OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4756/01

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

MONDAY 22 JUNE 2015: Morning

DURATION: 1 hour 30 minutes plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4756/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LEG) 6246067871 115852/2

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.

WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (54 marks)

1 (a) (i) A curve has polar equation $r = 2a\cos\theta + 2b\sin\theta$, where $a > 0$ and $b > 0$.

Show, by considering its cartesian equation, that the curve is a circle which passes through the origin. Find the centre and radius of the circle in terms of a and b. [5]

(ii) For the case $a = b = 1$, use integration to show that the region bounded by a minor arc of the circle and the lines $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{3}$ has area $1 + \frac{\pi}{3}$. [5]

(b) Given that $\mathrm{f}(t) = \ln(1+t)$, obtain expressions for $\mathrm{f}'(t)$, $\mathrm{f}''(t)$ and $\mathrm{f}'''(t)$. Hence show that the Maclaurin series for $\ln(1+t)$ begins

$$t - \frac{t^2}{2} + \frac{t^3}{3} \dots .$$

Deduce the first two non-zero terms of the Maclaurin series for $\ln\left(\frac{1+t}{1-t}\right)$. [8]

2 **(a)** **(i)** By considering $\left(z+\frac{1}{z}\right)^5$, where

$z = \cos\theta + \mathrm{j}\sin\theta$, show that

$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$. [5]

(ii) Use de Moivre's theorem to find an expression for $\cos 5\theta$ in terms of powers of $\cos\theta$. [5]

(b) **(i)** Obtain the roots of the equation $w^5 = 4\sqrt{2}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$. Show the points corresponding to these roots in an Argand diagram. [4]

(ii) For each root w, let $v = w\sqrt{2}\,\mathrm{e}^{\mathrm{j}\pi/10}$.

Show the points corresponding to the values of v on your Argand diagram.

Find, in simplified form, an equation for which the values of v are the roots. [4]

3 This question concerns the matrix $\mathbf{M}$ where

$$\mathbf{M} = \begin{pmatrix} 5 & -1 & 3 \\ 4 & -3 & -2 \\ 2 & 1 & 4 \end{pmatrix}.$$

(i) Obtain the characteristic equation of $\mathbf{M}$.

Find the eigenvalues of $\mathbf{M}$. [7]

These eigenvalues are denoted by λ_1, λ_2, λ_3, where $\lambda_1 < \lambda_2 < \lambda_3$.

(ii) Verify that an eigenvector corresponding to λ_1 is $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ and that an eigenvector corresponding to λ_2 is $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. Find an eigenvector of the form $\begin{pmatrix} a \\ 1 \\ c \end{pmatrix}$ corresponding to λ_3. [5]

(iii) Write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M} = \mathbf{PDP}^{-1}$. (You are not required to calculate $\mathbf{P}^{-1}$.)

Hence write down an expression for $\mathbf{M}^4$ in terms of $\mathbf{P}$ and a diagonal matrix. You should give the elements of the diagonal matrix explicitly. [3]

(iv) Use the Cayley-Hamilton theorem to obtain an expression for $\mathbf{M}^4$ as a linear combination of $\mathbf{M}$ and $\mathbf{M}^2$. [3]

SECTION B (18 marks)

4 (i) Starting with the relationship $\cosh^2 t - \sinh^2 t = 1$, deduce a relationship between $\tanh^2 t$ and $\operatorname{sech}^2 t$. [1]

You are given that $y = \operatorname{artanh} x$.

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{1 - x^2}$. [4]

(iii) Show, by integrating the result in part (ii), that

$y = \dfrac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right)$. [4]

(iv) Show that $\displaystyle\int_0^{\frac{\sqrt{3}}{6}} \frac{1}{1 - 3x^2}\,\mathrm{d}x = \frac{1}{\sqrt{3}}\operatorname{artanh}\frac{1}{2}$. Express this answer in logarithmic form. [4]

(v) Use integration by parts to find $\displaystyle\int \operatorname{artanh} x \,\mathrm{d}x$, giving your answer in terms of logarithms. [5]

END OF QUESTION PAPER