OCR RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4768/01 Statistics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4768/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 7 3 3 0 2 0 1 1 3 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A certain industrial process requires a supply of water. It has been found that, for best results, the mean water pressure in suitable units should be 7.8. The water pressure is monitored by taking measurements at regular intervals. On a particular day, a random sample of the measurements is as follows.

7.50 7.64 7.68 7.51 7.70 7.85 7.34 7.72 7.74

These data are to be used to carry out a hypothesis test concerning the mean water pressure.

(i) Why is a test based on the Normal distribution not appropriate in this case? **[2]**

(ii) What distributional assumption is needed for a test based on the t distribution? **[1]**

(iii) Carry out a t test, with a 2% level of significance, to see whether it is reasonable to assume that the mean pressure is 7.8. **[9]**

(iv) Explain what is meant by a 95% confidence interval. **[2]**

(v) Find a 95% confidence interval for the actual mean water pressure. **[4]**

2 A particular species of reed that grows up to 2 metres in length is used for thatching. The lengths in metres of the reeds when harvested are modelled by the random variable X which has the following probability density function, $\mathrm{f}(x)$.

$$\mathrm{f}(x) = \begin{cases} \frac{3}{16}(4x - x^2) & \text{for } 0 \leqslant x \leqslant 2 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Sketch $\mathrm{f}(x)$. **[3]**

(ii) Show that $\mathrm{E}(X) = \frac{5}{4}$ and find the standard deviation of the lengths of the harvested reeds. **[8]**

(iii) Find the standard error of the mean length for a random sample of 100 reeds. **[2]**

Once the harvested reeds have been collected, any that are shorter than 1 metre are discarded.

(iv) Find the proportion of reeds that should be discarded according to the model. **[2]**

(v) Reeds are harvested from a large area which is divided into several reed beds. A sample of the harvested reeds is required for quality control. How might the method of cluster sampling be used to obtain it? **[3]**

3 In the manufacture of child car seats, a resin made up of three ingredients is used. The ingredients are two polymers and an impact modifier. The resin is prepared in batches. Each ingredient is supplied by a separate feeder and the amount supplied to each batch, in kg, is assumed to be Normally distributed with mean and standard deviation as shown in the table below. The three feeders are also assumed to operate independently of each other.

	Mean	Standard deviation
Polymer 1	2025	44.6
Polymer 2	1565	21.8
Impact modifier	1410	33.8

(i) Find the probability that, in a randomly chosen batch of resin, there is no more than 2100 kg of polymer 1. **[3]**

(ii) Find the probability that, in a randomly chosen batch of resin, the amount of polymer 1 exceeds the amount of polymer 2 by at least 400 kg. **[4]**

(iii) Find the value of b such that the total amount of the ingredients in a randomly chosen batch exceeds b kg 95% of the time. **[4]**

(iv) Polymer 1 costs £1.20 per kg, polymer 2 costs £1.30 per kg and the impact modifier costs £0.80 per kg. Find the mean and variance of the total cost of a batch of resin. **[3]**

(v) Each batch of resin is used to make a large number of car seats from which a random sample of 50 seats is selected in order that the tensile strength (in suitable units) of the resin can be measured. From one such sample, the 99% confidence interval for the true mean tensile strength of the resin in that batch was calculated as (123.72, 127.38). Find the mean and standard deviation of the sample. **[4]**

[Question 4 is printed overleaf.]

4 (a) At a college, two examiners are responsible for marking, independently, the students' projects. Each examiner awards a mark out of 100 to each project. There is some concern that the examiners' marks do not agree, on average. Consequently a random sample of 12 projects is selected and the marks awarded to them are compared.

(i) Describe how a random sample of projects should be chosen. **[2]**

(ii) The marks given for the projects in the sample are as follows.

Project	1	2	3	4	5	6	7	8	9	10	11	12
Examiner A	58	37	72	78	67	77	62	41	80	60	65	70
Examiner B	73	47	74	71	78	96	54	27	97	73	60	66

Carry out a test at the 10% level of significance of the hypotheses H_0: $m = 0$, H_1: $m \neq 0$, where m is the population median difference. **[8]**

(b) A calculator has a built-in random number function which can be used to generate a list of random digits. If it functions correctly then each digit is equally likely to be generated. When it was used to generate 100 random digits, the frequencies of the digits were as follows.

