OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4753/01

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

QUESTION PAPER

FRIDAY 12 JUNE 2015:

Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4753/01
MEI Examination Formulae and Tables (MF2)
Insert for Question 9(iv)

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (RW/SG) 6232636622 115507

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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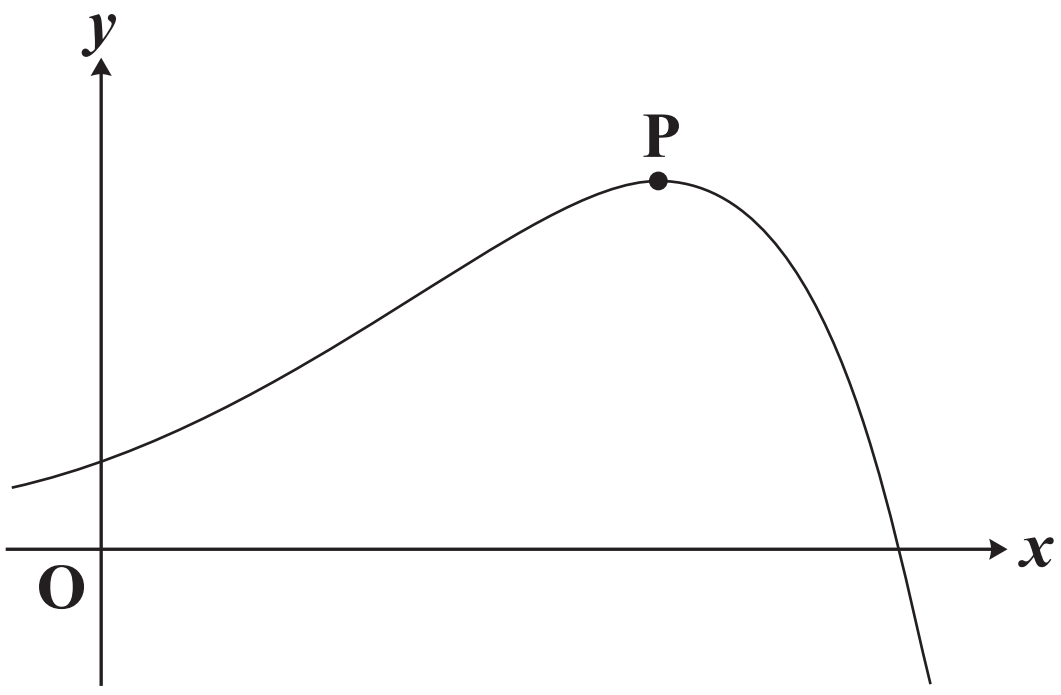
SECTION A (36 marks)

1 Fig. 1 shows part of the curve $y = e^{2x}\cos x$.

FIG. 1



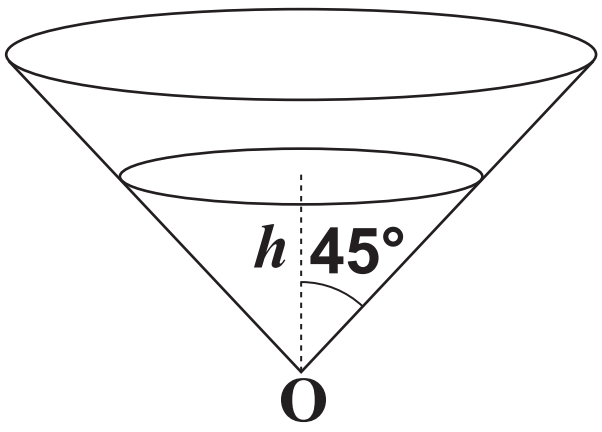
Find the coordinates of the turning point P. [6]

2 Find $\int \sqrt[3]{2x-1}\,dx$. [4]

3 Find the exact value of $\int_1^2 x^3 \ln x\,dx$. [5]

4 Fig. 4 shows a cone with its axis vertical. The angle between the axis and the slant edge is 45°. Water is poured into the cone at a constant rate of 5 cm^3 per second. At time t seconds, the height of the water surface above the vertex O of the cone is h cm, and the volume of water in the cone is V cm^3.

FIG. 4



Find V in terms of h.

Hence find the rate at which the height of water is increasing when the height is 10 cm.

[You are given that the volume V of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$]. [5]

5 A curve has implicit equation $y^2 + 2x \ln y = x^2$.

Verify that the point (1, 1) lies on the curve, and find the gradient of the curve at this point. [6]

6 Solve each of the following equations, giving your answers in exact form.

(i) $6\arcsin x - \pi = 0$. [2]

(ii) $\arcsin x = \arccos x$. [2]

7 (i) The function $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = \frac{1-x}{1+x},\ x \neq -1.$$

Show that $\mathrm{f}(\mathrm{f}(x)) = x$.

