OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755:** Further Concepts for Advanced Mathematics

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

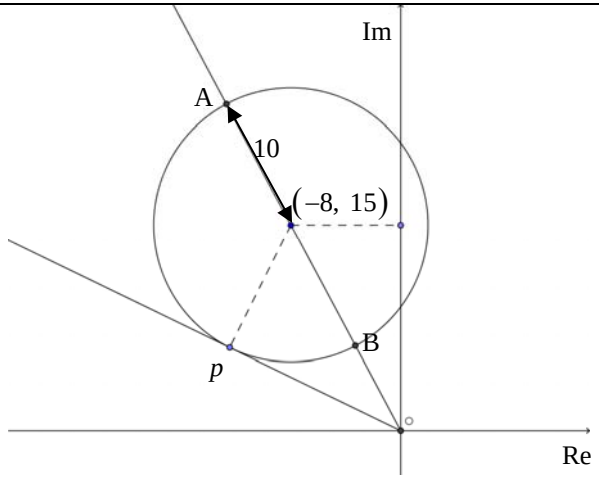
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		A is a reflection in the line $y = x$	B1		
			B is a two way stretch, (scale) factor 2 in the x-direction and (scale) factor 3 in the y-direction	B1 B1 [3]	Stretch, with attempt at details. Details correct.	
1	(ii)		$\mathbf{BA} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$	M1 A1 [2]	Attempt to multiply in correct order	
2			$\frac{z}{z^*} = \frac{a+b\mathrm{j}}{a-b\mathrm{j}} = \frac{(a+b\mathrm{j})^2}{(a-b\mathrm{j})(a+b\mathrm{j})}$	M1	Multiply top and bottom by $a + b$j and attempt to simplify	
			$= \frac{a^2+2ab\mathrm{j}-b^2}{a^2+b^2}$	M1	Using $j^2 = -1$	
			$\Rightarrow \mathrm{Re}\left(\frac{z}{z^*}\right) = \frac{a^2-b^2}{a^2+b^2}$ and $\mathrm{Im}\left(\frac{z}{z^*}\right) = \frac{2ab}{a^2+b^2}$	A1 A1 [4]	cao correctly labelled cao correctly labelled	
3			$z = 2 - \mathrm{j}$ is also a root	B1	Stated, not just used.	
			$\alpha\beta\gamma = \frac{15}{2}$, or $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{22}{2}$, with $\alpha\beta = (2+j)(2-j) = 5$ used.	M1 A1	Attempt to use roots in a relationship Correct equation obtained for γ .	Allow incorrect signs
			OR $(az+b)(z-2+j)(z-2-j) = 2z^3 + pz^2 + 22z - 15$	M1	Attempt use of complex factors.	Allow incorrect signs (z+…)
			$\Rightarrow (az+b)(z^2-4z+5) = 2z^3 + pz^2 + 22z - 15$	A1	Correct complex factors; one pair of factors correctly multiplied	
			OR $2(2+11j) + p(3+4j) + 22(2+j) - 15 = 0$	M1 A1	Substitution correct equation	Allow an incorrect sign
			Complete valid method for then obtaining the other unknown.	M1	Root relation, obtaining linear factor, equating real and imaginary parts	Signs correct
			real root $= \frac{3}{2}$, $p = -11$	A1 A1 [6]	FT one value	

