OXFORD CAMBRIDGE AND RSA EXAMINATIONS

ADVANCED SUBSIDIARY GCE

4776/01

MATHEMATICS (MEI)

Numerical Methods

WEDNESDAY 20 MAY 2009: Afternoon
DURATION: 1 hour 30 minutes

SUITABLE FOR VISUALLY IMPAIRED CANDIDATES

Candidates answer on the Answer Booklet

OCR SUPPLIED MATERIALS:

8 page Answer Booklet
MEI Examination Formulae and Tables (MF2)
Graph paper

OTHER MATERIALS REQUIRED:

None

READ INSTRUCTIONS OVERLEAF

DC (SM) OV91011 14524

INSTRUCTIONS TO CANDIDATES

- **Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.**
- **Use black ink. Pencil may be used for graphs and diagrams only.**
- **Read each question carefully and make sure that you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

INFORMATION FOR CANDIDATES

- **The number of marks is given in brackets [ ] at the end of each question or part question.**
- **You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**
- **The total number of marks for this paper is 72.**

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SECTION A (36 marks)

1 **A quadratic function, f(x), is to be fitted to the data shown in the following table.**

x	0	0.4	1
y	1.6	2.4	1.8

(i) Use Lagrange's method to find f(x), simplifying the coefficients. [6]

(ii) Explain why Newton's forward difference interpolation formula would not have been useful for this purpose. [1]

2 **Show that the equation**

$$x^2 + \frac{1}{x} = 3$$

has a root in the interval (1, 2).

Use the Newton-Raphson method to find this root, giving it correct to 6 significant figures. [7]

3 The numbers X and Y shown below are known to be correct to 3 decimal places.

$X = 2.718 \qquad Y = 3.142$

(i) State the maximum possible errors in X, $X + Y$, $X - Y$, $10X + 20Y$. [4]

(ii) Find the maximum possible relative errors in X and Y. Hence state approximately the maximum possible relative errors in XY and $\frac{X}{Y}$. [4]

4 You are given that, for A and B in radians and $A \approx B$,

$$\frac{\sin A - \sin B}{A - B} \approx \cos\frac{A + B}{2}. \qquad (*)$$

A computer program calculates values of sine and cosine correct to 6 decimal places.

(i) In the case $A = 1.01$, $B = 1$, find the values of the left and right sides of (*) as calculated by this program. [2]

(ii) Identify two distinct reasons for the difference in these two values. [2]

(iii) Explain briefly why the right side of (*) is likely to be evaluated more accurately than the left as A gets progressively closer to B. [2]

5 Sketch, on the same axes, the graphs $y = x$ and $y = 1 - x^4$ for $0 \leqslant x \leqslant 1$. You should use the same scale on each axis.

Show numerically that the iteration $x_{r+1} = 1 - x_r^{\,4}$, starting with $x_0 = 0.6$, diverges.

Illustrate this divergence on your sketch, showing x_0, x_1, x_2, x_3. [8]

SECTION B (36 marks)

6 The integral $\int_0^{0.8} \sqrt{3+x-x^2}\,dx$ is to be evaluated numerically.

(i) Find, as efficiently as possible, the mid-point rule estimates and the trapezium rule estimates for $h = 0.8$ and 0.4. [6]

(ii) Use the values in part (i) to show that the first Simpson's rule estimate is 1.427 959 (correct to 6 decimal places), and to find a second Simpson's rule estimate. [3]

(iii) Given that, for $h = 0.2$, the mid-point rule estimate is 1.428 782 and the trapezium rule estimate is 1.426 497, calculate a third Simpson's rule estimate. [2]

(iv) Show that the differences between successive mid-point rule estimates reduce by a factor of about 0.25 as h is halved. Find the corresponding factor for the Simpson's rule estimates. Hence give the integral to the accuracy that appears justified. [7]

7 **(i)** **Use Newton's forward difference interpolation formula to find the quadratic function that passes through the following data points. [8]**

x	1	1.2	1.4
f(x)	0.6	–0.1	0.4

(ii) **Use the quadratic function to estimate $f'(1.2)$. Show that the central difference formula gives exactly the same estimate. What does this suggest about the central difference formula? [5]**

(iii) **Use the quadratic function to estimate $f'(1)$. Show that the forward difference does not give the same value. What does this show about the forward difference method? Which of these two estimates is likely to be more accurate? [5]**