Hence write down $\mathrm{f}^{-1}(x)$. [3]

(ii) The function $\mathrm{g}(x)$ is defined for all real x by

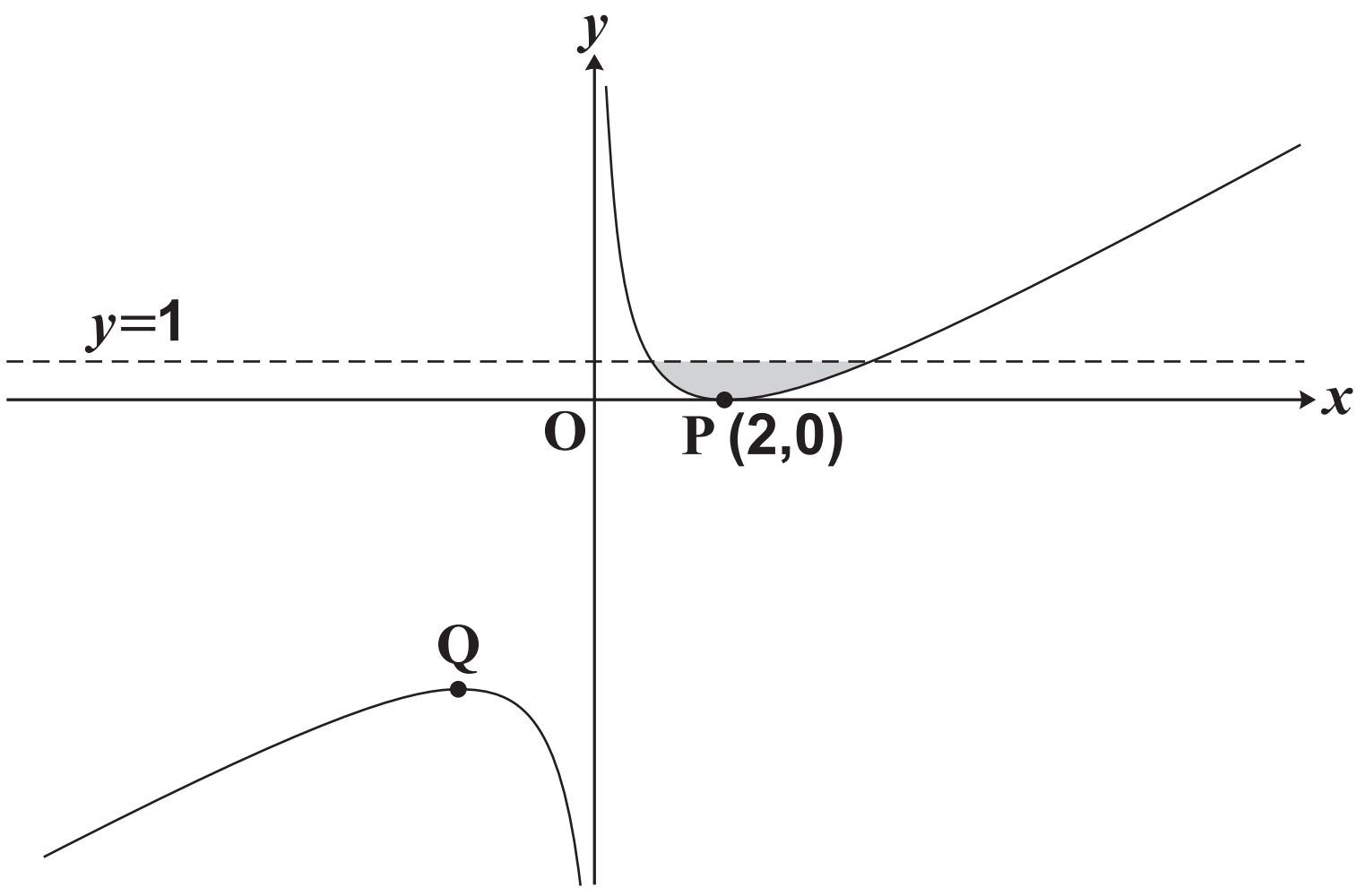
$$\mathrm{g}(x) = \frac{1-x^2}{1+x^2}.$$

Prove that $\mathrm{g}(x)$ is even. Interpret this result in terms of the graph of $y = \mathrm{g}(x)$. [3]

SECTION B (36 marks)

8 Fig. 8 shows the line $y = 1$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{(x-2)^2}{x}$. The curve touches the x-axis at $\mathrm{P}(2, 0)$ and has another turning point at the point Q.

