

ADVANCED GCE

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

4754A

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Monday 1 June 2009

Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

- 1 Express $4 \cos \theta - \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $4 \cos \theta - \sin \theta = 3$, for $0 \leq \theta \leq 2\pi$. [7]

- 2 Using partial fractions, find $\int \frac{x}{(x+1)(2x+1)} dx$. [7]

- 3 A curve satisfies the differential equation $\frac{dy}{dx} = 3x^2y$, and passes through the point $(1, 1)$. Find y in terms of x . [4]

- 4 The part of the curve $y = 4 - x^2$ that is above the x -axis is rotated about the y -axis. This is shown in Fig. 4.

Find the volume of revolution produced, giving your answer in terms of π . [5]

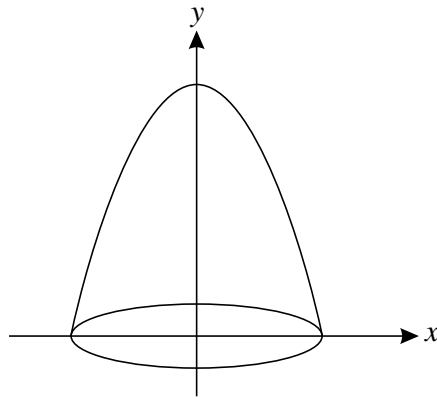


Fig. 4

- 5 A curve has parametric equations

$$x = at^3, \quad y = \frac{a}{1+t^2},$$

where a is a constant.

Show that $\frac{dy}{dx} = \frac{-2}{3t(1+t^2)^2}$.

Hence find the gradient of the curve at the point $(a, \frac{1}{2}a)$. [7]

- 6 Given that $\operatorname{cosec}^2 \theta - \cot \theta = 3$, show that $\cot^2 \theta - \cot \theta - 2 = 0$.

Hence solve the equation $\operatorname{cosec}^2 \theta - \cot \theta = 3$ for $0^\circ \leq \theta \leq 180^\circ$. [6]

Section B (36 marks)

- 7 When a light ray passes from air to glass, it is deflected through an angle. The light ray ABC starts at point A (1, 2, 2), and enters a glass object at point B (0, 0, 2). The surface of the glass object is a plane with normal vector \mathbf{n} . Fig. 7 shows a cross-section of the glass object in the plane of the light ray and \mathbf{n} .

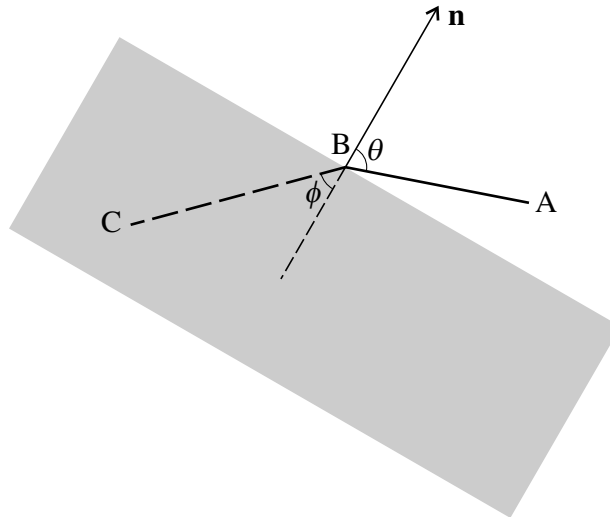


Fig. 7

- (i) Find the vector \overrightarrow{AB} and a vector equation of the line AB. [2]

The surface of the glass object is a plane with equation $x + z = 2$. AB makes an acute angle θ with the normal to this plane.

- (ii) Write down the normal vector \mathbf{n} , and hence calculate θ , giving your answer in degrees. [5]

The line BC has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$. This line makes an acute angle ϕ with the normal to the plane.

- (iii) Show that $\phi = 45^\circ$. [3]

- (iv) Snell's Law states that $\sin \theta = k \sin \phi$, where k is a constant called the refractive index. Find k . [2]

The light ray leaves the glass object through a plane with equation $x + z = -1$. Units are centimetres.

- (v) Find the point of intersection of the line BC with the plane $x + z = -1$. Hence find the distance the light ray travels through the glass object. [5]

[Question 8 is printed overleaf.]

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1PB.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

8 Archimedes, about 2200 years ago, used regular polygons inside and outside circles to obtain approximations for π .

- (i) Fig. 8.1 shows a regular 12-sided polygon inscribed in a circle of radius 1 unit, centre O. AB is one of the sides of the polygon. C is the midpoint of AB. Archimedes used the fact that the circumference of the circle is greater than the perimeter of this polygon.

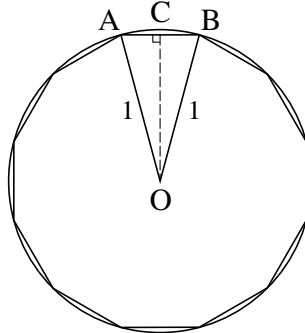


Fig. 8.1

- (A) Show that $AB = 2 \sin 15^\circ$. [2]
- (B) Use a double angle formula to express $\cos 30^\circ$ in terms of $\sin 15^\circ$. Using the exact value of $\cos 30^\circ$, show that $\sin 15^\circ = \frac{1}{2}\sqrt{2 - \sqrt{3}}$. [4]
- (C) Use this result to find an exact expression for the perimeter of the polygon.

Hence show that $\pi > 6\sqrt{2 - \sqrt{3}}$. [2]

- (ii) In Fig. 8.2, a regular 12-sided polygon lies outside the circle of radius 1 unit, which touches each side of the polygon. F is the midpoint of DE. Archimedes used the fact that the circumference of the circle is less than the perimeter of this polygon.

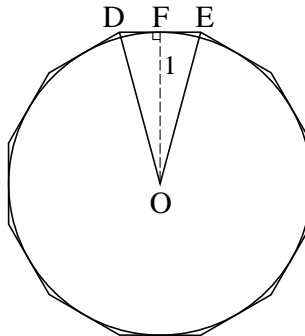


Fig. 8.2

- (A) Show that $DE = 2 \tan 15^\circ$. [2]
- (B) Let $t = \tan 15^\circ$. Use a double angle formula to express $\tan 30^\circ$ in terms of t .
Hence show that $t^2 + 2\sqrt{3}t - 1 = 0$. [3]
- (C) Solve this equation, and hence show that $\pi < 12(2 - \sqrt{3})$. [4]

- (iii) Use the results in parts (i)(C) and (ii)(C) to establish upper and lower bounds for the value of π , giving your answers in decimal form. [2]