OCR
Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Show that the equation

$$x = (\cos x)^3 \qquad (*)$$

where x is in radians, has a root in the interval $(0, 1)$. **[2]**

This root is denoted by α.

(ii) Show numerically that the iteration

$$x_{r+1} = (\cos x_r)^3$$

with $x_0 = 0.6$ does not converge to α. **[2]**

(iii) Show that

$$x = \sqrt{x(\cos x)^3}$$

is a rearrangement of (*). Use the corresponding iteration to find α correct to 4 decimal places. **[4]**

2 An estimate is required of I, where

$$I = \int_0^{0.6} \mathrm{f}(x)\,\mathrm{d}x.$$

The only available values of $\mathrm{f}(x)$ are as follows.

x	0	0.1	0.2	0.4
$\mathrm{f}(x)$	2.3063	2.2769	2.1883	1.8308

(i) Obtain the best possible estimates of $\int_0^{0.2} \mathrm{f}(x)\,\mathrm{d}x$ and $\int_{0.2}^{0.6} \mathrm{f}(x)\,\mathrm{d}x$. Hence give an estimate of I. **[5]**

(ii) State what you would do differently if $\mathrm{f}(0.6)$ became available. **[1]**

3 A computer program is used to calculate values of f where

$$\frac{1}{f} = \frac{1}{u} - \frac{1}{v}.$$

One possible formula for f is

$$f = \frac{1}{\left(\frac{1}{u} - \frac{1}{v}\right)}. \qquad (*)$$

(i) Show that another formula for f is

$$f = \frac{uv}{v-u}. \qquad (**)$$ **[1]**

The program stores and calculates all numbers rounded to 5 significant figures.

(ii) Find the values of f given by the program using (*) and (**) when $u = 11$ and $v = 11.05$.

Show that one of these values is exact and find the relative error in the other. **[6]**

(iii) State what process gives rise to the error in the inexact value. **[1]**

4 The table shows values of a function $\mathrm{f}(x)$ correct to 5 decimal places.

x	1	1.1	1.01	1.001	1.0001	1.00001
$\mathrm{f}(x)$	0.94404	0.96771	0.94641	0.94428	0.94406	0.94404

(i) Find five estimates of $\mathrm{f}'(1)$ using the forward difference method. **[3]**

(ii) Comment on the accuracy of these estimates.

Give a value for $\mathrm{f}'(1)$ to the accuracy that you consider appropriate. Justify your answer. **[3]**

5 An approximate formula for $\sqrt{x}$ of the form

$$\sqrt{x} \approx a + bx \qquad (*)$$

is required for values of x near to 1.

(i) Find the values of a and b for which (*) is exact when $x = 1$ and $x = 1.21$. **[4]**

(ii) With these values of a and b, find the absolute and relative errors in (*) when $x = 0.81$. **[4]**

Section B (36 marks)

6 The variables p and q are known to take the following values.

p	1	2	3
q	2.2	2.8	5.2

(i) Plot these points and draw, by eye, a smooth curve through them. **[2]**

(ii) Use Newton's forward difference interpolation formula to obtain a quadratic expression for q in terms of p. Give your answer in simplified form. **[6]**

(iii) Estimate q when $p = 2.5$. Let this estimate be denoted by α. **[2]**

(iv) Now suppose that a quadratic expression for p in terms of q is required. Explain why Newton's formula could not be used for this purpose. Use Lagrange's method to write down an expression for this quadratic. (You are not required to simplify this expression.) **[5]**

(v) Estimate p when q is equal to the value α found in part **(iii)**. Comment on your answer. **[3]**

7 **(i)** Show that the equation

$$3x^4 + x - 2 = 0 \qquad (*)$$

has a root, α, in the interval $(0, 1)$.

Show that (*) does not have any other positive roots. **[3]**

(ii) Sketch the curve $y = 3x^4 + x - 2$ for $0 \leqslant x \leqslant 1$. **[2]**

The secant method, with $x_0 = 0$ and $x_1 = 1$ is used to find α.

(iii) Find x_2 exactly, and calculate x_3 and x_4 correct to 6 significant figures.

Show, by means of appropriate lines on your graph, how the secant method produces these values. **[8]**

(iv) Iterate the secant method further to find α correct to 3 significant figures. Show that you have obtained the required accuracy. **[5]**

END OF QUESTION PAPER