OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4753/01

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

QUESTION PAPER

FRIDAY 20 JANUARY 2012: Afternoon
DURATION: 1 hour 30 minutes

SUITABLE FOR VISUALLY IMPAIRED CANDIDATES

Candidates answer on the Printed Answer Book, or any suitable paper provided by the Centre. The Printed Answer Book may be enlarged by the Centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4753/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (NF/SW) 12MEP01018 55628

INSTRUCTIONS TO CANDIDATES

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

INFORMATION FOR CANDIDATES

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**
- **The total number of marks for this paper is 72.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (36 marks)

1 Differentiate $x^2 \tan 2x$. [3]

2 The functions f(x) and g(x) are defined as follows.

$f(x) = \ln x, \quad x > 0$
$g(x) = 1 + x^2, \quad x \in \mathbb{R}$

Write down the functions fg(x) and gf(x), and state whether these functions are odd, even or neither. [4]

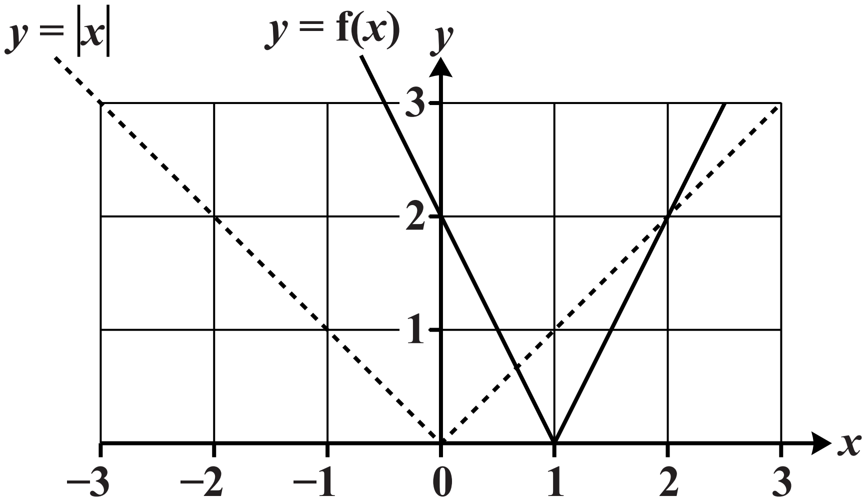
3 Show that $\int_0^{\frac{\pi}{2}} x \cos \tfrac{1}{2}x \, dx = \frac{\sqrt{2}}{2}\pi + 2\sqrt{2} - 4$. [5]

4 Prove or disprove the following statement:

'No cube of an integer has 2 as its units digit.' [2]

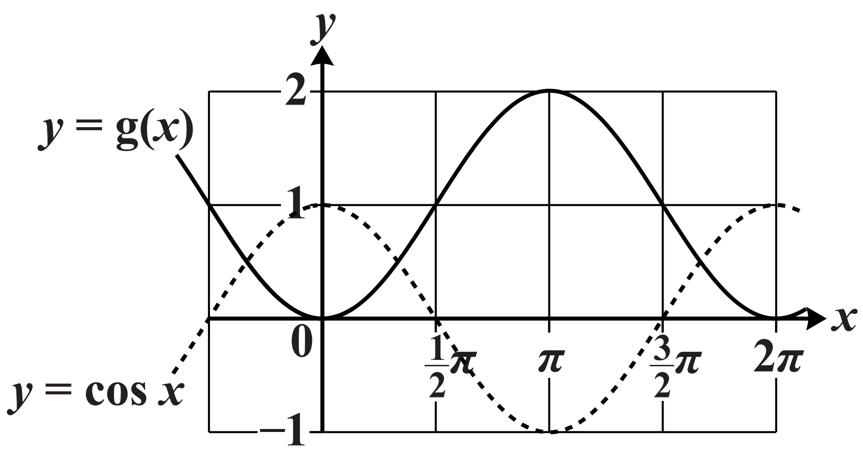
5 **Each of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for $\mathrm{f}(x)$ and $\mathrm{g}(x)$.**

(i)



[3]

(ii)



[3]

6 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius r metres of the oil slick t hours after the start of the leak is modelled by the equation $r = 20(1 - e^{-0.2t})$.

