

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**A2 GCE**

**4753/01**

**MATHEMATICS (MEI)**

**Methods for Advanced Mathematics (C3)**

**QUESTION PAPER**

**FRIDAY 20 JANUARY 2012: Afternoon**

**DURATION: 1 hour 30 minutes**

**SUITABLE FOR VISUALLY IMPAIRED CANDIDATES**

**Candidates answer on the Printed Answer Book, or any suitable paper provided by the Centre. The Printed Answer Book may be enlarged by the Centre.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4753/01**

**MEI Examination Formulae and Tables (MF2)**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

## **INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **ALL** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

## **INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **NO MARKS** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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**SECTION A (36 marks)**

**1 Differentiate  $x^2 \tan 2x$ . [3]**

**2 The functions  $f(x)$  and  $g(x)$  are defined as follows.**

$$\begin{aligned} f(x) &= \ln x, & x > 0 \\ g(x) &= 1 + x^2, & x \in \mathbb{R} \end{aligned}$$

**Write down the functions  $fg(x)$  and  $gf(x)$ , and state whether these functions are odd, even or neither. [4]**

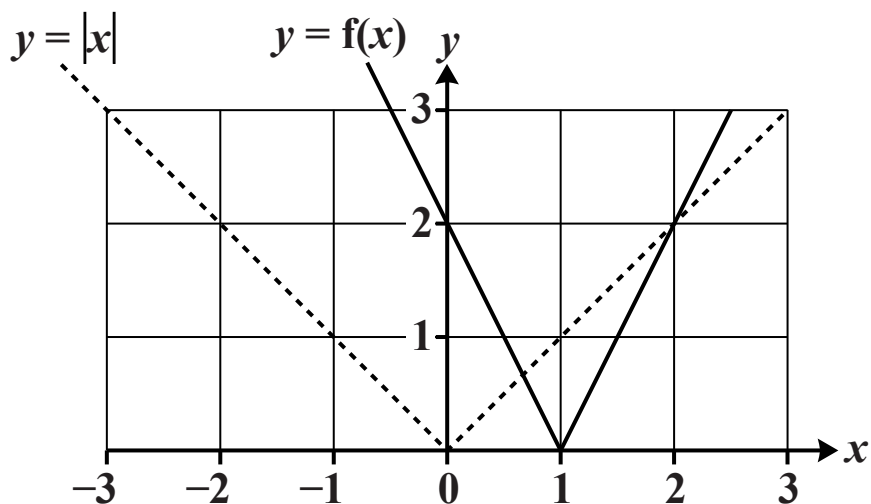
**3 Show that  $\int_0^{\frac{\pi}{2}} x \cos \frac{1}{2}x \, dx = \frac{\sqrt{2}}{2} \pi + 2\sqrt{2} - 4$ . [5]**

**4 Prove or disprove the following statement:**

**‘No cube of an integer has 2 as its units digit.’ [2]**

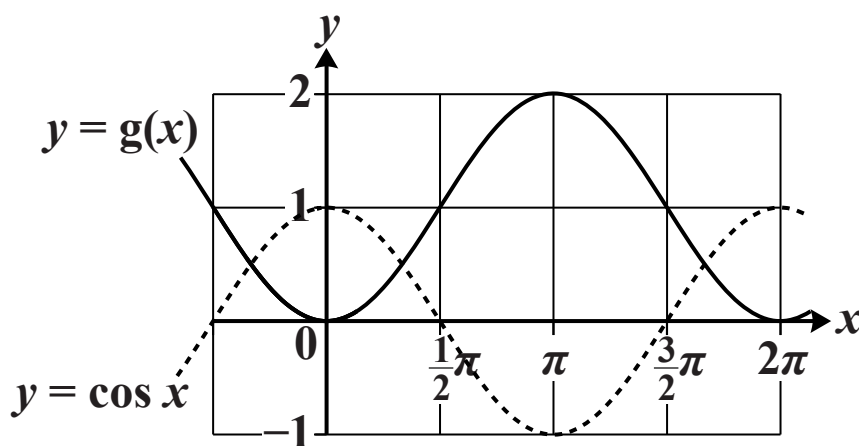
5 Each of the graphs of  $y=f(x)$  and  $y=g(x)$  below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for  $f(x)$  and  $g(x)$ .

