OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) — 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 1 June 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $4\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $4\cos\theta - \sin\theta = 3$, for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

2 Using partial fractions, find $\displaystyle\int \frac{x}{(x+1)(2x+1)}\,\mathrm{d}x$. **[7]**

3 A curve satisfies the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2y$, and passes through the point (1, 1). Find y in terms of x. **[4]**

4 The part of the curve $y = 4 - x^2$ that is above the x-axis is rotated about the y-axis. This is shown in Fig. 4.

Find the volume of revolution produced, giving your answer in terms of π. **[5]**

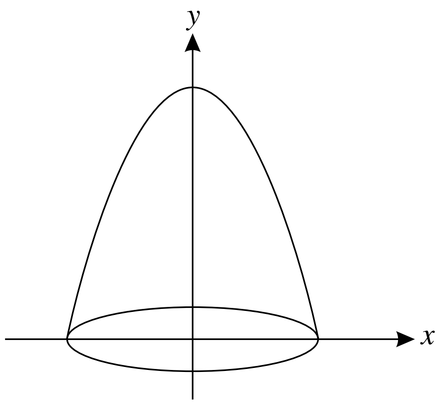

Fig. 4

5 A curve has parametric equations

$$x = at^3, \quad y = \frac{a}{1+t^2},$$

where a is a constant.

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2}{3t(1+t^2)^2}$.

Hence find the gradient of the curve at the point $(a, \frac{1}{2}a)$. **[7]**

6 Given that $\operatorname{cosec}^2\theta - \cot\theta = 3$, show that $\cot^2\theta - \cot\theta - 2 = 0$.

Hence solve the equation $\operatorname{cosec}^2\theta - \cot\theta = 3$ for $0° \leqslant \theta \leqslant 180°$. **[6]**

Section B (36 marks)

7 When a light ray passes from air to glass, it is deflected through an angle. The light ray ABC starts at point A (1, 2, 2), and enters a glass object at point B (0, 0, 2). The surface of the glass object is a plane with normal vector **n**. Fig. 7 shows a cross-section of the glass object in the plane of the light ray and **n**.

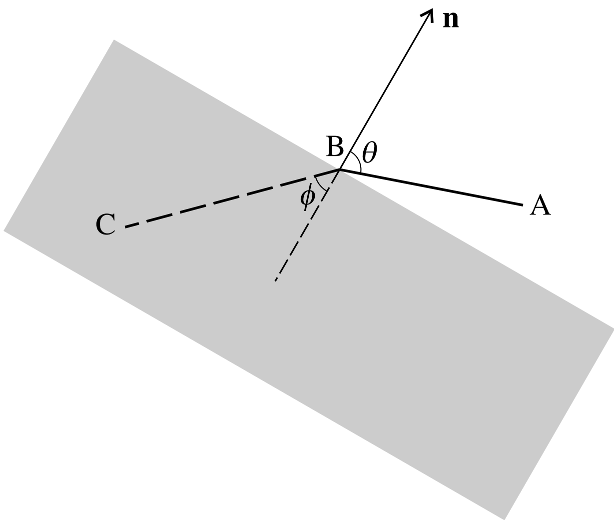

Fig. 7

(i) Find the vector $\overrightarrow{AB}$ and a vector equation of the line AB. **[2]**

The surface of the glass object is a plane with equation $x + z = 2$. AB makes an acute angle θ with the normal to this plane.

(ii) Write down the normal vector **n**, and hence calculate θ, giving your answer in degrees. **[5]**

The line BC has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$. This line makes an acute angle ϕ with the normal to the plane.

(iii) Show that $\phi = 45°$. **[3]**

(iv) Snell's Law states that $\sin\theta = k\sin\phi$, where k is a constant called the refractive index. Find k. **[2]**

The light ray leaves the glass object through a plane with equation $x + z = -1$. Units are centimetres.

(v) Find the point of intersection of the line BC with the plane $x + z = -1$. Hence find the distance the light ray travels through the glass object. **[5]**

[Question 8 is printed overleaf.]

8 Archimedes, about 2200 years ago, used regular polygons inside and outside circles to obtain approximations for π.

(i) Fig. 8.1 shows a regular 12-sided polygon inscribed in a circle of radius 1 unit, centre O. AB is one of the sides of the polygon. C is the midpoint of AB. Archimedes used the fact that the circumference of the circle is greater than the perimeter of this polygon.

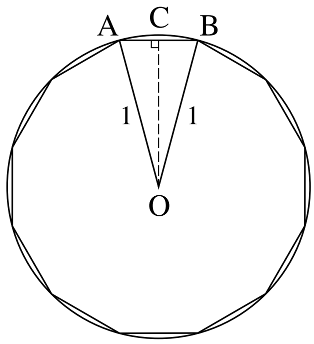

Fig. 8.1

(A) Show that AB $= 2\sin 15°$. **[2]**

(B) Use a double angle formula to express $\cos 30°$ in terms of $\sin 15°$. Using the exact value of $\cos 30°$, show that $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$. **[4]**

(C) Use this result to find an exact expression for the perimeter of the polygon.

Hence show that $\pi > 6\sqrt{2-\sqrt{3}}$. **[2]**

(ii) In Fig. 8.2, a regular 12-sided polygon lies outside the circle of radius 1 unit, which touches each side of the polygon. F is the midpoint of DE. Archimedes used the fact that the circumference of the circle is less than the perimeter of this polygon.

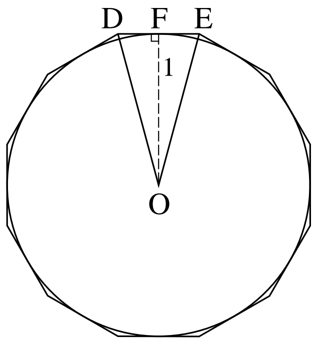

Fig. 8.2

(A) Show that DE $= 2\tan 15°$. **[2]**

(B) Let $t = \tan 15°$. Use a double angle formula to express $\tan 30°$ in terms of t.

Hence show that $t^2 + 2\sqrt{3}t - 1 = 0$. **[3]**

(C) Solve this equation, and hence show that $\pi < 12(2-\sqrt{3})$. **[4]**

(iii) Use the results in parts **(i)**(C) and **(ii)**(C) to establish upper and lower bounds for the value of π, giving your answers in decimal form. **[2]**