OCR
Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Solve the equation $\frac{5x}{2x+1} - \frac{3}{x+1} = 1$. **[5]**

2 Express $6\cos 2\theta + \sin\theta$ in terms of $\sin\theta$.

Hence solve the equation $6\cos 2\theta + \sin\theta = 0$, for $0° \leqslant \theta \leqslant 360°$. **[7]**

3 **(i)** Find the first three terms of the binomial expansion of $\frac{1}{\sqrt[3]{1-2x}}$. State the set of values of x for which the expansion is valid. **[5]**

(ii) Hence find a and b such that $\frac{1-3x}{\sqrt[3]{1-2x}} = 1 + ax + bx^2 + \ldots$. **[3]**

4 You are given that $\mathrm{f}(x) = \cos x + \lambda \sin x$ where λ is a positive constant.

(i) Express $\mathrm{f}(x)$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving R and α in terms of λ. **[4]**

(ii) Given that the maximum value (as x varies) of $\mathrm{f}(x)$ is 2, find R, λ and α, giving your answers in exact form. **[4]**

5 A curve has parametric equations $x = \sec\theta$, $y = 2\tan\theta$.

(i) Given that the derivative of $\sec\theta$ is $\sec\theta\tan\theta$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\,\mathrm{cosec}\,\theta$. **[3]**

(ii) Verify that the cartesian equation of the curve is $y^2 = 4x^2 - 4$. **[2]**

Fig. 5 shows the region enclosed by the curve and the line $x = 2$. This region is rotated through 180° about the x-axis.

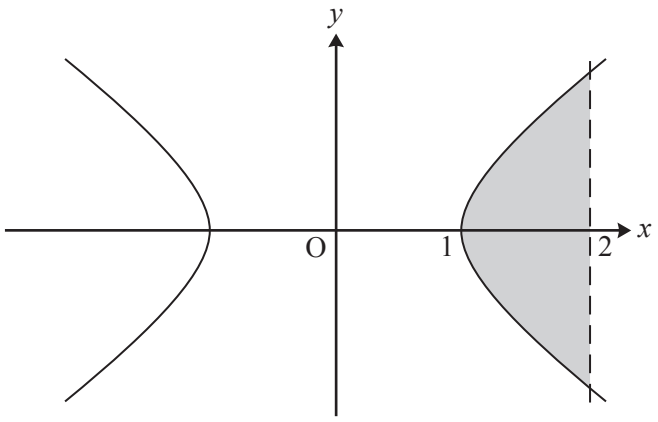

Fig. 5

(iii) Find the volume of revolution produced, giving your answer in exact form. **[3]**

Section B (36 marks)

6 Fig. 6 shows a lean-to greenhouse ABCDHEFG. With respect to coordinate axes O*xyz*, the coordinates of the vertices are as shown. All distances are in metres. Ground level is the plane $z = 0$.

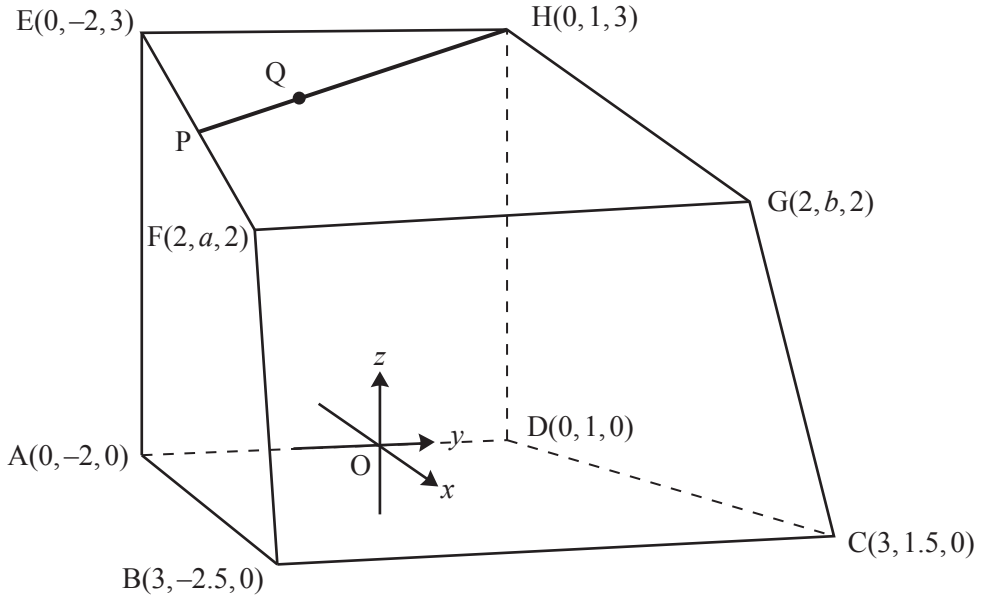

Fig. 6

(i) Verify that the equation of the plane through A, B and E is $x + 6y + 12 = 0$.

Hence, given that F lies in this plane, show that $a = -2\frac{1}{3}$. **[4]**

(ii) (*A*) Show that the vector $\begin{pmatrix} 1 \\ -6 \\ 0 \end{pmatrix}$ is normal to the plane DHC. **[2]**

(*B*) Hence find the cartesian equation of this plane. **[2]**

(*C*) Given that G lies in the plane DHC, find *b* and the length FG. **[2]**

(iii) Find the angle EFB. **[5]**

A straight wire joins point H to a point P which is half way between E and F. Q is a point two-thirds of the way along this wire, so that HQ = 2QP.

(iv) Find the height of Q above the ground. **[3]**

Question 7 begins on page 4.

7 A drug is administered by an intravenous drip. The concentration, x, of the drug in the blood is measured as a fraction of its maximum level. The drug concentration after t hours is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(1+x-2x^2),$$

where $0 \leqslant x < 1$, and k is a positive constant. Initially, $x = 0$.

(i) Express $\dfrac{1}{(1+2x)(1-x)}$ in partial fractions. **[3]**

(ii) Hence solve the differential equation to show that $\dfrac{1+2x}{1-x} = \mathrm{e}^{3kt}$. **[7]**

(iii) After 1 hour the drug concentration reaches 75% of its maximum value and so $x = 0.75$.

Find the value of k, and the time taken for the drug concentration to reach 90% of its maximum value. **[3]**

(iv) Rearrange the equation in part **(ii)** to show that $x = \dfrac{1-\mathrm{e}^{-3kt}}{1+2\mathrm{e}^{-3kt}}$.

Verify that in the long term the drug concentration approaches its maximum value. **[5]**

END OF QUESTION PAPER