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	6	8	11	14	12	9	15	5	14	6

Use a goodness of fit test, with a significance level of 10%, to investigate whether the random number function is generating digits with equal probability. **[8]**

OCR
RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4768/01 Statistics 3

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
- Question Paper 4768/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

Turn over

1 (i)	
1 (ii)	
1 (iii)	
	(answer space continued on next page)

1 (iii)	(continued)
1 (iv)	

1 (v)	
2 (i)	

2 (ii)	

2 (iii)

2 (iv)

2 (v)

3 (i)	
3 (ii)	

3 (iii)	
3 (iv)	

3 (v)

4 (a) (i)

4 (a) (ii)	

4 (b)	
	(answer space continued on next page)

4 (b)	(continued)

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4768:** Statistics 3

Mark Scheme for January 2013

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

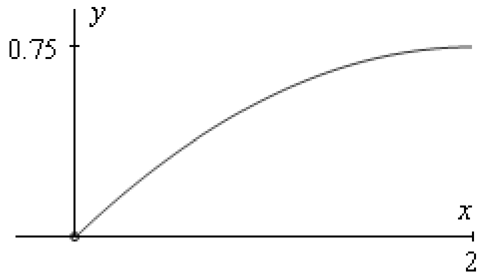
On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		A Normal test is not appropriate since … … the sample is small and … the population variance is not known (it must be estimated from the data).	E1 E1 **[2]**	Allow use of "σ", otherwise insist on "population".
1	(ii)		The sample is taken from a Normal population.	B1 **[1]**	
1	(iii)		H_0: $\mu = 7.8$ H_1: $\mu \neq 7.8$ where μ is the mean water pressure. $\bar{x} = 7.631 \quad s = 0.1547$ Test statistic is $\dfrac{7.631 - 7.8}{\frac{0.1547}{\sqrt{9}}}$ $= -3.27(7)$. Refer to t_8. Double-tailed 2% point is ±2.896. Significant. Sufficient evidence to suggest that the mean water pressure has changed.	B1 B1 B1 M1 A1 M1 A1 A1 A1 **[9]**	Both hypotheses. Hypotheses in words only must include "population". Do NOT allow "$\bar{X} = \ldots$" or similar unless $\bar{X}$ is clearly and explicitly stated to be a <u>population</u> mean. For adequate verbal definition. Allow absence of "population" if correct notation μ is used. $s_n = 0.1459$ but do <u>NOT</u> allow this here or in construction of test statistic, but ft from there. Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 7.8 + (c's –2.896) × $0.1547/\sqrt{9}$ (= 7.65…) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's –2.896) × $0.1547/\sqrt{9}$ (= 7.78…) for comparison with 7.8.) c.a.o. but ft from here in any case if wrong. Use of $\mu - \bar{x}$ scores M1A0. No ft from here if wrong. Must compare test statistic with <u>minus</u> 2.896 unless absolute values are being compared. No ft from here if wrong. Allow P(t < –3.27(7) or t > 3.27(7)) = 0.0113 for M1A1. ft only c's test statistic if both M's scored. ft only c's test statistic if both M's scored. Conclusion in context to include "average" o.e.

Question			Answer	Marks	Guidance
1	(iv)		In repeated sampling, 95% of all confidence intervals constructed in this way will contain the true mean.	 E1 E1 **[2]**	
1	(v)		CI is given by 7.631 ± 2·306 $\times \frac{0.1547}{\sqrt{9}}$ = 7.631 ± 0.118(9) = (7.512, 7.750)	M1 B1 M1 A1 **[4]**	ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_8 is OK. Allow c's $\overline{x}$. 2.306 seen. Allow c's s_{n-1}. c.a.o. Must be expressed as an interval.
2	(i)			G1 G1 G1 **[3]**	Curve with positive gradient, through the origin and in the first quadrant only. Correct shape for an inverted parabola ending at maximum point. End point (2, 3/4) labelled.

