OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4756

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 9 January 2009
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** **(i)** By considering the derivatives of $\cos x$, show that the Maclaurin expansion of $\cos x$ begins

$$1 - \tfrac{1}{2}x^2 + \tfrac{1}{24}x^4.$$ **[4]**

(ii) The Maclaurin expansion of $\sec x$ begins

$$1 + ax^2 + bx^4,$$

where a and b are constants. Explain why, for sufficiently small x,

$$\left(1 - \tfrac{1}{2}x^2 + \tfrac{1}{24}x^4\right)\left(1 + ax^2 + bx^4\right) \approx 1.$$

Hence find the values of a and b. **[5]**

(b) **(i)** Given that $y = \arctan\left(\dfrac{x}{a}\right)$, show that $\dfrac{dy}{dx} = \dfrac{a}{a^2 + x^2}$. **[4]**

(ii) Find the exact values of the following integrals.

(A) $\displaystyle\int_{-2}^{2} \frac{1}{4 + x^2}\, dx$ **[3]**

(B) $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{1 + 4x^2}\, dx$ **[3]**

2 **(i)** Write down the modulus and argument of the complex number $e^{j\pi/3}$. **[2]**

(ii) The triangle OAB in an Argand diagram is equilateral. O is the origin; A corresponds to the complex number $a = \sqrt{2}(1 + j)$; B corresponds to the complex number b.

Show A and the two possible positions for B in a sketch. Express a in the form $re^{j\theta}$. Find the two possibilities for b in the form $re^{j\theta}$. **[5]**

(iii) Given that $z_1 = \sqrt{2}e^{j\pi/3}$, show that $z_1^6 = 8$. Write down, in the form $re^{j\theta}$, the other five complex numbers z such that $z^6 = 8$. Sketch all six complex numbers in a new Argand diagram. **[6]**

Let $w = z_1 e^{-j\pi/12}$.

(iv) Find w in the form $x + jy$, and mark this complex number on your Argand diagram. **[3]**

(v) Find w^6, expressing your answer in as simple a form as possible. **[2]**

3 **(a)** A curve has polar equation $r = a\tan\theta$ for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$, where a is a positive constant.

(i) Sketch the curve. **[3]**

(ii) Find the area of the region between the curve and the line $\theta = \frac{1}{4}\pi$. Indicate this region on your sketch. **[5]**

(b) **(i)** Find the eigenvalues and corresponding eigenvectors for the matrix $\mathbf{M}$ where

$$\mathbf{M} = \begin{pmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{pmatrix}.$$ **[6]**

(ii) Give a matrix $\mathbf{Q}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M} = \mathbf{QDQ}^{-1}$. **[3]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(a)** **(i)** Prove, from definitions involving exponentials, that

$$\cosh^2 x - \sinh^2 x = 1.$$ **[2]**

(ii) Given that $\sinh x = \tan y$, where $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$, show that

(A) $\tanh x = \sin y$,

(B) $x = \ln(\tan y + \sec y)$. **[6]**

(b) **(i)** Given that $y = \operatorname{artanh} x$, find $\frac{dy}{dx}$ in terms of x.

Hence show that $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1-x^2}\,dx = 2\operatorname{artanh}\frac{1}{2}$. **[4]**

(ii) Express $\frac{1}{1-x^2}$ in partial fractions and hence find an expression for $\displaystyle\int \frac{1}{1-x^2}\,dx$ in terms of logarithms. **[4]**

(iii) Use the results in parts **(i)** and **(ii)** to show that $\operatorname{artanh}\frac{1}{2} = \frac{1}{2}\ln 3$. **[2]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 The limaçon of Pascal has polar equation $r = 1 + 2a\cos\theta$, where a is a constant.

(i) Use your calculator to sketch the curve when $a = 1$. (You need not distinguish between parts of the curve where r is positive and negative.) **[3]**

(ii) By using your calculator to investigate the shape of the curve for different values of a, positive and negative,

(A) state the set of values of a for which the curve has a loop within a loop,

(B) state, with a reason, the shape of the curve when $a = 0$,

(C) state what happens to the shape of the curve as $a \to \pm\infty$,

(D) name the feature of the curve that is evident when $a = 0.5$, and find another value of a for which the curve has this feature. **[7]**

(iii) Given that $a > 0$ and that a is such that the curve has a loop within a loop, write down an equation for the values of θ at which $r = 0$. Hence show that the angle at which the curve crosses itself is $2\arccos\left(\frac{1}{2a}\right)$.

