OCR
RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Given that $y = e^{-x} \sin 2x$, find $\frac{dy}{dx}$. **[3]**

(ii) Hence show that the curve $y = e^{-x} \sin 2x$ has a stationary point when $x = \frac{1}{2}\arctan 2$. **[3]**

2 A curve has equation $x^2 + 2y^2 = 4x$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y. **[3]**

(ii) Hence find the exact coordinates of the stationary points of the curve. [You need not determine their nature.] **[3]**

3 Express $1 < x < 3$ in the form $|x - a| < b$, where a and b are to be determined. **[2]**

4 The temperature θ °C of water in a container after t minutes is modelled by the equation

$$\theta = a - be^{-kt},$$

where a, b and k are positive constants.

The initial and long-term temperatures of the water are 15 °C and 100 °C respectively. After 1 minute, the temperature is 30 °C.

(i) Find a, b and k. **[6]**

(ii) Find how long it takes for the temperature to reach 80 °C. **[2]**

5 The driving force F newtons and velocity v km s^{-1} of a car at time t seconds are related by the equation $F = \frac{25}{v}$.

(i) Find $\frac{dF}{dv}$. **[2]**

(ii) Find $\frac{dF}{dt}$ when $v = 50$ and $\frac{dv}{dt} = 1.5$. **[3]**

6 Evaluate $\int_0^3 x(x+1)^{-\frac{1}{2}}\,dx$, giving your answer as an exact fraction. **[5]**

7 **(i)** Disprove the following statement:

$$3^n + 2 \text{ is prime for all integers } n \geqslant 0.$$ **[2]**

(ii) Prove that no number of the form 3^n (where n is a positive integer) has 5 as its final digit. **[2]**

Section B (36 marks)

8 Fig. 8 shows parts of the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, where $\mathrm{f}(x) = \tan x$ and $\mathrm{g}(x) = 1 + \mathrm{f}(x - \frac{1}{4}\pi)$.

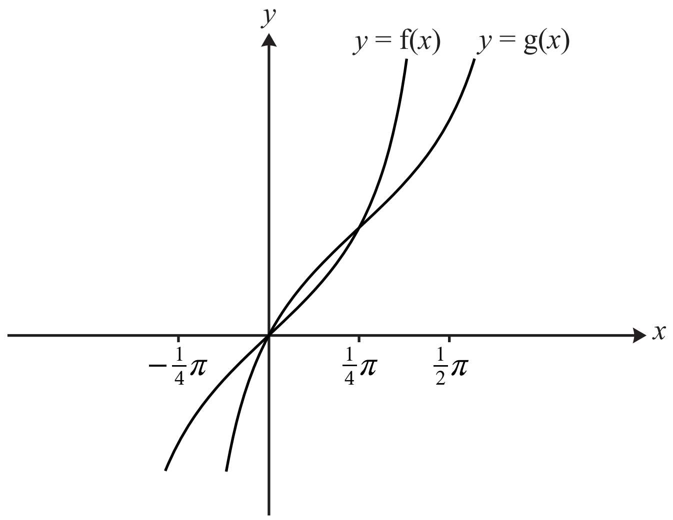

Fig. 8

(i) Describe a sequence of two transformations which maps the curve $y = \mathrm{f}(x)$ to the curve $y = \mathrm{g}(x)$. **[4]**

It can be shown that $\mathrm{g}(x) = \dfrac{2\sin x}{\sin x + \cos x}$.

(ii) Show that $\mathrm{g}'(x) = \dfrac{2}{(\sin x + \cos x)^2}$. Hence verify that the gradient of $y = \mathrm{g}(x)$ at the point $(\frac{1}{4}\pi, 1)$ is the same as that of $y = \mathrm{f}(x)$ at the origin. **[7]**

(iii) By writing $\tan x = \dfrac{\sin x}{\cos x}$ and using the substitution $u = \cos x$, show that $\displaystyle\int_0^{\frac{1}{4}\pi} \mathrm{f}(x)\,\mathrm{d}x = \int_{\frac{1}{\sqrt{2}}}^{1} \frac{1}{u}\,\mathrm{d}u$.

Evaluate this integral exactly. **[4]**

(iv) Hence find the exact area of the region enclosed by the curve $y = \mathrm{g}(x)$, the x-axis and the lines $x = \frac{1}{4}\pi$ and $x = \frac{1}{2}\pi$. **[2]**

9 Fig. 9 shows the line $y = x$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{1}{2}(\mathrm{e}^x - 1)$. The line and the curve intersect at the origin and at the point $\mathrm{P}(a, a)$.

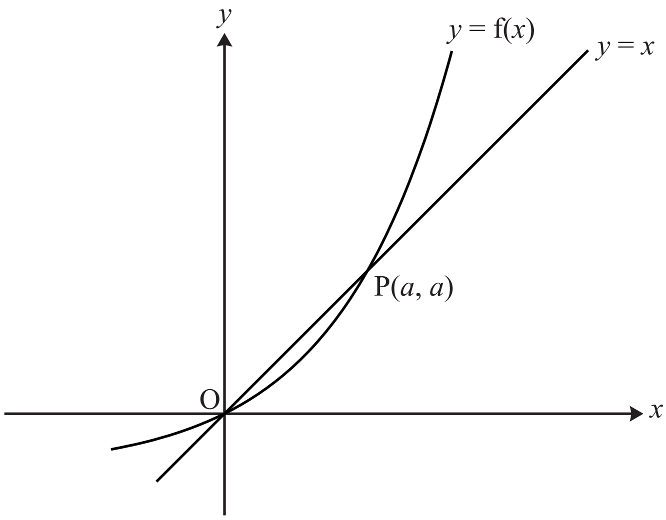

Fig. 9

(i) Show that $\mathrm{e}^a = 1 + 2a$. **[1]**

(ii) Show that the area of the region enclosed by the curve, the x-axis and the line $x = a$ is $\frac{1}{2}a$. Hence find, in terms of a, the area enclosed by the curve and the line $y = x$. **[6]**

(iii) Show that the inverse function of $\mathrm{f}(x)$ is $\mathrm{g}(x)$, where $\mathrm{g}(x) = \ln(1 + 2x)$. Add a sketch of $y = \mathrm{g}(x)$ to the copy of Fig. 9. **[5]**

(iv) Find the derivatives of $\mathrm{f}(x)$ and $\mathrm{g}(x)$. Hence verify that $\mathrm{g}'(a) = \dfrac{1}{\mathrm{f}'(a)}$.

Give a geometrical interpretation of this result. **[7]**