

**ADVANCED GCE  
MATHEMATICS (MEI)**

Statistics 4

**4769**

Candidates answer on the answer booklet.

**OCR supplied materials:**

- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Thursday 26 May 2011  
Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

*Option 1: Estimation*

- 1** The random variable  $X$  has the Normal distribution with mean 0 and variance  $\theta$ , so that its probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\theta}} e^{-x^2/2\theta}, \quad -\infty < x < \infty,$$

where  $\theta$  ( $\theta > 0$ ) is unknown. A random sample of  $n$  observations from  $X$  is denoted by  $X_1, X_2, \dots, X_n$ .

- (i) Find  $\hat{\theta}$ , the maximum likelihood estimator of  $\theta$ . [14]
- (ii) Show that  $\hat{\theta}$  is an unbiased estimator of  $\theta$ . [4]
- (iii) In large samples, the variance of  $\hat{\theta}$  may be estimated by  $\frac{2\hat{\theta}^2}{n}$ . Use this and the results of parts (i) and (ii) to find an approximate 95% confidence interval for  $\theta$  in the case when  $n = 100$  and  $\sum X_i^2 = 1000$ . [6]

*Option 2: Generating Functions*

- 2** The random variable  $X$  has the  $\chi_n^2$  distribution. This distribution has moment generating function  $M(\theta) = (1 - 2\theta)^{-\frac{1}{2}n}$ , where  $\theta < \frac{1}{2}$ .

- (i) Verify the expression for  $M(\theta)$  quoted above for the cases  $n = 2$  and  $n = 4$ , given that the probability density functions of  $X$  in these cases are as follows. [10]

$$n = 2: \quad f(x) = \frac{1}{2}e^{-\frac{1}{2}x} \quad (x > 0)$$

$$n = 4: \quad f(x) = \frac{1}{4}xe^{-\frac{1}{2}x} \quad (x > 0)$$

- (ii) For the general case, use  $M(\theta)$  to find the mean and variance of  $X$  in terms of  $n$ . [7]
- (iii)  $Y_1, Y_2, \dots, Y_k$  are independent random variables, each with the  $\chi_1^2$  distribution. Show that  $W = \sum_{i=1}^k Y_i$  has the  $\chi_k^2$  distribution. [4]

- (iv) Use the Central Limit Theorem to find an approximation for  $P(W < 118.5)$  for the case  $k = 100$ . [3]

## Option 3: Inference

- 3 (i) Explain the meaning of the following terms in the context of hypothesis testing: Type I error, Type II error, operating characteristic, power. [8]

- (ii) A market research organisation is designing a sample survey to investigate whether expenditure on everyday food items has increased in 2011 compared with 2010. For one of the populations being studied, the random variable  $X$  is used to model weekly expenditure, in £, on these items in 2011, where  $X$  is Normally distributed with mean  $\mu$  and variance  $\sigma^2$ . As the corresponding mean value in 2010 was 94, the hypotheses to be examined are

$$H_0: \mu = 94,$$

$$H_1: \mu > 94.$$

By comparison with the corresponding 2010 value,  $\sigma^2$  is assumed to be 25.

The following criteria for the survey are laid down.

- If in fact  $\mu = 94$ , the probability of concluding that  $\mu > 94$  must be only 2%
- If in fact  $\mu = 97$ , the probability of concluding that  $\mu > 94$  must be 95%

A random sample of size  $n$  is to be taken and the usual Normal test based on  $\bar{X}$  is to be used, with a critical value of  $c$  such that  $H_0$  is rejected if the value of  $\bar{X}$  exceeds  $c$ . Find  $c$  and the smallest value of  $n$  that is required. [13]

- (iii) Sketch the power function of an ideal test for examining the hypotheses in part (ii). [3]

## Option 4: Design and Analysis of Experiments

- 4 (a) Provide an example of an experimental situation where there is one factor of primary interest and where a suitable experimental design would be

(i) randomised blocks,

(ii) a Latin square.

In each case, explain carefully why the design is suitable and why the other design would not be appropriate. [12]

- (b) An industrial experiment to compare four treatments for increasing the tensile strength of steel is carried out according to a completely randomised design. For various reasons, it is not possible to use the same number of replicates for each treatment. The increases, in a suitable unit of tensile strength, are as follows.

Treatment A	Treatment B	Treatment C	Treatment D
10.1	21.1	9.2	22.6
21.2	20.3	8.8	17.4
11.6	16.0	15.2	23.1
13.6		15.0	19.2
		12.4	

[The sum of these data items is 256.8 and the sum of their squares is 4471.92.]

Construct the usual one-way analysis of variance table. Carry out the appropriate test, using a 5% significance level. [12]

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE**



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