Obtain the cartesian equations of the tangents at the point where the curve crosses itself. Explain briefly how these equations relate to the answer to part **(ii)**(A). **[8]**

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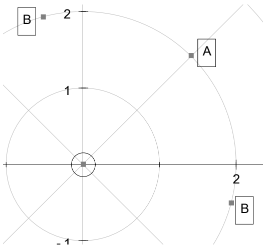
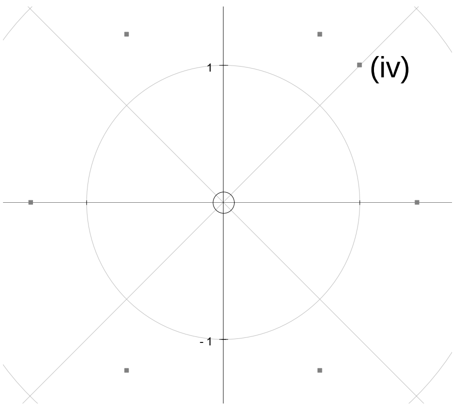
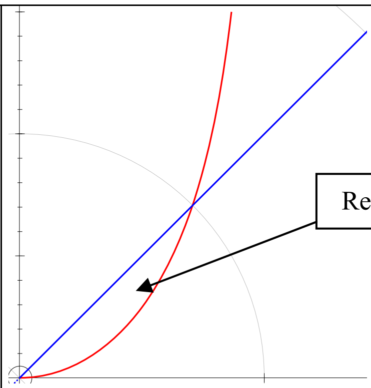
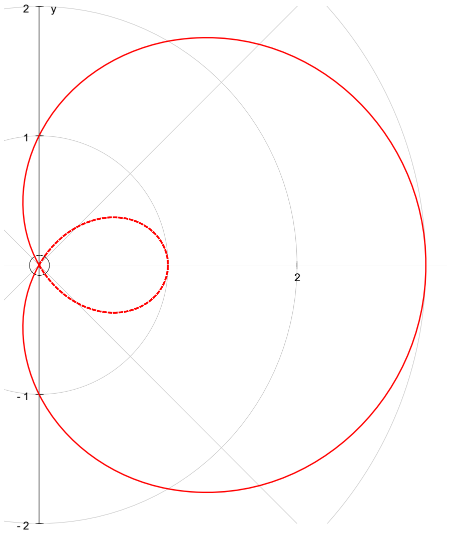
1 (a)(i)	$f(x) = \cos x$ $f(0) = 1$ $f'(x) = -\sin x$ $f'(0) = 0$ $f''(x) = -\cos x$ $f''(0) = -1$ $f'''(x) = \sin x$ $f'''(0) = 0$ $f''''(x) = \cos x$ $f''''(0) = 1$ $\Rightarrow \quad \cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 \ldots$	M1 A1 A1 A1 (ag) **4**	Derivatives cos, sin, cos, sin, cos Correct signs Correct values. Dep on previous A1 www
(ii)	$\cos x \times \sec x = 1$ $\Rightarrow \quad \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right)\left(1 + ax^2 + bx^4\right) = 1$ $\Rightarrow \quad 1 + \left(a - \frac{1}{2}\right)x^2 + \left(b - \frac{1}{2}a + \frac{1}{24}\right)x^4 = 1$ $\Rightarrow \quad a - \frac{1}{2} = 0, \; b - \frac{1}{2}a + \frac{1}{24} = 0$ $\Rightarrow \quad a = \frac{1}{2}$ $b = \frac{5}{24}$	E1 M1 A1 B1 B1 **5**	o.e. Multiply to obtain terms in x^2 and x^4 Terms correct in any form (may not be collected) Correctly obtained by any method: must not just be stated Correctly obtained by any method
(b)(i)	$y = \arctan \frac{x}{a}$ $\Rightarrow \quad x = a \tan y$ $\Rightarrow \quad \frac{dx}{dy} = a \sec^2 y$ $\Rightarrow \quad \frac{dx}{dy} = a(1 + \tan^2 y)$ $\Rightarrow \quad \frac{dy}{dx} = \frac{a}{a^2 + x^2}$	 M1 A1 A1 A1 (ag) **4**	 (a) tan y = and attempt to differentiate both sides Or $\sec^2 y \frac{dy}{dx} = \frac{1}{a}$ Use $\sec^2 y = 1 + \tan^2 y$ o.e. www SC1: Use derivative of arctan x and Chain Rule (properly shown)
(ii)(A)	$\int_{-2}^{2} \frac{1}{4 + x^2}\,dx = \left[\frac{1}{2}\arctan\frac{x}{2}\right]_{-2}^{2}$ $= \frac{\pi}{4}$	M1 A1 A1 **3**	arctan alone, or any tan substitution $\frac{1}{2}$ and $\frac{x}{2}$, or $\int \frac{1}{2}\,d\theta$ without limits Evaluated in terms of π
(ii)(B)	$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{1 + 4x^2}\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\frac{1}{4} + x^2}\,dx$ $= \left[2\arctan(2x)\right]_{-\frac{1}{2}}^{\frac{1}{2}}$ $= \pi$	M1 A1 A1 **3**	arctan alone, or any tan substitution 2 and $2x$, or $\int 2\,d\theta$ without limits Evaluated in terms of π **19**