Question			Answer	Marks	Guidance
2	(ii)		$\text{E}(X)=\frac{3}{16}\int_0^2(4x^2-x^3)\text{d}x$	M1	Correct integral for E(X) with limits (which may appear later).
			$=\frac{3}{16}\left[\frac{4x^3}{3}-\frac{x^4}{4}\right]_0^2$	M1	Correctly integrated. Dep on previous M1.
			$=\frac{3}{16}\left\{\left(\frac{32}{3}-\frac{16}{4}\right)-0\right\}$		
			$=\frac{5}{4}$	A1	Limits used correctly to obtain PRINTED ANSWER (BEWARE) convincingly. Condone absence of "–0".
			$\text{E}(X^2)=\frac{3}{16}\int_0^2(4x^3-x^4)\text{d}x$	M1	Correct integral for E(X) with limits (which may appear later).
			$=\frac{3}{16}\left[x^4-\frac{x^5}{5}\right]_0^2$	M1	Correctly integrated. Dep on previous M1.
			$=\frac{3}{16}\left\{\left(16-\frac{32}{5}\right)-0\right\}$		
			$=\frac{9}{5}$	A1	Limits used correctly to obtain result. Condone absence of "–0".
			$\text{Var}(X)=\frac{9}{5}-\left(\frac{5}{4}\right)^2=\frac{19}{80}$	M1	Use of Var(X) = E(X^2) – E(X)^2.
			$\text{sd}=\sqrt{\frac{19}{80}}=0.487(3)$	A1	cao
				[8]	
2	(iii)		$\text{SE}(\bar{X})=\frac{0.487}{\sqrt{100}}$	M1	
			$=0.0487$	A1	ft c's $\sigma/10$.
				[2]	

Question			Answer	Marks	Guidance
2	(iv)		$\mathrm{P}(X<1)=\frac{3}{16}\int_0^1(4x-x^2)\mathrm{d}x$	M1	Correct integral for P(X < 1) with limits (which may appear later).
			$=\frac{3}{16}\left[2x^2-\frac{x^3}{3}\right]_0^1$		
			$=\frac{3}{16}\left\{\left(2-\frac{1}{3}\right)-0\right\}$		
			$=\frac{5}{16}$	A1	cao. Condone absence of "–0" when limits applied.
				[2]	
2	(v)		Regard the reed beds as clusters.	E1	NB "Clusters of reeds" scores 0 unless clearly and correctly explained.
			Select a few clusters (maybe only one) at random.	E1	
			Take a (simple random) sample of reeds (or maybe all of them) from the selected cluster(s).	E1	
				[3]	
3			$P1 \sim \mathrm{N}(2025, 44.6^2)$ $P2 \sim \mathrm{N}(1565, 21.8^2)$ $I \sim \mathrm{N}(1410, 33.8^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.
3	(i)		$\mathrm{P}(P1<2100)=$	M1	For standardising. Award once, here or elsewhere.
			$\mathrm{P}\left(Z<\frac{2100-2025}{44.6}=1.681(6)\right)$	A1	
			$=0.9536/7$	A1	c.a.o.
				[3]	

Question			Answer	Marks	Guidance
3	(ii)		Require P($P1 - P2 > 400$) $P1 - P2 \sim (2025 - 1565 = 460,$ $44.6^2 + 21.8^2 = 2464.4)$ P(this > 400) = $\mathrm{P}\left(Z > \frac{400-460}{\sqrt{2464.4}} = -1.208(6)\right) = 0\cdot 8864/5$	M1 B1 B1 A1 **[4]**	 Mean. Variance. Accept sd (= 49.64). cao
3	(iii)		$T = P1 + P2 + I \sim \mathrm{N}(5000,$ $\sigma^2 = 44.6^2 + 21.8^2 + 33.8^2 = 3606.84)$ Require b s.t. P($T > b$) = 0.95 $\therefore \frac{b - 5000}{\sqrt{3606.84}} = -1.645$ $\therefore b = 5000 - 1.645 \times \sqrt{3606.84} = 4901.2..$	B1 B1 B1 A1 **[4]**	Mean. Variance. Accept sd (= 60.056…). –1.645 seen. c.a.o.
3	(iv)		Mean = (1.2 × 2025) + (1.3 × 1565) + (0.8 × 1410) = £5592.50 Var = $(1.2^2 \times 44.6^2) + (1.3^2 \times 21.8^2) +$ $(0.8^2 \times 33.8^2) = 4398.7076 \approx £^2 4399$	B1 M1 A1 **[3]**	Condone absence of £. Use of at least one of $(1.2^2 \times 44.6^2)$ etc… Condone absence of $£^2$.
3	(v)		Mean = (123.72 + 127.38)/2 = 125.55 $s = \frac{127.38 - 125.55}{2.576/\sqrt{50}} = 5.02(3)$	B1 B1 M1 A1 **[4]**	Cao Sight of 2.576. Or equivalent. cao