(i)



[3]

(ii)



[3]

6 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius  $r$  metres of the oil slick  $t$  hours after the start of the leak is modelled by the equation  $r = 20(1 - e^{-0.2t})$ .

(i) Find the radius of the slick when  $t = 2$ , and the rate at which the radius is increasing at this time. [4]

(ii) Find the rate at which the area of the slick is increasing when  $t = 2$ . [4]

7 Fig. 7 below shows the curve  $x^3 + y^3 = 3xy$ . The point P is a turning point of the curve.

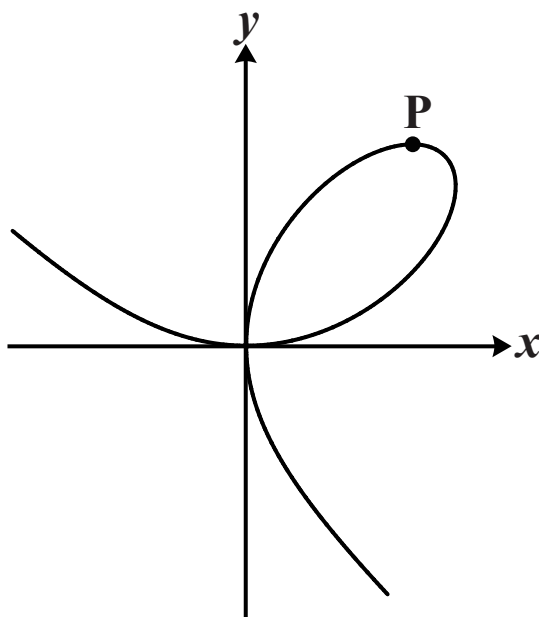


Fig. 7

(i) Show that  $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$ . [4]

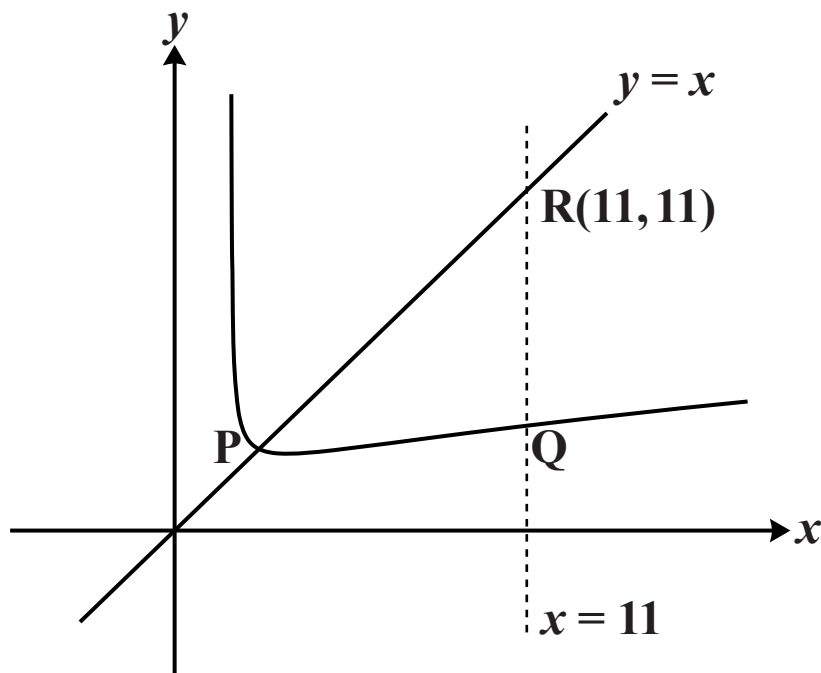
(ii) Hence find the exact  $x$ -coordinate of P. [4]

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**SECTION B (36 marks)**

- 8** Fig. 8 below shows the curve  $y = \frac{x}{\sqrt{x-2}}$ , together with the lines  $y = x$  and  $x = 11$ .

The curve meets these lines at P and Q respectively. R is the point (11, 11).



