OCR

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A:** Applications of Advanced Mathematics: Paper A

Mark Scheme for June 2011

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

1 $\dfrac{1}{(2x+1)(x^2+1)} = \dfrac{A}{2x+1} + \dfrac{Bx+C}{x^2+1}$ $\Rightarrow\quad 1 = A(x^2 + 1) + (Bx + C)(2x + 1)$ $x = -½$: $1 = 1¼\,A \Rightarrow A = 4/5$ coeff of x^2: $0 = A + 2B \Rightarrow B = -2/5$ constants: $1 = A + C \Rightarrow C = 1/5$	M1 M1 B1 B1 B1 [5]	correct form of partial fractions mult up and equating or substituting oe soi www www www	for omission of B or C on numerator, M0, M1, then (x= -1/2, A= 4/5) B1, B0, B0 is possible. for $\dfrac{A+Dx}{2x+1}+\dfrac{Bx+C}{x^2+1}$, M1,M1 then B1 for both A=4/5 and D=0, B1, B1 is possible. isw for incorrect assembly of final partial fractions following correct A,B & C. condone omission of brackets for second M1 only if the brackets are implied by subsequent working.
2 $(1+3x)^{\frac{1}{3}} = 1 + \frac{1}{3}(3x) + \dfrac{\frac{1}{3}.(-\frac{2}{3})}{2!}(3x)^2 + \ldots$ $= 1 + x - x^2 + \ldots$ Valid for $-1 \le 3x \le 1$ $\Rightarrow\quad -1/3 \le x \le 1/3$	M1 A1 A1 M1 A1 [5]	correct binomial coefficients $1 + x \ldots$ $\ldots - x^2$ or $\|3x\| \le 1$ oe or $\|x\| \le 1/3$ (correct final answer scores M1A1)	ie 1, 1/3, (1/3)(-2/3)/2 not nCr form simplified www in this part simplified www in this part, ignore subsequent terms using $(3x)^2$ as $3x^2$ can score M1B1B0 condone omission of brackets if $3x^2$ is used as $9x^2$ do not allow MR for power 3 or -1/3 or similar **condone inequality signs throughout** or say < at one end and ≤ at the other condone $-1/3 \le \|x\| \le 1/3$, $x \le 1/3$ is M0A0 the last two marks are not dependent on the first three
3 $2\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$ $= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 2,\ R\sin\alpha = 3$ $\Rightarrow R^2 = 2^2 + 3^2 = 13, R = \sqrt{13}$ $\tan\alpha = 3/2$, $\Rightarrow \alpha = 0.983$ minimum $1 - \sqrt{13}$, maximum $1 + \sqrt{13}$	M1 B1 M1 A1 B1 B1 [6]	correct pairs $R = \sqrt{13}$ or 3.61 or better 0.98 or better or –2.61, 4.61 or better	condone wrong sign at this stage correct division, ft from first M1 radians only accept multiples of π that round to 0.98 allow B1, B1ft for $1-\sqrt{R}$ and $1+\sqrt{R}$ for their R to 2dp or better

4(i) $x = 2\sin\theta$, $y = \cos 2\theta$ When $\theta = \pi/3$, $x = 2\sin \pi/3 = \sqrt{3}$ $y = \cos 2\pi/3 = -½$ **EITHER** $dx/d\theta = 2\cos\theta$, $dy/d\theta = -2\sin 2\theta$ $\Rightarrow \frac{dy}{dx} = \frac{-\sin 2\theta}{\cos\theta}$ $= \frac{-\sin 2\pi/3}{\cos\pi/3} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$ ……………………………………………………….... **OR** expressing y in terms of x, $y=1-x^2/2$ $\frac{dy}{dx}=-x$ or $-2\sin\theta$ $= -\sqrt{3}$	B1 B1 M1 A1 A1 …….. M1 A1 A1 [5]	$x = \sqrt{3}$ $y = -½$ $dy/dx = (dy/d\theta) / (dx/d\theta)$ used any correct equivalent form exact www ………………………………………… exact www	exact only (isw all dec answers following exact ans) ft their derivatives if right way up (condone one further minor slip if intention clear) condone poor notation can isw if incorrect simplification
(ii) $y = 1 - 2\sin^2\theta = 1 - 2(x/2)^2 = 1 - ½x^2$	M1A1 [2]	or reference to (i) if used there	for M1, need correct trig identity and attempt to substitute for x allow SC B1 for $y=\cos 2\arcsin(x/2)$ or equivalent

