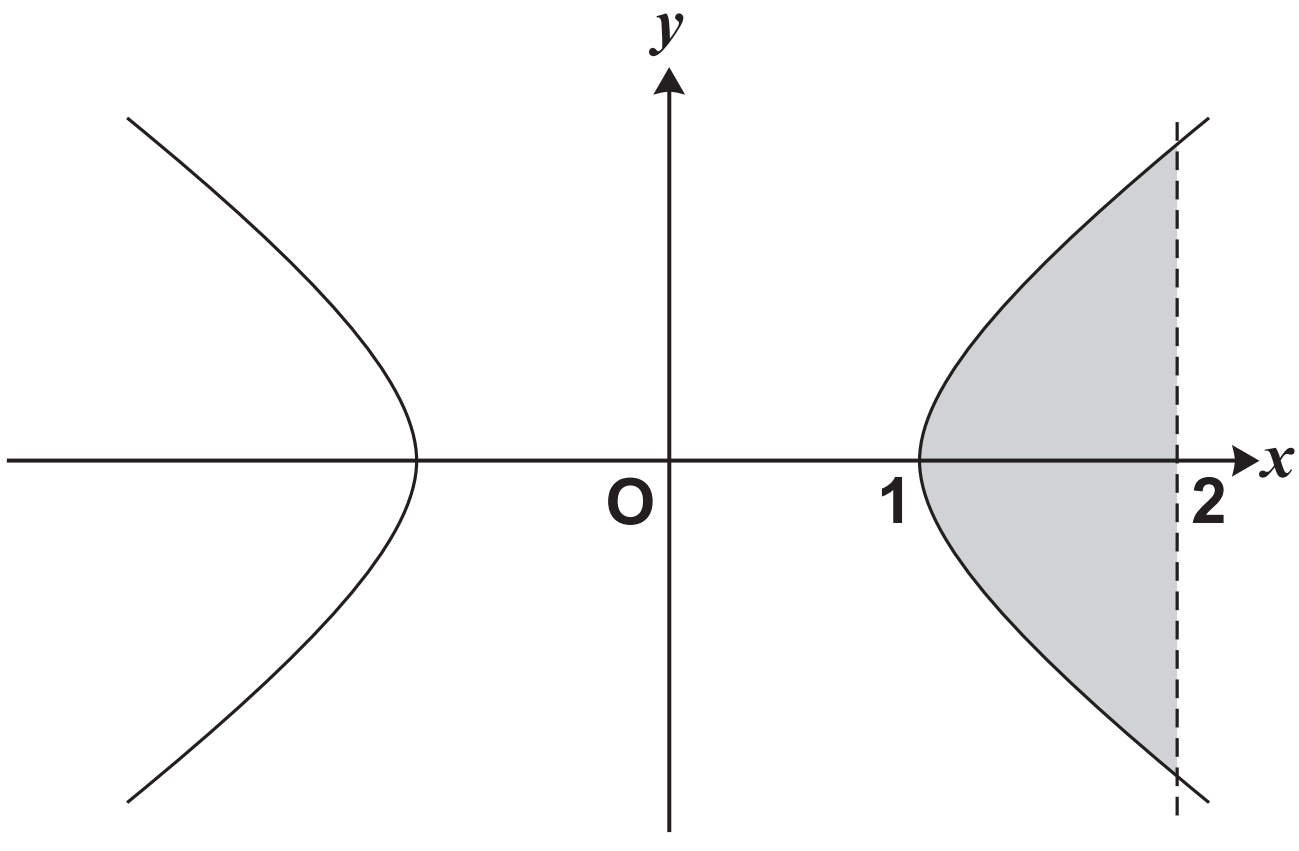
5 A curve has parametric equations $x = \sec\theta$, $y = 2\tan\theta$.

(i) Given that the derivative of $\sec\theta$ is $\sec\theta\tan\theta$, show that $\dfrac{dy}{dx} = 2\,\text{cosec}\,\theta$. [3]

(ii) Verify that the cartesian equation of the curve is $y^2 = 4x^2 - 4$. [2]

Fig. 5 below shows the region enclosed by the curve and the line $x = 2$. This region is rotated through 180° about the x-axis.