Question			Answer	Marks	Guidance
4	(a)	(i)	Number all the projects to be marked. (Sampling frame.) Use a form of random number generator to select the projects in the sample until 12 projects have been selected.	E1 E1 **[2]**	Do not award if candidate subsequently describes a different method of sampling (eg systematic sampling). Condone absence of 12.
4	(a)	(ii)	H_0: $m = 0$ H_1: $m \neq 0$ where m is the population median difference between the examiners' marks.		This is given in the question.
			Diff: 15, 10, 2, –7, 11, 19, –8, –14, 17, 13, –5, –4 Rank: 10, 6, 1, 4, 7, 12, 5, 9, 11, 8, 3, 2	M1 M1 A1	For differences. ZERO (out of 8) in this section if differences not used. For ranks. ft from here if ranks wrong.
			$W_- = 2 + 3 + 4 + 5 + 9 = 23$	B1	(or $W_+ = 1 + 6 + 7 + 8 + 10 + 11 + 12 = 55$)
			Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 12$.	M1	No ft from here if wrong.
			Lower (or upper if 55 used) 5% tail is 17 (or 61 if 55 used).	A1	i.e. a 2-tail test. No ft from here if wrong.
			Result is not significant.	A1	ft only c's test statistic.
			Insufficient evidence to suggest a difference in the marks awarded, on average.	A1 **[8]**	ft only c's test statistic. Conclusion in context to include "average" o.e.

Diff	15	10	2	–7	11	19	–8	–14	17	13	–5	–4
Rank	10	6	1	4	7	12	5	9	11	8	3	2

Question			Answer	Marks	Guidance
4	(b)		H_0: The random number function is performing as it should. H_1: The random number function is not performing as it should.	B1	Both hypotheses. Must be the right way round. Allow use of the uniform distribution/model. Do not accept "data fit model" oe.
			All expected frequencies are 10	B1	
			$X^2 = 1.6 + 0.4 + 0.1 + 1.6 + 0.4 + 0.1 + 2.5 + 2.5 + 1.6 + 1.6$	M1	Calculation of X^2.
			$= 12.4$	A1	c.a.o.
			Refer to χ_9^2.	M1	Allow correct df (= cells – 1) from wrongly grouped table and ft. Otherwise, no ft if wrong. $P(X^2 > 12.4) = 0.1916$.
			Upper 10% point is 14.68.	A1	No ft from here if wrong.
			Not significant.	A1	ft only c's test statistic.
			Insufficient evidence to suggest that the random number function is not performing as it should.	A1	ft only c's test statistic. Conclusion in context. Allow in terms of the uniform distribution/model. Do not accept "data fit model" oe.
				[8]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

January 2013

4768 Statistics 3

General Comments

As might be expected on a paper at this level, the scripts indicated that most candidates knew what they were doing most of the time. In addition, there were very few scripts which showed evidence of candidates running out of time. That being the case, it is disappointing to report that a large number of scripts suffered from a lack of precision that manifested itself in a number of ways across the paper. Examples include final answers being given to more than 5 significant figures, not enough accuracy being used in calculations, hypotheses and conclusions being given without context, conclusions to hypothesis tests being too assertive, and other examples which will be commented on below. The cumulative effect of these errors was significant for many candidates.

Comments on Individual Questions

1 Water pressure - *t* test

1(i)(ii) Well understood by most candidates, but a lack of precision meant that some candidates did not state that it was the *population* variance that was unknown, and others stated that the data had to be Normally distributed.

(iii) The hypotheses were usually well stated. A few candidates did not define μ, and a few gave a description without context. Of the small proportion who gave their hypotheses in words, the majority used mean rather than population mean. It was pleasing to see the overwhelming majority of candidates correctly opted to use s_{n-1} rather than s_n, but a number of candidates used a truncated value for $\bar{x}$ in the calculation and lost accuracy as a result. Virtual all candidates correctly calculated the test statistic. The correct point of t_8 was usually used, although some candidates opted for t_9. It was not always possible to see if this was through a misunderstanding or a misreading of the table. Most candidates correctly rejected the null hypothesis, but too many gave conclusions which were too assertive or lacked context.

