

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS  
AS GCE**

**4776/01**

**MATHEMATICS (MEI)**

**Numerical Methods**

**QUESTION PAPER**

**FRIDAY 12 JUNE 2015:**

**Morning**

**DURATION: 1 hour 30 minutes  
plus your additional time allowance**

**MODIFIED ENLARGED 24pt**

**Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4776/01**

**MEI Examination Formulae and Tables  
(MF2)**

**Insert for question 6(i)**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

## **INSTRUCTIONS TO CANDIDATES**

**Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**

**IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED.**

**Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**

**Use black ink. HB pencil may be used for graphs and diagrams only.**

**Read each question carefully. Make sure you know what you have to do before starting your answer.**

**Answer ALL the questions.**

**You are permitted to use a scientific or graphical calculator in this paper.**

**Final answers should be given to a degree of accuracy appropriate to the context.**

## **INFORMATION FOR CANDIDATES**

**The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**

**You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**

**The total number of marks for this paper is 72.**

**Any blank pages are indicated.**

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## SECTION A (36 marks)

1 (i) Show that the equation

$$x = (\cos x)^3 \quad (*)$$

where  $x$  is in radians, has a root in the interval  $(0, 1)$ . [2]

This root is denoted by  $\alpha$ .

(ii) Show numerically that the iteration

$$x_{r+1} = (\cos x_r)^3$$

with  $x_0 = 0.6$  does not converge to  $\alpha$ . [2]

(iii) Show that

$$x = \sqrt{x(\cos x)^3}$$

is a rearrangement of (\*). Use the corresponding iteration to find  $\alpha$  correct to 4 decimal places. [4]

**2 An estimate is required of  $I$ , where**

$$I = \int_0^{0.6} f(x) dx.$$

**The only available values of  $f(x)$  are as follows.**

$x$	<b>0</b>	<b>0.1</b>	<b>0.2</b>	<b>0.4</b>
$f(x)$	<b>2.3063</b>	<b>2.2769</b>	<b>2.1883</b>	<b>1.8308</b>

**(i) Obtain the best possible estimates of**

**$\int_0^{0.2} f(x) dx$  and  $\int_{0.2}^{0.6} f(x) dx$ . Hence give an estimate of  $I$ . [5]**

**(ii) State what you would do differently if  $f(0.6)$  became available. [1]**

- 3 A computer program is used to calculate values of  $f$  where**

$$\frac{1}{f} = \frac{1}{u} - \frac{1}{v}.$$

**One possible formula for  $f$  is**

$$f = \frac{1}{\left(\frac{1}{u} - \frac{1}{v}\right)}. \quad (*)$$

- (i) Show that another formula for  $f$  is**

$$f = \frac{uv}{v - u}. \quad (**) \quad [1]$$

**The program stores and calculates all numbers rounded to 5 significant figures.**

- (ii) Find the values of  $f$  given by the program using (\*) and (\*\*) when  $u = 11$  and  $v = 11.05$ .**

**Show that one of these values is exact and find the relative error in the other.**

**[6]**

- (iii) State what process gives rise to the error in the inexact value. [1]**

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**4 The table opposite shows values of a function  $f(x)$  correct to 5 decimal places.**

**(i) Find five estimates of  $f'(1)$  using the forward difference method. [3]**

**(ii) Comment on the accuracy of these estimates.**

**Give a value for  $f'(1)$  to the accuracy that you consider appropriate. Justify your answer. [3]**

$x$	1	1.1	1.01	1.001	1.0001	1.00001
$f(x)$	0.94404	0.96771	0.94641	0.94428	0.94406	0.94404

**5 An approximate formula for  $\sqrt{x}$  of the form**

$$\sqrt{x} \approx a + bx \quad (*)$$

**is required for values of  $x$  near to 1.**

- (i) Find the values of  $a$  and  $b$  for which (\*) is exact when  $x = 1$  and  $x = 1.21$ . [4]**
- (ii) With these values of  $a$  and  $b$ , find the absolute and relative errors in (\*) when  $x = 0.81$ . [4]**

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## SECTION B (36 marks)

6 The variables  $p$  and  $q$  are known to take the following values.

$p$	1	2	3
$q$	2.2	2.8	5.2

- (i) Plot these points and draw, by eye, a smooth curve through them. [2]
- (ii) Use Newton's forward difference interpolation formula to obtain a quadratic expression for  $q$  in terms of  $p$ . Give your answer in simplified form. [6]
- (iii) Estimate  $q$  when  $p = 2.5$ . Let this estimate be denoted by  $\alpha$ . [2]

- (iv) Now suppose that a quadratic expression for  $p$  in terms of  $q$  is required. Explain why Newton's formula could not be used for this purpose. Use Lagrange's method to write down an expression for this quadratic. (You are not required to simplify this expression.) [5]**
- (v) Estimate  $p$  when  $q$  is equal to the value  $\alpha$  found in part (iii). Comment on your answer. [3]**

**7 (i) Show that the equation**

$$3x^4 + x - 2 = 0 \quad (*)$$

**has a root,  $\alpha$ , in the interval  $(0, 1)$ .**

**Show that (\*) does not have any other positive roots. [3]**

**(ii) Sketch the curve  $y = 3x^4 + x - 2$  for  $0 \leq x \leq 1$ . [2]**

**The secant method, with  $x_0 = 0$  and  $x_1 = 1$  is used to find  $\alpha$ .**

**(iii) Find  $x_2$  exactly, and calculate  $x_3$  and  $x_4$  correct to 6 significant figures.**

**Show, by means of appropriate lines on your graph, how the secant method produces these values. [8]**

**(iv) Iterate the secant method further to find  $\alpha$  correct to 3 significant figures. Show that you have obtained the required accuracy. [5]**

**END OF QUESTION PAPER**

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