

Mathematics (MEI)

Advanced GCE 4777

Numerical Computation

Mark Scheme for June 2010

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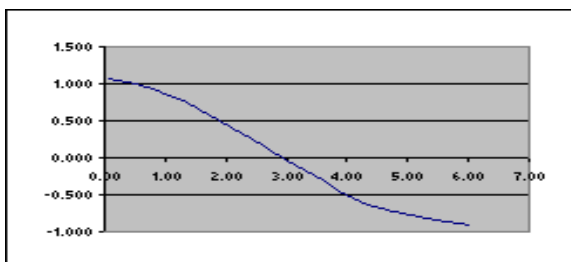
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1 (i) The data are not evenly spaced so (ordinary) differences will not work [E1]
 Lagrange's method is not well suited to increasing the degree of the [E1]
 approximating polynomial because it requires complete recalculation [E1]
 [subtotal 3]

x	f
0.09	1.076
0.93	0.897
1.91	0.498
4.10	-0.544
4.91	-0.740
6.04	-0.900



[G2]

[subtotal 2]

(iii) x	f	1DD	2DD	3DD	4DD	5DD
1.91	0.498					
4.10	-0.544	-0.4758				
4.91	-0.740	-0.24198	0.077941			
0.93	0.897	-0.41131	0.053417	0.025025		
0.09	1.076	-0.2131	-0.04112	0.023576	0.000796	
6.04	-0.900	-0.3321	-0.02329	0.015782	-0.00402	-0.00117

re-order: [M1A1]
 table: [M1A1]

f(3)						
=	0.498					
+	-0.51862	-0.021		linear		[M1A1]
+	-0.09345	-0.114		quadratic		[M1A1]
+	0.057309	-0.057		cubic		[M1A1]
+	0.003774	-0.053		quartic		[M1A1]

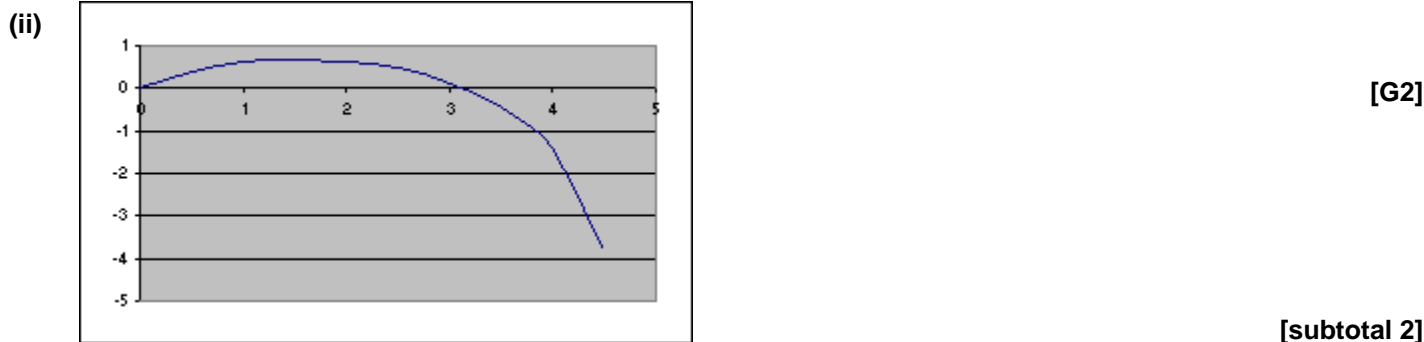
f(3) approximately zero, but difficult to say whether -0.05 or -0.06, -0.1 or 0.0. [E1E1]
 [subtotal 14]

