OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4758/01**

Differential Equations

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 May 2009
Afternoon

Duration: 1 hour 30 minutes

4758010

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A car travels over a rough surface. The vertical motion of the front suspension is modelled by the differential equation

$$\frac{d^2y}{dt^2} + 25y = 20\cos 5t,$$

where y is the vertical displacement of the top of the suspension and t is time.

(i) Find the general solution. **[8]**

Initially $y = 1$ and $\frac{dy}{dt} = 0$.

(ii) Find the solution subject to these conditions. **[4]**

(iii) Sketch the solution curve for $t \geqslant 0$. **[4]**

A refined model of the motion of the suspension is given by

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 25y = 20\cos 5t.$$

(iv) Verify that $y = 2\sin 5t$ is a particular integral for this differential equation. Hence find the general solution. **[6]**

(v) Compare the behaviour of the suspension predicted by the two models. **[2]**

2 The differential equation

$$x\frac{dy}{dx} + 3y = \frac{\sin x}{x}$$

is to be solved for $x > 0$.

(i) Find the general solution for y in terms of x. **[9]**

As $x \to 0$, y tends to a finite limit.

(ii) Use the approximations $\sin x \approx x - \frac{1}{6}x^3$ and $\cos x \approx 1 - \frac{1}{2}x^2$ (both valid for small x) to find the value of the arbitrary constant and the limiting value of y as $x \to 0$. Hence state the particular solution. **[6]**

(iii) Show that, when $y = 0$, $\tan x = x$. **[2]**

An alternative method of investigating the behaviour of y for small x is to use the approximation $\sin x \approx x - \frac{1}{6}x^3$ in the differential equation, giving

$$x\frac{dy}{dx} + 3y = \frac{x - \frac{1}{6}x^3}{x}.$$

(iv) Solve this differential equation and, given that y tends to a finite limit as $x \to 0$, show that the value of the limit is the same as that found in part **(ii)**. **[7]**

3 **(a)** An electric circuit has an inductor and a resistor in series with an alternating power source. The circuit is switched on and after t seconds the current is I amps. The current satisfies the differential equation

$$2\frac{\mathrm{d}I}{\mathrm{d}t} + 4I = 3\cos 2t.$$

(i) Find the complementary function and a particular integral. Hence state the general solution for I in terms of t. **[8]**

Initially the current is zero.

(ii) Find the particular solution. **[2]**

(iii) Calculate the amplitude of the current for large values of t. Sketch the solution curve for large values of t. **[4]**

(b) The displacement, y, of a particle at time t satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2 - 2y + \mathrm{e}^{-t}.$$

You are **not** required to solve this differential equation.

The particle initially has displacement zero. The displacement has only one stationary value, which is where $y = \frac{9}{8}$. Also the velocity of the particle tends to zero as $t \to \infty$.

(i) Without solving the differential equation, use it to find

(A) the gradient of the solution curve when $t = 0$; **[2]**

(B) the value of t at the stationary value of y; **[3]**

(C) the limit of y as $t \to \infty$. **[2]**

(ii) Hence sketch the solution curve for $t \geqslant 0$, illustrating these results. **[3]**

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 7x + 6y + 2\mathrm{e}^{-3t}$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = -12x - 10y + 5\sin t$$

are to be solved for $t \geqslant 0$.

(i) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 14\mathrm{e}^{-3t} + 30\sin t.$$ **[5]**

(ii) Show that this differential equation has a particular integral of the form $x = a\mathrm{e}^{-3t} - 9\cos t + 3\sin t$, where a is a constant to be determined.

Hence find the general solution for x in terms of t. **[8]**

(iii) Find the corresponding general solution for y. **[4]**

(iv) Show that, for large values of t, $x = y$ when $\tan t \approx k$, where k is a constant to be determined. **[4]**

(v) Find the ratio of the amplitudes of y and x for large values of t. **[3]**