2 (i)	Modulus = 1 Argument = $\frac{\pi}{3}$	B1 B1 **2**	Must be separate Accept 60°, 1.05^c
(ii)	 $a = 2e^{\frac{j\pi}{4}}$ $\arg b = \frac{\pi}{4} \pm \frac{\pi}{3}$ $b = 2e^{-\frac{j\pi}{12}}, 2e^{\frac{7j\pi}{12}}$	G2,1,0 B1 M1 A1ft **5**	G2: A in first quadrant, argument $\approx \frac{\pi}{4}$ B in second quadrant, same mod B′ in fourth quadrant, same mod Symmetry G1: 3 points and at least 2 of above, or B, B′ on axes, or BOB′ straight line, or BOB′ reflex Must be in required form (accept $r = 2$, $\theta = \pi/4$) Rotate by adding (or subtracting) $\pi/3$ to (or from) argument. Must be $\pi/3$ Both. Ft value of r for a. Must be in required form, but don't penalise twice
(iii)	$z_1^6 = \left(\sqrt{2}e^{\frac{j\pi}{3}}\right)^6 = \left(\sqrt{2}\right)^6 e^{2j\pi}$ $= 8$ Others are $re^{j\theta}$ where $r = \sqrt{2}$ and $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{2\pi}{3}, \pi$  	M1 A1 (ag) M1 A1 G1 G1 **6**	$\left(\sqrt{2}\right)^6 = 8$ or $\frac{\pi}{3} \times 6 = 2\pi$ seen www "Add" $\frac{\pi}{3}$ to argument more than once Correct constant r and five values of θ. Accept θ in [0, 2π] or in degrees 6 points on vertices of regular hexagon Correctly positioned (2 roots on real axis). Ignore scales SC1 if G0 and 5 points correctly plotted
(iv)	$w = z_1 e^{-\frac{j\pi}{12}} = \sqrt{2}e^{\frac{j\pi}{3}}e^{-\frac{j\pi}{12}} = \sqrt{2}e^{\frac{j\pi}{4}}$ $= \sqrt{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)$ $= 1 + j$	M1 A1 G1 **3**	$\arg w = \frac{\pi}{3} - \frac{\pi}{12}$ Or B2 Same modulus as z_1
(v)	$w^6 = \left(\sqrt{2}e^{\frac{j\pi}{4}}\right)^6 = 8e^{\frac{3j\pi}{2}}$ $= -8j$	M1 A1 **2**	Or $z_1^6 e^{-\frac{j\pi}{2}} = 8e^{-\frac{j\pi}{2}}$ cao. Evaluated **18**