FIG. 5



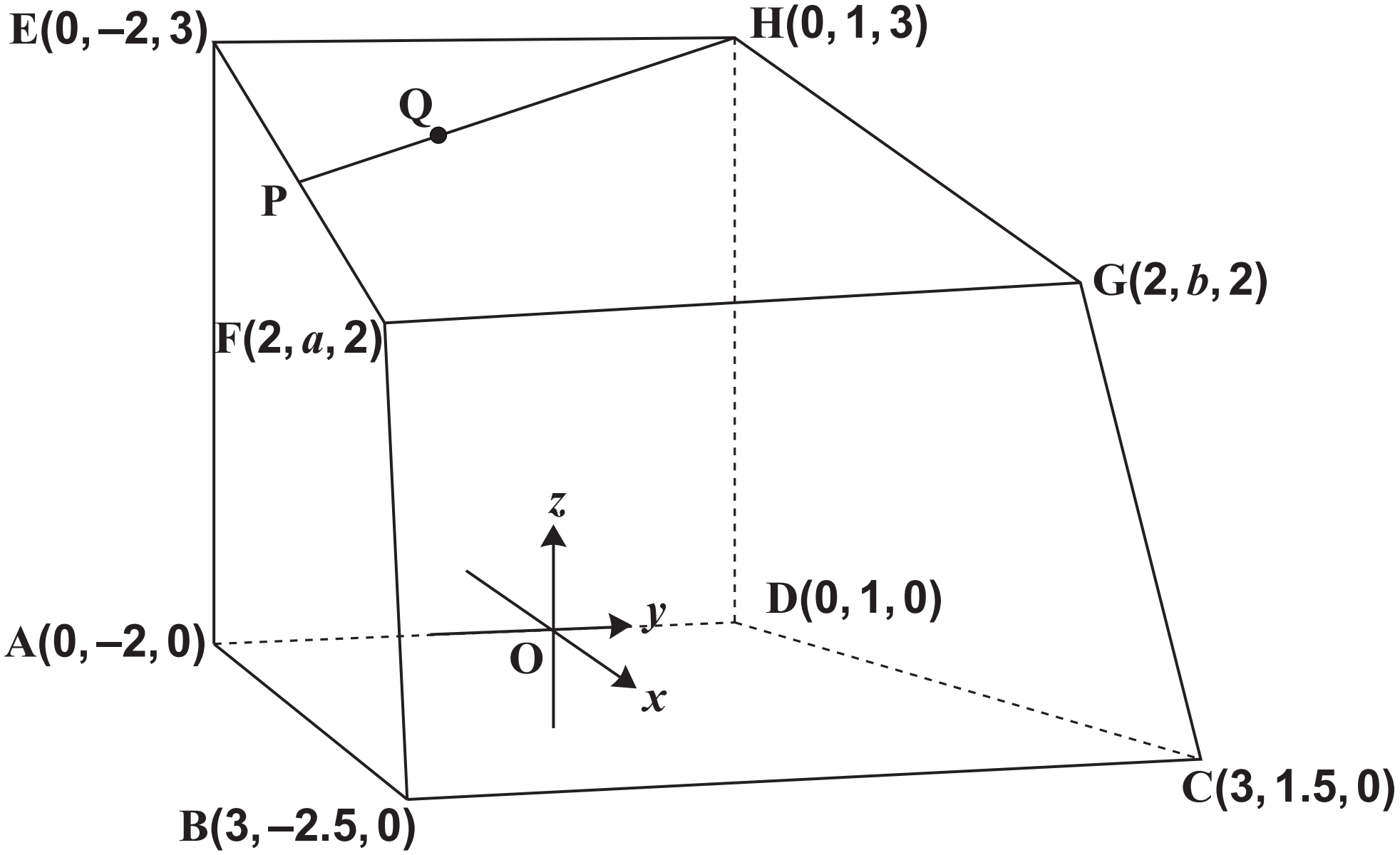
(iii) Find the volume of revolution produced, giving your answer in exact form. [3]

SECTION B (36 marks)

6 Fig. 6 below shows a lean-to greenhouse $\mathbf{ABCDHEFG}$. The diagram is a two-dimensional representation of a three-dimensional shape.

With respect to coordinate axes $\mathbf{O}xyz$, the coordinates of the vertices are as shown. All distances are in metres. Ground level is the plane $z = 0$.

FIG. 6



(i) Verify that the equation of the plane through A, B and E is $x + 6y + 12 = 0$.

Hence, given that F lies in this plane, show that $a = -2\frac{1}{3}$. [4]

(ii) (*A*) Show that the vector $\begin{pmatrix} 1 \\ -6 \\ 0 \end{pmatrix}$ is normal to the plane DHC. [2]

(*B*) Hence find the cartesian equation of this plane. [2]

(*C*) Given that G lies in the plane DHC, find b and the length FG. [2]

(iii) Find the angle EFB. [5]

A straight wire joins point H to a point P which is half way between E and F. Q is a point two-thirds of the way along this wire, so that $\mathrm{HQ} = 2\mathrm{QP}$.

(iv) Find the height of Q above the ground. [3]

7 A drug is administered by an intravenous drip. The concentration, x, of the drug in the blood is measured as a fraction of its maximum level. The drug concentration after t hours is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(1 + x - 2x^2),$$

where $0 \leqslant x < 1$, and k is a positive constant. Initially, $x = 0$.

(i) Express $\dfrac{1}{(1+2x)(1-x)}$ in partial fractions. [3]

(ii) Hence solve the differential equation to show that $\dfrac{1+2x}{1-x} = \mathrm{e}^{3kt}$. [7]

(iii) After 1 hour the drug concentration reaches 75% of its maximum value and so $x = 0.75$.

Find the value of k, and the time taken for the drug concentration to reach 90% of its maximum value. [3]

(iv) Rearrange the equation in part (ii) to show that $x = \dfrac{1 - \mathrm{e}^{-3kt}}{1 + 2\mathrm{e}^{-3kt}}$.

Verify that in the long term the drug concentration approaches its maximum value. [5]

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