**Fig. 8**

- (i) Verify that the  $x$ -coordinate of P is 3. [2]
- (ii) Show that, for the curve,  $\frac{dy}{dx} = \frac{x-4}{2(x-2)^{\frac{3}{2}}}$ .

Hence find the gradient of the curve at P. Use the result to show that the curve is NOT symmetrical about  $y = x$ . [7]



**(iii) Using the substitution  $u = x - 2$ , show that**

$$\int_3^{11} \frac{x}{\sqrt{x-2}} dx = 25\frac{1}{3}.$$

**Hence find the area of the region PQR bounded by the curve and the lines  $y = x$  and  $x = 11$ . [9]**

9 Fig. 9 below shows the curves  $y = f(x)$  and  $y = g(x)$ .

The function  $y = f(x)$  is given by  $f(x) = \ln\left(\frac{2x}{1+x}\right)$ ,  $x > 0$ .

The curve  $y = f(x)$  crosses the  $x$ -axis at P, and the line  $x = 2$  at Q.

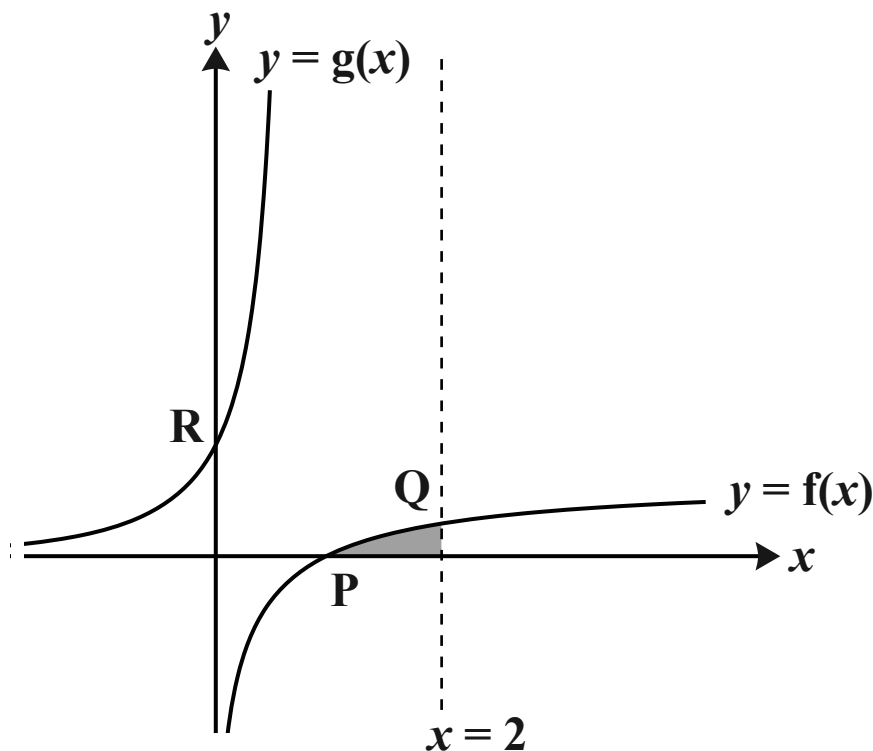


Fig. 9

(i) Verify that the  $x$ -coordinate of P is 1.

Find the exact  $y$ -coordinate of Q. [2]

(ii) Find the gradient of the curve at P.

[Hint: use  $\ln \frac{a}{b} = \ln a - \ln b$ .] [4]

The function  $g(x)$  is given by  $g(x) = \frac{e^x}{2 - e^x}$ ,  $x < \ln 2$ .

The curve  $y = g(x)$  crosses the  $y$ -axis at the point R.

**(iii) Show that  $g(x)$  is the inverse function of  $f(x)$ .**

**Write down the gradient of  $y = g(x)$  at  $R$ . [5]**

**(iv) Show, using the substitution  $u = 2 - e^x$  or otherwise, that**

$$\int_0^{\ln \frac{4}{3}} g(x) dx = \ln \frac{3}{2}.$$

**Using this result, show that the exact area of the shaded region shown in Fig. 9 is  $\ln \frac{32}{27}$ .**

**[Hint: consider its reflection in  $y = x$ .] [7]**



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