OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

4754/01B

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper B: Comprehension

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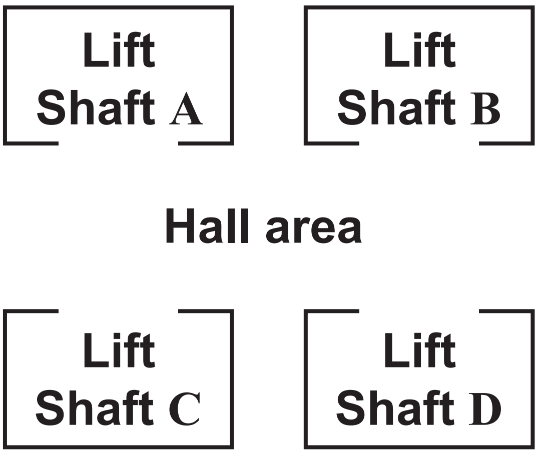
SCHEDULING LIFTS

Introduction

Most modern high-rise office buildings have several lift shafts located together around a central point in the building. Fig. 1 below illustrates the plan view of a possible layout for each floor of a building having four lifts arranged around a hall area.

FIG. 1



In each lift shaft, a car travels up and down between the floors of the building.

When calling a car from the hall area, the passenger presses either an ‘up’ button or a ‘down’ button to indicate the direction in which he or she wishes to travel. The way in which the lifts are programmed will determine which lift stops for the passenger. The time that elapses between calling a car and the car arriving is called the ‘wait time’. On entering the car the passenger selects the floor number to which he or she wishes to travel.

To operate effectively, high-rise buildings depend on the efficiency of their lifts. Some buildings have cars that can stop at all floors; in other buildings some of the cars are restricted to certain floors. This article looks at several ways in which lifts are programmed in office buildings.

Buildings with a single lift

In a building served by just one lift, a typical upward then downward journey will be as follows. When moving upwards, the car will only stop to pick up passengers if they wish to travel upwards. When all these passengers have reached their destinations, the car will travel to the highest floor from which a downward request has been made. It will then travel downwards, satisfying other downward requests as it does so.

In some buildings, when there are no further requests the car is programmed to return to the ground floor since that is where most calls are made; this should be expected to minimise the average wait time for passengers.

Other methods can be employed in buildings with just one lift shaft to improve the efficiency of the lift. An example of such a method is that a lift could be programmed to stop at the even-numbered floors only. In such a building, a passenger who wants to reach the seventh floor would need to leave at the sixth or eighth floor. If this decision is taken during the construction of the building it has the financial benefit of not requiring investment in entrances on the odd-numbered floors. In addition it reduces ‘transfer time’; this is the time spent opening doors, transferring people in and out of the car and closing the doors. One obvious drawback of this arrangement is the problem it causes for people who cannot negotiate stairs, such as those who use wheelchairs.

Average number of stops

Imagine that 7 people enter a car on the ground floor of a building which has 10 floors above the ground floor; these will be called ‘upper floors’. Assuming that the requests are independent and all floors are equally likely to be selected, the probability that these 7 people will all request different floors is

$$\frac{10}{10}\times\frac{9}{10}\times\frac{8}{10}\times\frac{7}{10}\times\frac{6}{10}\times\frac{5}{10}\times\frac{4}{10}.$$

This is approximately equal to 0.06. So the probability that there will be at least one floor where more than one person leaves the car is approximately 0.94. Thus it is very likely that the car will actually be required to stop at fewer than 7 floors. This raises the question: what is the average number of floors at which the car will stop?

To answer this question we assume that the requests are independent, that all floors are equally likely to be selected and that the car will not be making any stops to pick up additional passengers; this final assumption is realistic at the start of a working day. Letting n represent the number of passengers and f the number of upper floors, under the given assumptions the probability that nobody selects a particular floor is

$$\left(\frac{f-1}{f}\right)^{n}.$$

Therefore the probability that at least one person will select a particular floor is

$$1 - \left(\frac{f-1}{f}\right)^{n}.$$

This probability is the same for all f floors. It gives the average proportion of floors at which the car stops. It follows that the average number of floors at which the car stops is

$$f\left\{1 - \left(\frac{f-1}{f}\right)^{n}\right\}.$$

Table 2 opposite shows the average numbers of floors at which cars carrying up to 15 passengers would stop in buildings with up to 10 upper floors; each value is given to an accuracy of one decimal place.

For example, in a building with 10 upper floors, a car carrying 15 passengers would, on average, stop at approximately 8 floors.

Buildings with multiple lifts

It is not uncommon for more than 1000 people to work in a single high-rise office building. When designing these buildings, the architect has to reach a compromise between allocating space to lift shafts and space to offices. It is normal to find a hall area in such buildings that is occupied by several lift shafts. These lifts are programmed to operate in different ways depending on the expected demands at different times of day.

Upward peak-time operation

During the morning rush hour, when people arrive for work, or at the end of the lunch break, the cars are programmed to return to the ground floor. Each car leaves when it reaches its maximum passenger load or it has had its doors open for a certain period of time.

For the rest of this article we will consider an office building with a ground floor and 10 upper floors served by 4 lifts, A, B, C and D, which are grouped together around a hall area. Each car can hold 15 people. 30 people work on each of the upper floors.