(i) Find the radius of the slick when $t = 2$, and the rate at which the radius is increasing at this time. [4]

(ii) Find the rate at which the area of the slick is increasing when $t = 2$. [4]

7 Fig. 7 below shows the curve $x^3 + y^3 = 3xy$. The point P is a turning point of the curve.

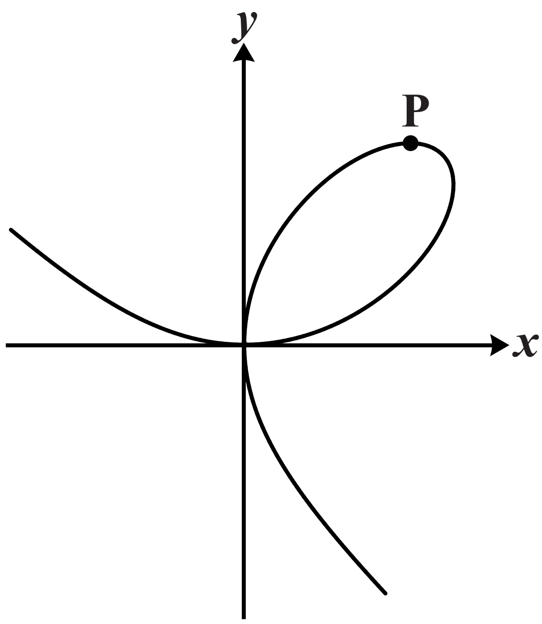


Fig. 7

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$. [4]

(ii) Hence find the exact x-coordinate of P. [4]

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SECTION B (36 marks)

8 **Fig. 8 below shows the curve $y = \dfrac{x}{\sqrt{x-2}}$, together with the lines $y = x$ and $x = 11$.**

The curve meets these lines at P and Q respectively. R is the point (11, 11).

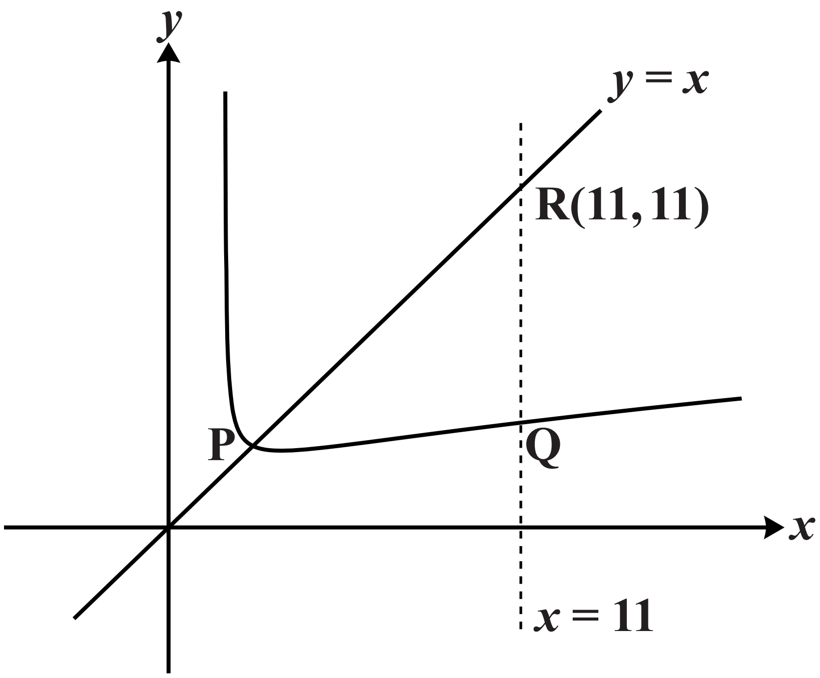

Fig. 8

(i) Verify that the x-coordinate of P is 3. [2]

(ii) Show that, for the curve, $\dfrac{dy}{dx} = \dfrac{x-4}{2(x-2)^{\frac{3}{2}}}$.

