OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4763/01

MATHEMATICS (MEI)

Mechanics 3

QUESTION PAPER

WEDNESDAY 20 MAY 2015: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4763/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 (i) Give the dimensions of force, work and power.   [3]

The force due to air resistance acting on a car is given by λv^2, where v is the speed and λ is a constant for that car.

(ii) Find the dimensions of λ. [2]

The power P of the car and its maximum speed U are related by the equation $P = \lambda U^3$.

(iii) Show that this equation is dimensionally consistent.   [2]

The time t taken for the car to accelerate from speed $\frac{1}{3}U$ to speed $\frac{2}{3}U$ is given by $t = km^\alpha P^\beta \lambda^\gamma$, where m is the mass of the car and k is a dimensionless constant.

(iv) Find α, β and γ.   [4]

Car C has mass $800\,\text{kg}$, power $35\,\text{kW}$, maximum speed $45\,\text{m s}^{-1}$, and takes $9.18\,\text{s}$ to accelerate from $15\,\text{m s}^{-1}$ to $30\,\text{m s}^{-1}$.

(v) Find the value of λ for Car C
   (A)   in SI units (based on kilograms, metres and seconds),
   (B)   in a system of units based on pounds, miles and hours, given that
      1 pound = 0.454 kg,
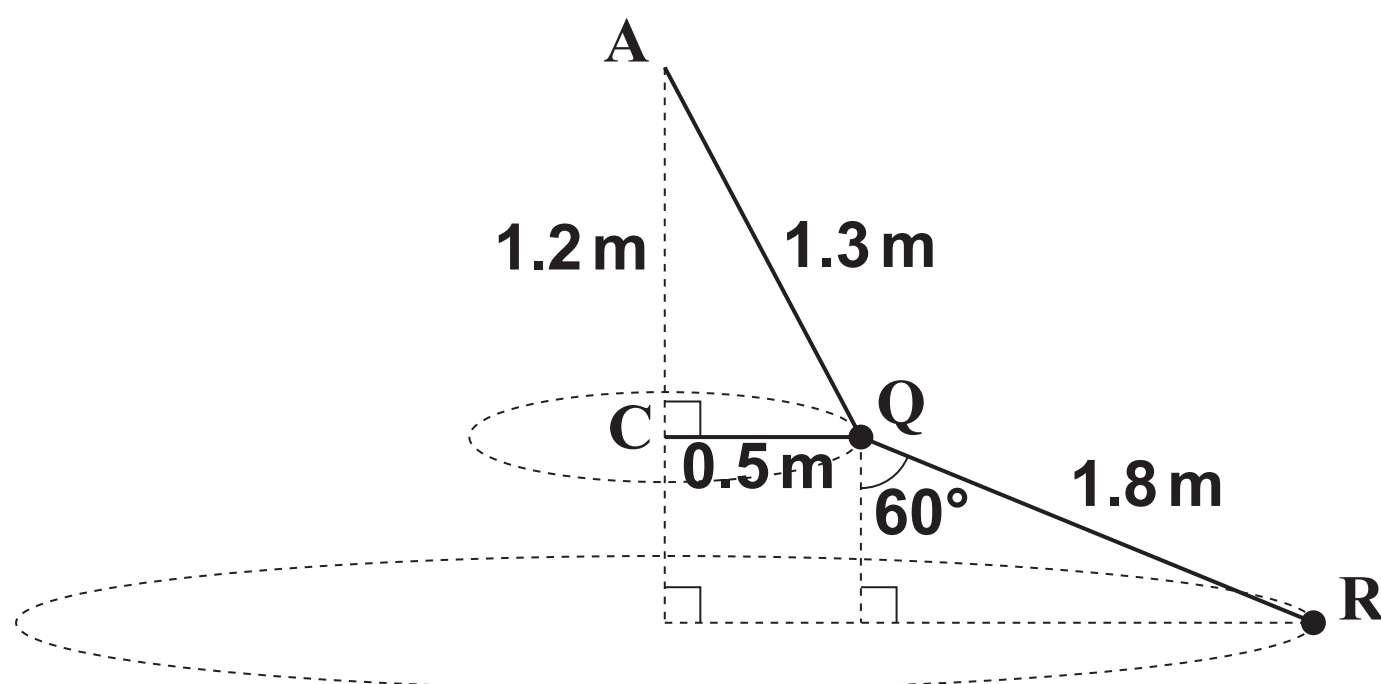   1 mile = 1609 m,
      1 hour = 3600 s.   [3]

(vi) Car D has mass $1250\,\text{kg}$, power $75\,\text{kW}$ and maximum speed $54\,\text{m s}^{-1}$. Find the time taken for Car D to accelerate from $18\,\text{m s}^{-1}$ to $36\,\text{m s}^{-1}$.   [4]