TABLE 2

Number of upper floors, f \ Number of passengers, n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.0	1.5	1.8	1.9	1.9	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
3	1.0	1.7	2.1	2.4	2.6	2.7	2.8	2.9	2.9	2.9	3.0	3.0	3.0	3.0	3.0
4	1.0	1.8	2.3	2.7	3.1	3.3	3.5	3.6	3.7	3.8	3.8	3.9	3.9	3.9	3.9
5	1.0	1.8	2.4	3.0	3.4	3.7	4.0	4.2	4.3	4.5	4.6	4.7	4.7	4.8	4.8
6	1.0	1.8	2.5	3.1	3.6	4.0	4.3	4.6	4.8	5.0	5.2	5.3	5.4	5.5	5.6
7	1.0	1.9	2.6	3.2	3.8	4.2	4.6	5.0	5.3	5.5	5.7	5.9	6.1	6.2	6.3
8	1.0	1.9	2.6	3.3	3.9	4.4	4.9	5.3	5.6	5.9	6.2	6.4	6.6	6.8	6.9
9	1.0	1.9	2.7	3.4	4.0	4.6	5.1	5.5	5.9	6.2	6.5	6.8	7.1	7.3	7.5
10	1.0	1.9	2.7	3.4	4.1	4.7	5.2	5.7	6.1	6.5	6.9	7.2	7.5	7.7	7.9

To model the use of the lifts at upward peak-time, we make the following simplifying assumptions.

On the ground floor there are always 15 people waiting to enter any given car.

For any car on the ground floor, the total time taken for its doors to open, 15 people to enter and the doors to close is 20 seconds; this is called the ‘loading time’.

Each car takes 3 seconds to move between adjacent floors.

On an upper floor the transfer time, to open the car doors, let passengers out and close the doors, is 15 seconds. (It is assumed nobody enters a car on an upper floor during this peak time.)

Nobody uses the stairs to walk between floors.

There are many ways in which the lifts could be programmed to transfer these 300 people to their offices. For most journeys, more than half the journey time is composed of load time and transfer time; minimising this is a major consideration when designing an effective strategy. Three different strategies are considered below.

Strategy 1

All four cars serve every floor. On the return journey from the top floor to the ground floor, nobody enters the car and so the descent takes 30 seconds. Table 3 opposite gives the timings for a round trip for one car that is required to stop at every floor.

TABLE 3

	Arrival time (seconds)	Departure time (seconds)
Ground floor	0	20
Floor 1	23	38
Floor 2	41	56
Floor 3	59	74
Floor 4	77	92
Floor 5	95	110
Floor 6	113	128
Floor 7	131	146
Floor 8	149	164
Floor 9	167	182
Floor 10	185	200
Return to ground floor	230	

With this strategy, and assuming every car stops at every floor in each trip, it will require 5 trips for each of the 4 cars to transport the 300 people to their floors. It will take 19 minutes 10 seconds for the cars to complete these trips and return to the ground floor.

However, from Table 2 it can be seen that on average there will be approximately 8 stops per trip. A round trip with 8 stops would take between 188 and 200 seconds.

Strategy 2

All 4 cars serve the ground floor. In addition, cars A and B serve floors 1 to 6 and cars C and D serve floors 7 to 10.

Table 4 below gives the timings for round trips in which the cars are required to stop at every floor they serve; Table 2 suggests this is a common occurrence in this case.

TABLE 4

	Cars A and B		Cars C and D	
	Arrival time (seconds)	Departure time (seconds)	Arrival time (seconds)	Departure time (seconds)
Ground floor	0	20	0	20
Floor 1	23	38		
Floor 2	41	56		
Floor 3	59	74		
Floor 4	77	92		
Floor 5	95	110		
Floor 6	113	128		
Floor 7			41	56
Floor 8			59	74
Floor 9			77	92
Floor 10			95	110
Return to ground floor	146		140	

With this strategy, and assuming every car stops at every floor it serves in each trip, it will take 14 minutes 36 seconds for the 4 cars to transport the 300 people to their floors and return to the ground floor.

Strategy 3

All 4 cars serve the ground floor. In addition, car A serves floors 1 to 3, car B serves floors 4 to 6, car C serves floors 7 and 8, and car D serves floors 9 and 10.

Assuming the cars are required to stop at every floor they serve, with this strategy car B is going to take longer than car A to complete each of its trips, and car D is going to take longer than car C.

One round trip for each of cars B and D is summarised in Table 5 below.

TABLE 5

	Car B		Car D	
	Arrival time (seconds)	Departure time (seconds)	Arrival time (seconds)	Departure time (seconds)
Ground floor	0	20	0	20
Floor 1				
Floor 2				
Floor 3				
Floor 4	32	47		
Floor 5	50	65		
Floor 6	68	83		
Floor 7				
Floor 8				
Floor 9			47	62
Floor 10			65	80
Return to ground floor	101		110	

Car B will be the last to complete its trips, returning to the ground floor after 10 minutes 6 seconds. This would seem to be the most efficient strategy but the assumption that the cars will always be filled in 20 seconds at the ground floor despite the fact that they serve only 2 or 3 floors is, perhaps, not realistic.

Other strategies are possible. In taller buildings, express lifts are often used. For example, in a building with 50 floors, an express lift might take people from the ground floor to floor 40 without stopping, and another lift then serves all higher floors.

Downward peak-time operation

At the start of the lunch break or the end of the working day, a high percentage of the office workers will need to travel to the ground floor in a short period of time. At these times, cars can be programmed to occupy the highest floors, each car on a different floor, to await a call.

Off-peak operation

During the working day, the office workers move between floors and in and out of the building. Whereas at peak times the cars mainly carry passengers in one direction, at off-peak times this is not the case; round trips often involve similar numbers of people travelling up and travelling down. In order to minimise wait time for passengers, different strategies are needed and these depend on several factors including the distribution of the population and the relative attraction of each floor.

Conclusion

In this article several simplifying assumptions have been made. In reality, to improve the efficiency of the lifts, it is necessary to carry out an in-depth study of the ways in which the users of the building use the lifts. Complex mathematical models are created to simulate lift use at different times of day and these provide optimal strategies for the lifts so that buildings can function efficiently.

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