OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE
4755/01
MATHEMATICS (MEI)
Further Concepts for Advanced Mathematics (FP1)

THURSDAY 14 MAY 2015: Morning

DURATION: 1 hour 30 minutes plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4755/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LEG) 5316604966 109287

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (36 marks)

1 Given that $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, where $\mathbf{M} = \begin{pmatrix} 4 & -3 \\ 8 & 21 \end{pmatrix}$, find x and y. [6]

2 Find the roots of the quadratic equation $z^2 - 4z + 13 = 0$.

Find the modulus and argument of each root. [5]

3 The equation $2x^3 + px^2 + qx + r = 0$ has a root at $x = 4$. The sum of the roots is 6 and the product of the roots is -10. Find p, q and r. [6]

4 Indicate, on a single Argand diagram

(i) the set of points for which $\arg(z-(-1-\mathrm{j}))=\frac{\pi}{4}$, [2]

(ii) the set of points for which $|z-(1+2\mathrm{j})|=2$, [2]

(iii) the set of points for which $|z-(1+2\mathrm{j})|\geqslant 2$ and $0\leqslant\arg(z-(-1-\mathrm{j}))\leqslant\frac{\pi}{4}$. [2]

5 (i) Show that $\sum_{r=1}^{n}(2r-1)=n^2$. [3]

(ii) Show that $\dfrac{\sum_{r=1}^{n}(2r-1)}{\sum_{r=n+1}^{2n}(2r-1)}=k$, where k is a constant to be determined. [4]

6 A sequence is defined by $u_1=3$ and $u_{n+1}=3u_n-5$. Prove by induction that $u_n=\dfrac{3^{n-1}+5}{2}$. [6]

SECTION B (36 marks)

7 A curve has equation $y = \dfrac{(3x+2)(x-3)}{(x-2)(x+1)}$.

(i) Write down the equations of the three asymptotes and the coordinates of the points where the curve crosses the axes. [4]

(ii) Sketch the curve, justifying how it approaches the horizontal asymptote. [5]

(iii) Find the set of values of x for which $y \geqslant 3$. [3]

8 The complex number 5 + 4j is denoted by α.

(i) Find α^2 and α^3, showing your working. [3]

(ii) The real numbers q and r are such that $\alpha^3 + q\alpha^2 + 11\alpha + r = 0$. Find q and r. [4]

Let $\mathrm{f}(z) = z^3 + qz^2 + 11z + r$, where q and r are as in part (ii).

(iii) Solve the equation $\mathrm{f}(z) = 0$. [3]

(iv) Solve the equation $z^4 + qz^3 + 11z^2 + rz = z^3 + qz^2 + 11z + r$. [2]

9 The triangle ABC has vertices at $A(0,0)$, $B(0,2)$ and $C(4,1)$. The matrix $\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ represents a transformation T.

(i) The transformation T maps triangle ABC onto triangle $A'B'C'$. Find the coordinates of A', B' and C'. [3]

Triangle $A'B'C'$ is now mapped onto triangle $A''B''C''$ using the matrix $M = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$.

(ii) Describe fully the transformation represented by M. [3]

(iii) Triangle $A''B''C''$ is now mapped back onto ABC by a single transformation. Find the matrix representing this transformation. [3]

(iv) Calculate the area of $A''B''C''$. [3]

END OF QUESTION PAPER

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