3(a)(i)		G1 G1 G1 **3**	r increasing with θ Correct for $0 \le \theta \le \pi/3$ (ignore extra) Gradient less than 1 at O
(ii)	Area $= \int_0^{\frac{\pi}{4}} \frac{1}{2} r^2 d\theta = \frac{1}{2}a^2 \int_0^{\frac{\pi}{4}} \tan^2\theta \, d\theta$ $= \frac{1}{2}a^2 \int_0^{\frac{\pi}{4}} \sec^2\theta - 1 \, d\theta$ $= \frac{1}{2}a^2 \left[\tan\theta - \theta\right]_0^{\frac{\pi}{4}}$ $= \frac{1}{2}a^2 \left(1 - \frac{\pi}{4}\right)$	M1 M1 A1 A1 G1 **5**	Integral expression involving $\tan^2\theta$ Attempt to express $\tan^2\theta$ in terms of $\sec^2\theta$ $\tan\theta - \theta$ and limits 0, $\frac{\pi}{4}$ A0 if e.g. triangle − this answer Mark region on graph
(b)(i)	Characteristic equation is $(0.2 - \lambda)(0.7 - \lambda) - 0.24 = 0$ $\Rightarrow \lambda^2 - 0.9\lambda - 0.1 = 0$ $\Rightarrow \lambda = 1, -0.1$ When $\lambda = 1$, $\begin{pmatrix} -0.8 & 0.8 \\ 0.3 & -0.3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow -0.8x + 0.8y = 0,\ 0.3x - 0.3y = 0$ $\Rightarrow x - y = 0$, eigenvector is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ o.e. When $\lambda = -0.1$, $\begin{pmatrix} 0.3 & 0.8 \\ 0.3 & 0.8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow 0.3x + 0.8y = 0$ $\Rightarrow$ eigenvector is $\begin{pmatrix} 8 \\ -3 \end{pmatrix}$ o.e.	M1 A1 M1 A1 M1 A1 **6**	$(\mathbf{M} - \lambda\mathbf{I})\boldsymbol{x} = \boldsymbol{x}$ M0 below At least one equation relating x and y At least one equation relating x and y
(ii)	$\mathbf{Q} = \begin{pmatrix} 1 & 8 \\ 1 & -3 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & -0.1 \end{pmatrix}$	B1ft B1ft B1 **3**	B0 if **Q** is singular. Must label correctly If order consistent. Dep on B1B1 earned **17**