Question			Answer	Marks	Guidance	
4	(i)		$x^2 - x + 2$ has discriminant -7, so $x^2 - x + 2 \neq 0$ and when e.g. $x = 0$, $x^2 - x + 2 > 0$ so positive for all x	E2,1,0	Discriminant < 0 shown **and** sign of $x^2 - x + 2$ or curve position discussed.	Allow complex roots found, with discussion
			OR $x^2 - x + 2 = (x - \frac{1}{2})^2 + \frac{7}{4} \geq \frac{7}{4} > 0$ for all x.	E2,1,0	Completing square and minimum value discussed	
			OR using $y = x^2 - x + 2$ $\frac{dy}{dx} = 2x - 1 = 0$ when $x = \frac{1}{2}$ and $y = \frac{7}{4}$; $\frac{d^2y}{dx^2} = 2 > 0$	E2,1,0	Calculus, showing minimum value>0.	
			Hence y has minimum value, and $y \geq \frac{7}{4} > 0$ for all x.	**[2]**		
4	(ii)		$\frac{2x}{x^2 - x + 2} > x$			
			$\Rightarrow 2x > x^3 - x^2 + 2x$	M1	Valid attempt to eliminate fraction	Or combine to one fraction > or<0
			$\Rightarrow 0 > x^3 - x^2 \Rightarrow 0 > x^2(x - 1)$	M1	Simplification and factors	In numerator
			0, 1 critical values	A1	Both, no other values given.	
			$x < 1$	A1		
			$\Rightarrow x < 0$ or $0 < x < 1$ or $x < 1, x \neq 0$	A1	cao	
				[5]		
			OR Graphical approach by sketching $y = \frac{2x}{x^2 - x + 2}$ and $y = x$ or $y = \frac{2x}{x^2 - x + 2} - x$	M2,1,0	Accuracy of sketch	
			Critical values 0 and 1	A1	Both	
			$x < 1$	A1		
			$\Rightarrow x < 0$ or $0 < x < 1$ or $x < 1, x \neq 0$	A1	cao	
				[5]		

Question			Answer	Marks	Guidance	
5	(i)		$\sum_{r=1}^{100}\frac{1}{(5+3r)(2+3r)}=k\sum_{r=1}^{100}\left[\frac{1}{2+3r}-\frac{1}{5+3r}\right]$	M1		
			$=k[\left(\frac{1}{5}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{11}\right)+...$	M1	Write out terms (at least first and last terms in full)	
			$+\left(\frac{1}{302}-\frac{1}{305}\right)]$	A1		
			$=k\left(\frac{1}{5}-\frac{1}{305}\right)$	M1	Cancelling inner terms	
			$=\frac{20}{305}=\frac{4}{61}$, oe	A1	cao	
				[5]		
	(ii)		$\frac{1}{15}$	B1		
				[1]		
6			When $n = 1$, $(-1)^0\frac{1\times 2}{2}=1$			
			and $1^2=1$, so true for $n = 1$	B1		
			Assume true for $n = k$			
			$\Rightarrow 1^2-2^2+3^2-.....+(-1)^{k-1}k^2=(-1)^{k-1}\frac{k(k+1)}{2}$	E1	Assuming true result for some n.	Condone series shown incomplete
			$\Rightarrow 1^2-2^2+3^2-.....+(-1)^{k-1}k^2+(-1)^{k+1-1}(k+1)^2$			
			$=(-1)^{k-1}\frac{k(k+1)}{2}+(-1)^{k+1-1}(k+1)^2$	M1*	Adding $(k+1)$th term to both sides.	
			$=(-1)^k\left[\frac{-k(k+1)}{2}+(k+1)^2\right]$	M1 Dep*	Attempt to factorise (at least one valid factor)	
			$=(-1)^k(k+1)\left(\frac{-k}{2}+k+1\right)$	A1	Correct factorisation Accept $(-1)^{k\pm m}$ provided expression correct.	

Question			Answer	Marks	Guidance	
			$=(-1)^k(k+1)\left(\frac{k+2}{2}\right)$	A1	Valid simplification with $(-1)^k$	
			$=(-1)^{[n-1]}\frac{n(n+1)}{2},\ n=k+1$	E1	Or target seen	
			Therefore if true for $n = k$ it is also true for $n = k+1$		Dependent on A1 and previous E1	
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1	Dependent on B1 and previous E1	
				[8]		
7	**(i)**		Asymptotes			
			$y = 0$,	B1		
			$x = 5$, $x = 8$	B1	both	
			Crosses axes at $(4,\ 0)$, $(0, -\frac{1}{10})$	B1 B1		
			$\frac{x-4}{(x-5)(x-8)} > 0 \Rightarrow x > 8$ or $4 < x < 5$	B1 B1		
				[6]		
7	**(ii)**		$\frac{x-4}{(x-5)(x-8)} = k \Rightarrow x-4 = kx^2 - 13kx + 40k$	M1	Attempt to remove fraction and simplify	
			$\Rightarrow kx^2 - (13k+1)x + 40k + 4 = 0$	A1	3 term quadratic (= 0)	
			$b^2 - 4ac = (13k+1)^2 - 4k(40k+4)$	M1	Attempt to use discriminant	
			$= 9k^2 + 10k + 1$	A1	Correct 3-term quadratic	
			Critical values –1, –1/9	A1	Roots found or factors shown	
			For no solutions to exist, $9k^2 + 10k + 1 < 0$	E1		
			$\Rightarrow -1 < k < -\frac{1}{9}$			
			No point on the graph has a y coordinate in the range $\Rightarrow -1 < y < -\frac{1}{9}$	E1	Accept equivalent statement	
				[7]		