5 $\text{cosec}^2\theta = 1 + \cot^2\theta$			(use of $1-\cot^2\theta$ could lead to M0 M1 M1 B1)
$\Rightarrow$ $1 + \cot^2\theta = 1 + 2\cot\theta$	M1	correct trig identity used	
$\Rightarrow$ $\cot^2\theta - 2\cot\theta = 0$			
$\Rightarrow$ $\cot\theta(\cot\theta - 2) = 0$	M1	factorising oe	allow if $\cot\theta=0$ not seen (ie quadratic equation followed by $\cot\theta-2=0$ or $\cot\theta=2$)
$\Rightarrow$ $\cot\theta = 0$, **and** $\cot\theta = 2$, $\tan\theta = ½$	M1	**both** needed and $\cot\theta = 1/\tan\theta$ soi	
$\Rightarrow$ $\theta = 26.6°, -153.4°, -90°, 90°$	B3,2,1,0	$-90°, 90°, 27°, -153°$ or better www	(omission of $\cot\theta=0$ could gain M1, M1, M0, B1)
……………………………………………………..		……………………………………	………………………………………...
OR $\dfrac{1}{\sin^2\theta} = 1 + \dfrac{2\cos\theta}{\sin\theta} = \dfrac{\sin\theta + 2\cos\theta}{\sin\theta}$			
$\Rightarrow \sin^2\theta + 2\sin\theta\cos\theta - 1 = 0$	M1	correct trig equivalents and a one line equation (or common denominator) formed	as above
$\Rightarrow 2\sin\theta\cos\theta - \cos^2\theta = 0$			
$\Rightarrow \cos\theta\ (2\sin\theta - \cos\theta) = 0$	M1	use of Pythagoras and factorising	allow if $\cos\theta=0$ not seen (as above)
$\Rightarrow \cos\theta = 0$, **and** $\tan\theta = ½$	M1	**both** needed and $\tan\theta = \sin\theta/\cos\theta$ oe soi	………………………………………
$\theta = 26.6°, -153.4°, -90°, 90°$	B3,2,1,0	accept 27º, -153º as above	**in both cases**, -1 if **extra** solutions in the range are given (dependent on at least B1 being scored)-not their incorrect solutions eg 26.6°,-153.4°, 0°,180°,-180° would obtain B1
	[6]	……………………………………… answers, no working, award B3,2,1,0 (it is possible to score say M1 then B3 ow)	-1 MR if answers given in radians (-π/2,π/2,0.464, -2.68 (-1.57.1.57) or multiples of π that round to these, or better) (dependent on at least B1 being scored) to lose both of these, at least B2 would need to be scored.
6 Vol = vol of rev of curve + vol of rev of line	M1	(soi) at any stage	
vol of rev of curve = $\int_0^2 \pi x^2\, dy$			
$= \int_0^2 \pi \dfrac{y}{2}\, dy$	M1	substituting $x^2 = y/2$	for M1 need π, substitution for x^2, (dy soi), intention to integrate and correct limits
$= \pi\left[\dfrac{y^2}{4}\right]_0^2$	B1	$\left[\dfrac{y^2}{4}\right]$	even if π missing or limits incorrect or missing
$= \pi$	A1		cao
height of cone = 3 – 2 = 1	B1	h=1 soi	OR $\pi\int_2^3 (3-y)^2 dy$ M1(even if expanded incorrectly)
so vol of cone = $1/3\ \pi\ 1^2 \times 1$ $= \pi/3$	B1		$=\pi/3$ A1 www
so total vol = $4\pi/3$	A1 [7]	www cao	

