

ADVANCED GCE

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

4754A

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

- 1 Express $\frac{x}{x^2-1} + \frac{2}{x+1}$ as a single fraction, simplifying your answer. [3]
- 2 Fig. 2 shows the curve $y = \sqrt{1+x^2}$.

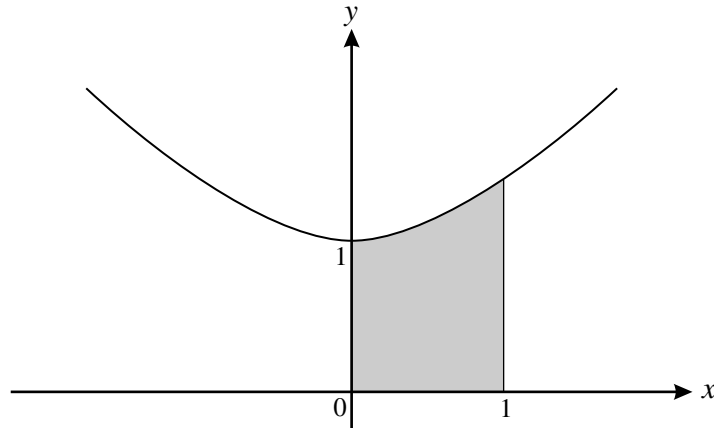


Fig. 2

- (i) The following table gives some values of x and y .

x	0	0.25	0.5	0.75	1
y	1	1.0308		1.25	1.4142

Find the missing value of y , giving your answer correct to 4 decimal places.

Hence show that, using the trapezium rule with four strips, the shaded area is approximately 1.151 square units. [3]

- (ii) Jenny uses a trapezium rule with 8 strips, and obtains a value of 1.158 square units. Explain why she must have made a mistake. [2]

- (iii) The shaded area is rotated through 360° about the x -axis. Find the exact volume of the solid of revolution formed. [3]

- 3 The parametric equations of a curve are

$$x = \cos 2\theta, \quad y = \sin \theta \cos \theta \quad \text{for } 0 \leq \theta < \pi.$$

Show that the cartesian equation of the curve is $x^2 + 4y^2 = 1$.

Sketch the curve. [5]

- 4 Find the first three terms in the binomial expansion of $\sqrt{4+x}$ in ascending powers of x .

State the set of values of x for which the expansion is valid. [5]

5 (i) Express $\frac{3}{(y-2)(y+1)}$ in partial fractions. [3]

(ii) Hence, given that x and y satisfy the differential equation

$$\frac{dy}{dx} = x^2(y-2)(y+1),$$

show that $\frac{y-2}{y+1} = Ae^{x^3}$, where A is a constant. [5]

6 Solve the equation $\tan(\theta + 45^\circ) = 1 - 2 \tan \theta$, for $0^\circ \leq \theta \leq 90^\circ$. [7]

Section B (36 marks)

7 A straight pipeline AB passes through a mountain. With respect to axes Oxyz, with Ox due East, Oy due North and Oz vertically upwards, A has coordinates $(-200, 100, 0)$ and B has coordinates $(100, 200, 100)$, where units are metres.

(i) Verify that $\overrightarrow{AB} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ and find the length of the pipeline. [3]

(ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. [6]

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is $x + 2y + 3z = 320$.

(iii) Find the coordinates of the point where the pipeline meets the layer of rock. [4]

(iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. [5]

[Question 8 is printed overleaf.]

- 8 Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin \theta, \quad y = 4 \cos \theta \quad \text{for } 0 \leq \theta \leq 2\pi.$$

This is shown in Fig. 8. B is a minimum point, and BC is vertical.

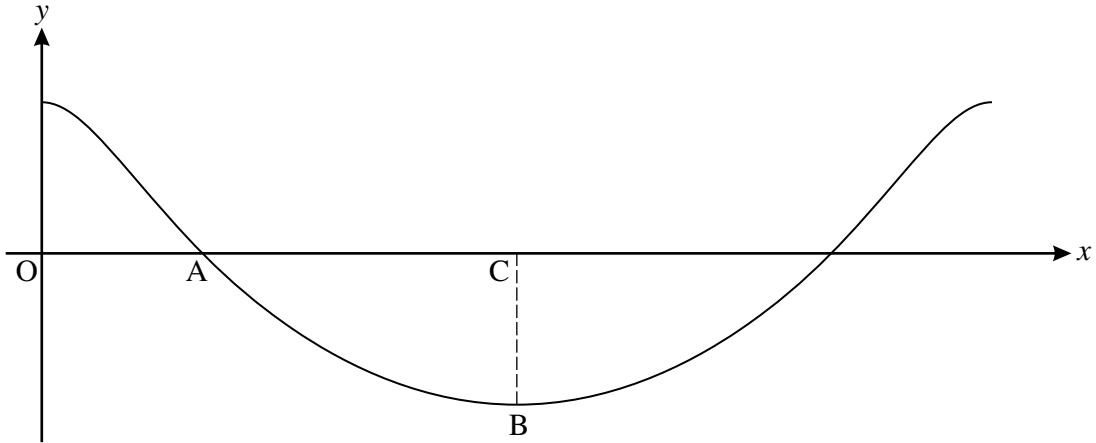


Fig. 8

- (i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$. [5]

- (ii) Find $\frac{dy}{dx}$ in terms of θ . Find the gradient of the track at A. [4]

- (iii) Show that, when the gradient of the track is 1, θ satisfies the equation

$$\cos \theta - 4 \sin \theta = 2. \quad [2]$$

- (iv) Express $\cos \theta - 4 \sin \theta$ in the form $R \cos(\theta + \alpha)$.

Hence solve the equation $\cos \theta - 4 \sin \theta = 2$ for $0 \leq \theta \leq 2\pi$. [7]

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.