FIG. 8



(i) Show that $f'(x) = 1 - \frac{4}{x^2}$, and find $f''(x)$.

Hence find the coordinates of Q and, using $f''(x)$, verify that it is a maximum point. [7]

(ii) Verify that the line $y = 1$ meets the curve $y = f(x)$ at the points with x-coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve. [6]

The curve $y = f(x)$ is now transformed by a translation with vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. The resulting curve has equation $y = g(x)$.

(iii) Show that $g(x) = \frac{x^2 - 3x}{x + 1}$. [3]

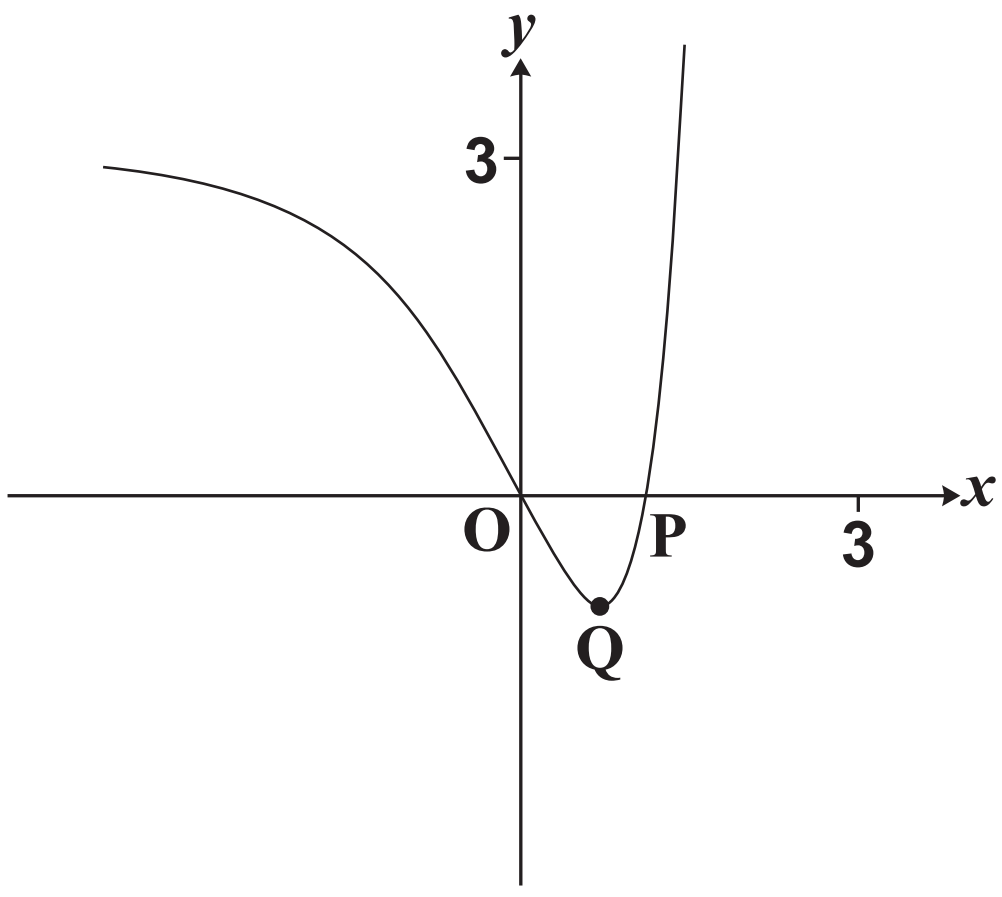
(iv) Without further calculation, write down the value of $\int_0^3 g(x)\,dx$, justifying your answer. [2]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where

$\mathrm{f}(x) = (\mathrm{e}^x - 2)^2 - 1, \quad x \in \mathbb{R}$.

The curve crosses the x-axis at O and P, and has a turning point at Q.

FIG. 9



(i) Find the exact x-coordinate of P. [2]

(ii) Show that the x-coordinate of Q is $\ln 2$ and find its y-coordinate. [4]

(iii) Find the exact area of the region enclosed by the curve and the x-axis. [5]

The domain of $\mathrm{f}(x)$ is now restricted to $x \geqslant \ln 2$.

(iv) Find the inverse function $\mathrm{f}^{-1}(x)$. Write down its domain and range, and sketch its graph on the copy of Fig. 9. [7]

END OF QUESTION PAPER

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