Question			Answer	Marks	Guidance	
8	(i)					The circle should be reasonably circular. The radius should be shown to be 10 by annotation as in the diagram or by other positions marked. The centre point should be indicated and correct. The region should be shown by a key or by description. Accept a "dotty" outline to a shaded interior.
			The set of points for which $\lvert z-(-8+15\mathrm{j})\rvert<10$ is all points inside the circle, radius 10, centre $(-8,\ 15)$, excluding the points on the circumference.	B4 **[4]**	Circle, B1; radius 10, B1; centre $(-8,\ 15)$, B1;all points inside but not on circumference of the correctly placed circle, B1	Correctly placed: the "circle" must lie above the Re axis and intersect the Im axis twice as in the diagram.

Question			Answer	Marks	Guidance	
8	(ii)		Origin to centre of circle = $\sqrt{(-8)^2+15^2}=17$.	M1		Allow centre at $\pm 8 \pm 15j$ and FT
			Origin to centre of the circle $\pm$ 10	M1	Use of radius of circle	
			Point A is the point on the circle furthest from the origin. Since the radius of the circle is 10, OA = 27. Point B is the point on the circle closest to the origin. Since the radius of the circle is 10, OB=7. Hence for z in the circle $7<\lvert z\rvert<27$	E1	Correct explanation for both	
				[3]		
8	(iii)		P is the point where a line from the origin is a tangent to the circle giving the greatest argument θ, $-\pi<\theta\leq\pi$	B1	Correctly positioned on circle	Allow circles centred as in (ii)
			$\lvert p\rvert=\sqrt{17^2-10^2}=\sqrt{189}=13.7$ (3 s.f.)	B1	Accept $\sqrt{189}$ or $3\sqrt{21}$ or 13.7	
			$\arg p=\frac{\pi}{2}+\arcsin\frac{8}{17}+\arcsin\frac{10}{17}$	M1	Attempt to calculate the correct angle.	Correct circle only
			$=2.69$ (3 s.f.)	A1	cao Accept 154°	
				[4]		
9	(i)		$(8\times4)-(7\times5)-(12\times1)=-15$	M1	Any valid method soi	
			$\Rightarrow k=-\frac{1}{15}$	A1	No working or wrong working SC B1	
				[2]		
9	(ii)		$\begin{pmatrix}x\\y\\z\end{pmatrix}=-\frac{1}{15}\begin{pmatrix}4&2&3\\5&4&0\\1&-1&2\end{pmatrix}\begin{pmatrix}14\\-25\\3\end{pmatrix}=\begin{pmatrix}-1\\2\\-3\end{pmatrix}$	B1	Use of $\mathbf{A}^{-1}$ in correct position(s)	Condone missing k
				M1	Attempt to multiply matrices to obtain column vector	
			$x=-1, y=2, z=-3$	A2	–1 each error	
				[4]		
9	(iii)		$(1\times a)+(-8\times-4)+(-21\times2)=0\Rightarrow a=10$	M1	Attempt to multiply $\mathbf{BB}^{-1}$ matrices to find a or b soi	
			$(-7\times5)+(5\times1)+(15\times b)=0\Rightarrow b=2$	A1	For both	
				[2]		

Question			Answer	Marks	Guidance	
9	(iv)		$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	B1	By notation or explicitly	
			$= \frac{1}{3}\begin{pmatrix} 1 & 0 & 5 \\ -4 & -3 & 1 \\ 2 & 1 & 2 \end{pmatrix} \times -\frac{1}{15}\begin{pmatrix} 4 & 2 & 3 \\ 5 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix}$	M1	Attempt to multiply in correct sequence, may be implied by the answer (at least 7 elements correct)	Must include k
			$= -\frac{1}{45}\begin{pmatrix} 9 & -3 & 13 \\ -30 & -21 & -10 \\ 15 & 6 & 10 \end{pmatrix}$	A2	–1 each error FT their value of b.	
				[4]		

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