Section B

7(i) $\overrightarrow{AB} = \begin{pmatrix}-4\\0\\-2\end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix}-2\\4\\1\end{pmatrix}$	B1B1		condone rows
$\cos BAC = \dfrac{\begin{pmatrix}-4\\0\\-2\end{pmatrix}.\begin{pmatrix}-2\\4\\1\end{pmatrix}}{AB.AC} = \dfrac{(-4).(-2)+0.4+(-2).1}{\sqrt{20}\sqrt{21}}$ $= 0.293$	M1 M1 A1	dot product evaluated **cos** BAC= dot product / |AB|. |AC| 0.293 or cos ABC=correct numerical expression as RHS above, or better	**substituted,** ft their vectors AB, AC for method only need to see method for modulae as far as √… use of vectors BA and CA could obtain B0 B0 M1 M1 A1 A1
⇒ BAC = 73.0°	A1 [6]	or rounds to 73.0º (accept 73º www)	(or 1.27 radians)
(ii) A: $x + y - 2z + d = 2 - 6 + d = 0$ ⇒ $d = 4$ B: $-2 + 0 - 2\times1 + 4 = 0$ C: $0 + 4 - 2\times4 + 4 = 0$	 M1 DM1 A1	 substituting one point evaluating for other two points $d = 4$ www	alternatively, finding the equation of the plane using any valid method (eg from vector equation, M1 A1 for using valid equation and eliminating both parameters, A1 for required form, or using vector cross product to get x+y-2z =c oe M1 A1,finding c and required form, A1, or showing that two vectors in the plane are perpendicular to normal vector M1 A1 and finding d, A1) oe
Normal $\mathbf{n} = \begin{pmatrix}1\\1\\-2\end{pmatrix}$	B1	stated or used as normal anywhere in part (ii)	
$\mathbf{n}.\begin{pmatrix}0\\0\\1\end{pmatrix} = \dfrac{-2}{\sqrt{6}} = \cos\theta$	M1 A1	finding angle between normal vector and **k** allow ±2/√6 or 144.7º for A1	(may have deliberately made +ve to find acute angle)
⇒ θ = 144.7° ⇒ acute angle = 35.3°	 A1 [7]	 or rounds to 35.3º	do not need to find 144.7º explicitly (or 0.615 radians)
(iii) At D, $-2 + 4 - 2k + 4 = 0$ ⇒ $2k = 6$, $k = 3$ *	M1 A1	substituting into plane equation **AG**	
$\overrightarrow{CD} = \begin{pmatrix}-2\\0\\-1\end{pmatrix} = \frac{1}{2}\overrightarrow{AB}$	M1	$\overrightarrow{CD} = \begin{pmatrix}-2\\0\\-1\end{pmatrix}$	finding vector CD (or vector DC)
⇒ CD is parallel to AB	A1		or DC parallel to AB or BA oe (or hence two parallel sides, if clear which) but A0 if their vector CD is vector DC
CD : AB = 1 : 2	B1 [5]	mark final answer **www** allow CD:AB=1/2, √5:√20 oe, AB is twice CD oe	for B1 allow vector CD used as vector DC

8(i) $\frac{dV}{dt} = -kx$ $V = 1/3\, x^3 \Rightarrow dV/dx = x^2$ $\frac{dV}{dt} = \frac{dV}{dx}.\frac{dx}{dt} = x^2\frac{dx}{dt}$ $\Rightarrow x^2\frac{dx}{dt} = -kx$ $\Rightarrow x\frac{dx}{dt} = -k$ *	B1 M1 A1 [3]	oe eg $dx/dt = dx/dV \cdot dV/dt = 1/x^2 \cdot -kx = -k/x$ **AG**	
(ii) $x\frac{dx}{dt} = -k \Rightarrow \int x\,dx = \int -k\,dt$ $\Rightarrow \frac{1}{2}x^2 = -kt + c$ When $t = 0, x = 10 \Rightarrow 50 = c$ $\Rightarrow \frac{1}{2}x^2 = 50 - kt$ $\Rightarrow x = \sqrt{(100 - 2kt)}$ *	M1 A1 B1 A1 [4]	separating variables and intention to integrate condone absence of c finding c correctly ft their integral of form $ax^2 = bt + c$ where a, b non zero constants **AG**	
(iii) When $t = 50, x = 0$ $\Rightarrow 0 = 100 - 100k \Rightarrow k = 1$	M1 A1 [2]		
(iv) $dV/dt = 1 - kx = 1 - x$ $\Rightarrow x^2 dx/dt = 1 - x$ $\Rightarrow \frac{dx}{dt} = \frac{1-x}{x^2}$ *	M1 A1 [2]	for $dV/dt = 1 - kx$ or better **AG**	
(v) $\frac{1}{1-x} - x - 1 = \frac{1-(1-x)x-(1-x)}{1-x}$ $= \frac{1-x+x^2-1+x}{1-x} = \frac{x^2}{1-x}$ * $\int \frac{x^2}{1-x}dx = \int dt \Rightarrow \int (\frac{1}{1-x} - x - 1)dx = t + c$ $\Rightarrow -\ln(1-x) - \frac{1}{2}x^2 - x = t + c$ When $t = 0, x = 0 \Rightarrow c = -\ln 1 - 0 - 0 = 0$ $\Rightarrow t = \ln\left(\frac{1}{1-x}\right) - \frac{1}{2}x^2 - x$ *	M1 A1 M1 A1 B1 A1 [6]	combining to single fraction **AG** separating variables & subst for $x^2/(1-x)$ and intending to integrate condone absence of c finding c for equation of correct form eg $c = 0$, or $\pm$ ln1 (allow c=0 without evaluation here) cao **AG**	or long division or cross multiplying check signs need both sides of integral accept ln $(1/(1-x))$ as $-\ln(1-x)$ www ie $a\ln(1-x)+bx^2+dx=et+c$ a,b,d,e non zero constants do not allow if c=0 without evaluation
(vi) understanding that ln (1/0) or 1/0 is undefined oe	B1 [1]	www	ln (1/0) = ln 0, 1/0 = ∝ and ln (1/0) = ∝ are all B0

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