(iv) x	f	1DD	2DD	3DD	4DD	5DD
1.91	0.498					
4.10	-0.544	-0.4758				
4.91	-0.740	-0.24198	0.077941			
0.93	0.897	-0.41131	0.053417	0.025025		
0.09	1.076	-0.2131	-0.04112	0.023576	0.000796	
6.04	-0.900	-0.3321	-0.02329	0.015782	-0.00402	-0.00117

user-specified x:	2.89	0.498					
			-0.46628	0.032			adjust SS to allow user-specified x: [M1A1]
			-0.09242	-0.061			
			0.056679	-0.004			trial and error: [M1A1]
			0.003738	0.000			answer: [A1]

[subtotal 5]
 [TOTAL 24]

- 2 (i) $T_n - I = A_2h^2 + A_4h^4 + A_6h^6 + \dots$
 $T_{2n} - I = A_2(h/2)^2 + A_4(h/2)^4 + A_6(h/2)^6 + \dots$ [M1A1]
 $4(T_{2n} - I) - (T_n - I) = b_4h^4 + b_6h^6 + \dots$ [M1]
 $4T_{2n} - T_n - 3I = b_4h^4 + b_6h^6 + \dots$ [A1]
 $(4T_{2n} - T_n)/3 - I = B_4h^4 + B_6h^6 + \dots$ [A1]
 $(T_n^* = (4T_{2n} - T_n)/3$ has error of order h^4 as given)
 $T_n^{**} = (16T_{2n}^* - T_n^*)/15$ has error of order h^6 [B1]
 [subtotal 6]



(iii)

x	f(x)	T	T*	T**	T***	(T****)		
0	0							
3.141593	2.22E-16	3.49E-16						
1.570796	0.693147	1.088793	1.451724					
0.785398	0.5348						f:	[A1]
2.356194	0.5348	1.384458	1.483014	1.485099			T:	[M1A2]
0.392699	0.324026						T*:	[M1A1]
1.178097	0.654344						T**:	[M1A1]
1.963495	0.654344						T***:	[M1A1]
2.748894	0.324026	1.460639	1.486033	1.486234	1.486252			[M1A1]
0.19635	0.178222							[M1A1]
0.589049	0.441842							
0.981748	0.605119						answer:	[A1]
1.374447	0.683493							
1.767146	0.683493							
2.159845	0.605119							
2.552544	0.441842							
2.945243	0.178222	1.479855	1.48626	1.486275	1.486276	1.486276		

[subtotal 11]

- (iv) Spreadsheet as above, but seen to work for user-specified c in place of 3.141593 [M2]
- Sequence of values representing trial and error towards solution:
- | c | 4 | 4.5 | 4.4 | 4.45 | 4.44 | 4.442 |
|---|----------|----------|----------|----------|----------|---------|
| I | 0.977343 | -0.20713 | 0.133659 | -0.02687 | 0.006681 | 0.00003 |
- [M1A1]
- Answer 4.442 to 3 decimal places [A1]

[subtotal 5]
 [TOTAL 24]

3 (i) Modified Euler method

h	x	y	k1	k2	new y		
0.1	1	1	0.141421	0.150185	1.145803		
	1.1	1.145803	0.150346	0.159856	1.300904	setup:	[M2]
	1.2	1.300904	0.160034	0.170271	1.466056		
	1.3	1.466056	0.170466	0.181415	1.641997	first run:	[A2]
	1.4	1.641997	0.181626	0.193273	1.829446		
	1.5	1.829446	0.193499	0.205833	2.029112		
	1.6	2.029112	0.206072	0.219085	2.24169		
	1.7	2.24169	0.219337	0.23302	2.467869		
	1.8	2.467869	0.233284	0.247633	2.708328		
	1.9	2.708328	0.247908	0.262916	2.963739		
	2	2.963739					

h	α	diffs	ratio		
0.1	2.963739		of diffs		
0.05	2.964219	0.000480		further runs:	[A1A1A1]
0.025	2.964341	0.000122	0.254789		
0.0125	2.964372	0.000031	0.252418	differences:	[M1]
0.00625	2.964380	0.000008	0.251215	ratios:	[M1A1]