(iv)(v) A majority of candidates knew the meaning of a 95% confidence interval, although some definitions were clearer than others. A minority of candidates gave definitions in terms of just one interval and others had a definition which included the capture of sample means. The calculation of the confidence interval was well done – a few candidates used 1.96 or 2.326 and some changed to t_9.

2 Reed beds - probability density function

(i) A wide range of sketches was seen. A fully correct sketch was the most common outcome, but some sketches extended the parabola well beyond the defined range, others clearly did not have zero gradient at $x = 2$. Some sketches were unlabelled, and some had an increasing, rather than a decreasing gradient. A few sketches reached the maximum point at $x = 2$ and then continued with a horizontal line.

(ii) This was extremely well done by almost all candidates. Apart from a few arithmetic slips, the only errors which occurred were presenting $\mathrm{E}(X^2)$ as the variance, and forgetting to take the square root for the final answer.

(iii) A large number of candidates did not know the meaning of the term *standard error.* Many gave an interval as their response and a significant number did not respond at all.

(iv) This was well done by most candidates, but occasionally the wrong limits were seen and there were some attempts at Normal approximations.

(v) Only a minority of candidates gained full marks here. Defining the reed beds as clusters was rarely seen and many candidates talked about clusters of reeds. Many responses lacked context.

3 Child car seat - linear combinations of Normal distributions

(i) This part was almost invariably correct. Virtually all of those who did not score full marks here had selected the wrong tail.

(ii) This part was again almost invariably correct.

(iii) Most candidates were able to calculate the mean and variance of the distribution correctly. Most of these candidates then correctly identified the correct *z* value of –1.645. The most common error was the omission of the minus sign, but 1.96 was occasionally seen. Many candidates gave answers to 6 or more significant figures.

(iv) Virtually all candidates correctly calculated the mean cost. Many candidates also knew how to calculate the variance, but a few used multipliers of 1.2, 1.3 and 0.8 instead of their squares. Many variances were given to 6 significant figures, and even 8 significant figures were regularly seen.

3(v) Many candidates were able to find the mean and standard deviation, but often not very efficiently. The correct value of 2.576 was usually used, but various other *z* values were also seen. A surprising number of candidates were unable to multiply both sides of their equation by $\sqrt{50}$ correctly.

4 Wilcoxon paired test and goodness of fit test

(i) Some candidates simply gave a definition of a random sample and others described other sampling methods, but most candidates gave a correct description. It was surprising how many candidates wanted to pick numbers out of a hat as a method of random selection. Surely, at this level, a random number generator is a better choice.

(ii) This question was done extremely well by most candidates. Very few errors were made in calculating the differences and the ranks. The Wilcoxon statistic was almost always correctly calculated. Virtually all candidates gave the correct critical value of 17. A small minority of candidates decided that the result was significant, but more common faults were conclusions either not in context or too assertive.

(b)(iii) This question was well done by most candidates. Most candidates gave acceptable hypotheses and were able to calculate X^2 correctly. A very few candidates confused expected and observed values or combined groups. The correct point of the chi squared distribution was usually quoted, although 8, 11 and 12 degrees of freedom were all seen. A small minority of candidates decided that the result was significant, but more common faults were conclusions either not in context or too assertive.

6 The LP question was generally done reasonably well.

The summer 2012 report on 4771 contained the following quotation: "Far too many candidates, if they remembered to define their variables, neglected that essential phrase "the number of ...". The issue remains live! Again, in this examination, far too many candidates failed explicitly to define their variables. The phrase "*x* is hats", and variants of it, scored zero.

The insistence on the phrase "number of ..." secures the definition of units in the case of continuous variables, eg "Let *x* be the number of litres of ...", and points to the need for integer values in other cases. In this question most candidates failed adequately to deal with that integer requirement in part (ii). The majority of candidates were happy to round the LP solution to (13, 18). Few looked at nearby lattice points, and only a handful found the optimal integer point, (12, 19).