Hence find the gradient of the curve at P. Use the result to show that the curve is NOT symmetrical about $y = x$. [7]

(iii) Using the substitution $u = x - 2$, show that

$$\int_3^{11} \frac{x}{\sqrt{x-2}}\,\mathrm{d}x = 25\tfrac{1}{3}.$$

Hence find the area of the region PQR bounded by the curve and the lines $y = x$ and $x = 11$. [9]

9 **Fig. 9 below shows the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.**

The function $y = \mathrm{f}(x)$ is given by $\mathrm{f}(x) = \ln\left(\dfrac{2x}{1+x}\right), \; x > 0$.

The curve $y = \mathrm{f}(x)$ crosses the x-axis at P, and the line $x = 2$ at Q.

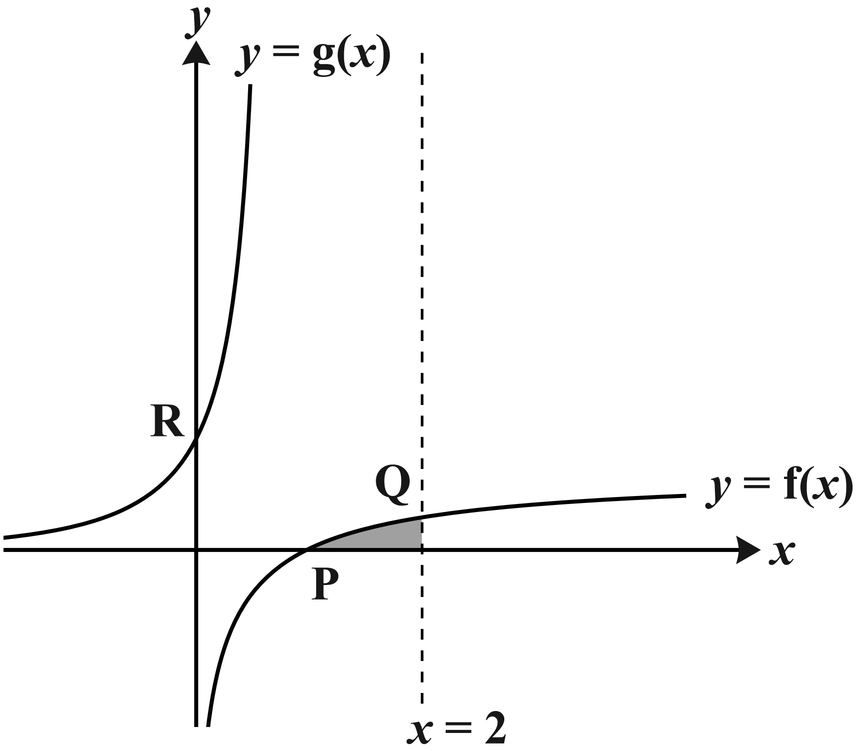

Fig. 9

(i) Verify that the x-coordinate of P is 1.

Find the exact y-coordinate of Q. [2]

(ii) Find the gradient of the curve at P.
[Hint: use $\ln \dfrac{a}{b} = \ln a - \ln b$.] [4]

The function g(x) is given by $\mathrm{g}(x) = \dfrac{\mathrm{e}^x}{2 - \mathrm{e}^x}, \quad x < \ln 2$.

The curve $y = \mathrm{g}(x)$ crosses the y-axis at the point R.

(iii) **Show that g(x) is the inverse function of f(x).**

Write down the gradient of y = g(x) at R. [5]

(iv) **Show, using the substitution $u = 2 - e^x$ or otherwise, that**

$$\int_0^{\ln \frac{4}{3}} g(x)\,dx = \ln \tfrac{3}{2}.$$

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln \frac{32}{27}$.

[Hint: consider its reflection in $y = x$.] [7]