4

2 (a) A particle P of mass m is attached to a fixed point O by a light inextensible string of length a. P is moving without resistance in a complete vertical circle with centre O and radius a. When P is at the highest point of the circle, the tension in the string is T_1. When OP makes an angle θ with the upward vertical, the tension in the string is T_2. Show that

$$T_2 = T_1 + 3mg(1 - \cos\theta). \quad [6]$$

(b) The fixed point A is 1.2 m vertically above the fixed point C. A particle Q of mass 0.9 kg is joined to A, to C, and to a particle R of mass 1.5 kg, by three light inextensible strings of lengths 1.3 m, 0.5 m and 1.8 m respectively. The particle Q moves in a horizontal circle with centre C, and R moves in a horizontal circle at the same constant angular speed as Q, in such a way that A, C, Q and R are always coplanar. The string QR makes an angle of 60° with the downward vertical. This situation is shown in Fig. 2.
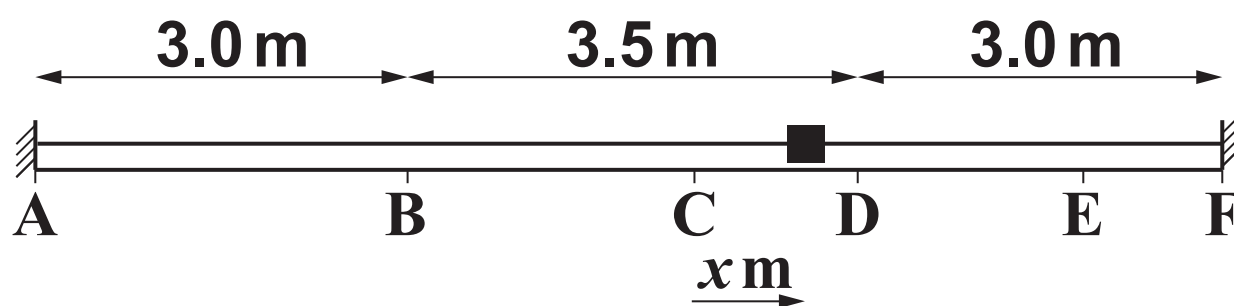
Fig. 2

(i) Find the tensions in the strings QR and AQ. [5]

(ii) Find the angular speed of the system. [3]

(iii) Find the tension in the string CQ. [4]

5

3 Fig. 3 shows the fixed points A and F which are 9.5 m apart on a smooth horizontal surface and points B and D on the line AF such that $AB = DF = 3.0$ m. A small block of mass 10.5 kg is joined to A by a light elastic string of natural length 3.0 m and stiffness $12\,N\,m^{-1}$; the block is joined to F by a light elastic string of natural length 3.0 m and stiffness $30\,N\,m^{-1}$. The block is released from rest at B and then slides along part of the line AF. The block has zero acceleration when it is at a point C, and it comes to instantaneous rest at a point E.

Fig. 3

(i) Find the distance BC. [3]

At time $t\,\mathrm{s}$ the displacement of the block from C is $x\,\mathrm{m}$, measured in the direction AF.

(ii) Show that, when the block is between B and D,

$$\frac{d^2 x}{dt^2} = -4x. \quad [4]$$

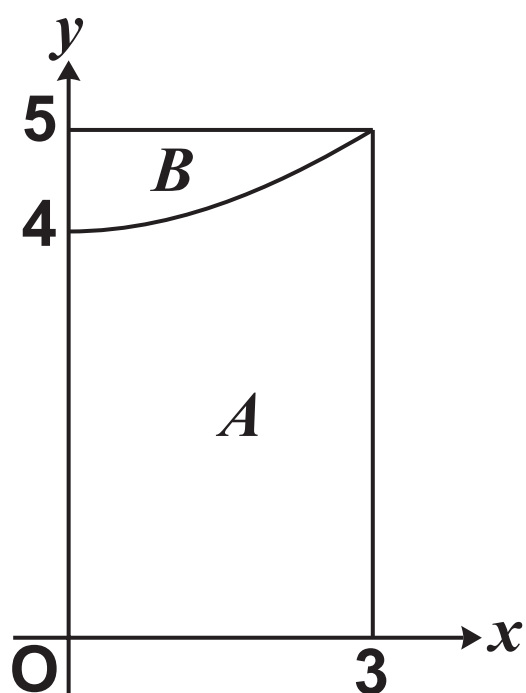(iii) Find the maximum speed of the block. [2]

(iv) Find the distance of the block from C when its speed is $4.8\,\mathrm{m\,s^{-1}}$. [2]

(v) Find the time taken for the block to travel from B to D. [4]

(vi) Find the distance DE. [3]

4 (a) A uniform lamina occupies the region bounded by the x-axis and the curve $y = \dfrac{x^2(a-x)}{a^2}$ for $0 \leqslant x \leqslant a$. Find the coordinates of the centre of mass of this lamina. **[9]**

(b) The region A is bounded by the x-axis, the y-axis, the curve $y = \sqrt{x^2 + 16}$ and the line $x = 3$. The region B is bounded by the y-axis, the curve $y = \sqrt{x^2 + 16}$ and the line $y = 5$. These regions are shown in Fig. 4.

Fig. 4

(i) Find the x-coordinate of the centre of mass of the uniform solid of revolution formed when the region A is rotated through 2π radians about the x-axis. **[5]**

(ii) Using your answer to part (i), or otherwise, find the x-coordinate of the centre of mass of the uniform solid of revolution formed when the region B is rotated through 2π radians about the x-axis. **[4]**

END OF QUESTION PAPER