Correct to 4 dp, $\alpha = 2.9644$ [A1]
 Ratio of differences indicates 2nd order convergence [E1]
 [subtotal 12]

(ii) Predictor corrector method

h	x	y	y pred	y corr1	y corr2	y corr3		
0.1	1	1	1.141421	1.145803	1.145884	1.145885		
	1.1	1.145885	1.296234	1.300989	1.301078	1.30108	setup:	[M2]
	1.2	1.30108	1.46112	1.466239	1.466336	1.466338		
	1.3	1.466338	1.636815	1.64229	1.642395	1.642397	first run:	[A2]
	1.4	1.642397	1.824039	1.829862	1.829975	1.829978		
	1.5	1.829978	2.023497	2.029664	2.029784	2.029786		
	1.6	2.029786	2.235885	2.242392	2.242518	2.24252		
	1.7	2.24252	2.461889	2.468732	2.468864	2.468866		
	1.8	2.468866	2.702189	2.709364	2.709501	2.709504		
	1.9	2.709504	2.957457	2.964961	2.965104	2.965107		
	2	2.965107						

h	α	diffs	ratio		
0.1	2.965107		of diffs		
0.05	2.964564	-0.000543		further runs:	[A1A1A1]
0.025	2.964428	-0.000136	0.250154		
0.0125	2.964394	-0.000034	0.250039	these -->	differences
0.00625	2.964385	-0.000008	0.25001	may appear in (iii)	and ratios:

(iii) The rate of convergence (see ratio of differences) is the same for both methods. [E1]
 Magnitude of errors about the same for a given h [E1]
 More programming required for predictor-corrector [E1]
 Modified Euler (at least in this case) is preferable [E1]
 [subtotal 4]
 [TOTAL 24]

4 (i)

7.1	6	5	4	1	$x_1 = 0.320827$	<i>Gauss elim:</i>
6	5.1	4	3	1		[M2A2]
5	4	3.1	2	1		<i>pivoting:</i>
4	3	2	1.1	1		[M1A2]
<hr/>						
	0.029577	-0.22535	-0.38028	0.15493		
	-0.22535	-0.42113	-0.8169	0.295775		<i>back subn:</i>
	-0.38028	-0.8169	-1.15352	0.43662	$x_2 = 0.103317$	[M1A2]
		-0.28889	-0.47	0.188889	$x_3 = -0.11419$	
		0.062963	-0.13333	0.037037		<i>solutions:</i>
			-0.23577	0.078205	$x_4 = -0.3317$	[A2]

product of pivots: -0.18390 magnitude of determinant: 0.18390

[M1A1]
[subtotal 14]

(ii)

$\alpha = 0.01$		$\beta = 0.01$				
7.01	6	5	4	1.01	$x_1 = 0.599796$	
6	5.01	4	3	1		
5	4	3.01	2	1		
4	3	2	1.01	1		
<hr/>						
	-0.12552	-0.2796	-0.42368	0.135521		
	-0.2796	-0.55633	-0.85307	0.279601		
	-0.42368	-0.85307	-1.27245	0.42368	$x_2 = -0.2999$	
		-0.02687	-0.0467	0.01	$x_3 = -0.1996$	
		0.006633	-0.01333	0		
			-0.02486	0.002469	$x_4 = -0.09929$	

product of pivots: -0.00198 magnitude of determinant: 0.001984

[M1A1]

$\alpha =$		$(B)\beta =$	
0.01	$(A)\beta = 0$	0.1	
x_1	0.302	0.600	
x_2	0.100	-0.300	
x_3	-0.101	-0.200	
x_4	-0.303	-0.099	

solutions:
[M1A1]
[M1A1]

Very large changes in the solution for small change in one coefficient. **[E1E1]**
 The determinant is very small in relation to the magnitude of the coefficients. **[E1E1]**
[subtotal 10]
[TOTAL 24]

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