4 (a)(i)	$\cosh^2 x = \left[\frac{1}{2}\left(e^x + e^{-x}\right)\right]^2 = \frac{1}{4}\left(e^{2x} + 2 + e^{-2x}\right)$ $\sinh^2 x = \left[\frac{1}{2}\left(e^x - e^{-x}\right)\right]^2 = \frac{1}{4}\left(e^{2x} - 2 + e^{-2x}\right)$ $\cosh^2 x - \sinh^2 x = \frac{1}{4}(2+2) = 1$	M1 A1 (ag) **2**	Both expressions (M0 if no "middle" term) and subtraction www
	OR $\cosh x + \sinh x = e^x$ $\cosh x - \sinh x = e^{-x}$ $\cosh^2 x - \sinh^2 x = e^x \times e^{-x} = 1$ A1		Both, and multiplication Completion
(ii)(A)	$\cosh x = \sqrt{1+\sinh^2 x} = \sqrt{1+\tan^2 y}$ $= \sec y$ $\Rightarrow \tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{\tan y}{\sec y} = \sin y$	M1 A1 A1 (ag) **3**	Use of $\cosh^2 x = 1 + \sinh^2 x$ and $\sinh x = \tan y$ www
(ii)(B)	$\operatorname{arsinh} x = \ln(x + \sqrt{1+x^2})$ $\Rightarrow \operatorname{arsinh}(\tan y) = \ln(\tan y + \sqrt{1+\tan^2 y})$ $\Rightarrow x = \ln(\tan y + \sec y)$	M1 A1 A1 (ag) **3**	Attempt to use ln form of arsinh www
	OR $\sinh x = \tan y \Rightarrow \dfrac{e^x - e^{-x}}{2} = \tan y$ $\Rightarrow e^{2x} - 2e^x \tan y - 1 = 0$ M1 $\Rightarrow e^x = \tan y \pm \sqrt{\tan^2 y + 1}$ A1 $\Rightarrow x = \ln(\tan y + \sec y)$ A1		 Arrange as quadratic and solve for e^x o.e. www
(b)(i)	$y = \operatorname{artanh} x \Rightarrow x = \tanh y$ $\Rightarrow \dfrac{dx}{dy} = \operatorname{sech}^2 y$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\operatorname{sech}^2 y} = \dfrac{1}{1-\tanh^2 y} = \dfrac{1}{1-x^2}$ Integral $= \left[\operatorname{artanh} x\right]_{-\frac{1}{2}}^{\frac{1}{2}}$ $= 2 \operatorname{artanh}\dfrac{1}{2}$	M1 A1 M1 A1 (ag) **4**	tanh y = and attempt to differentiate Or $\operatorname{sech}^2 y \dfrac{dy}{dx} = 1$ Or B2 for $\dfrac{1}{1-x^2}$ www artanh or any tanh substitution www
(ii)	$\dfrac{1}{1-x^2} = \dfrac{1}{(1-x)(1+x)} = \dfrac{A}{1-x} + \dfrac{B}{1+x}$ $\Rightarrow 1 = A(1+x) + B(1-x)$ $\Rightarrow A = ½, B = ½$ $\Rightarrow \int \dfrac{1}{1-x^2}\,dx = \int \dfrac{\frac{1}{2}}{1-x} + \dfrac{\frac{1}{2}}{1+x}\,dx$ $= -\frac{1}{2}\ln\lvert 1-x\rvert + \frac{1}{2}\ln\lvert 1+x\rvert + c$ or $\frac{1}{2}\ln\left\lvert\frac{1+x}{1-x}\right\rvert + c$ o.e.	 M1 A1 M1 A1 **4**	 Correct form of partial fractions and attempt to evaluate constants Log integrals www. Condone omitted modulus signs and constant After 0 scored, SC1 for correct answer
(iii)	$\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1-x^2}\,dx = \left[-\tfrac{1}{2}\ln\lvert 1-x\rvert + \tfrac{1}{2}\ln\lvert 1+x\rvert\right]_{-\frac{1}{2}}^{\frac{1}{2}} = \ln 3$ $\Rightarrow 2\operatorname{artanh}\dfrac{1}{2} = \ln 3 \Rightarrow \operatorname{artanh}\dfrac{1}{2} = \dfrac{1}{2}\ln 3$	M1 A1 (ag) **2**	Substitution of ½ **and** –½ seen anywhere (or correct use of 0, ½) www **18**

5 (i)		G1 G1 G1 3	Symmetry in horizontal axis (3, 0) to (0, 0) (0, 0) to (0, 1)
(ii)(A) (ii)(B) (ii)(C) (ii)(D)	$a > 0.5$ $a < -0.5$ Circle: r is constant The two loops get closer together The shape becomes more nearly circular Cusp $a = -0.5$	B1 B1 B1 B1 B1 B1 B1 7	 Shape and reason
(iii)	$1 + 2a\cos\theta = 0 \Rightarrow \cos\theta = -\frac{1}{2a}$ If $a > 0.5$, $-1 < -\frac{1}{2a} < 0$ and there are two values of θ in $[0, 2\pi]$, $\pi - \arccos\left(\frac{1}{2a}\right)$ and $\pi + \arccos\left(\frac{1}{2a}\right)$ These differ by $2\arccos\left(\frac{1}{2a}\right)$ $\arccos\left(\frac{1}{2a}\right) = \arctan\sqrt{4a^2-1}$ Tangents are $y = x\sqrt{4a^2-1}$ and $y = -x\sqrt{4a^2-1}$ $\sqrt{4a^2-1}$ is real for $a > 0.5$ if $a > 0$	B1 M1 A1 (ag) M1 A1 A1 A1ft E1 8	Equation Relating arccos to arctan by triangle or $\tan^2\theta = \sec^2\theta - 1$ Negative of above 18

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General Comments

The candidates generally responded positively to this paper and very many good scripts were seen, with very few scoring fewer than 20 marks: there were also enough challenging questions for the most able. These candidates were able to display their skills by deploying elegant methods, especially in Question 4. This was certainly a challenging question and some candidates who chose it could not complete it, but this was sometimes because they had used very long methods in, for example, the integral in Question 3(a)(ii), or the Maclaurin series in Q1(a)(ii). The "standard" integrals in Q1(b) and the eigenvalues and eigenvectors in Q3(b) were done very well by the vast majority of candidates, and there were many good responses to the whole of Question 2.

As is always the case, some scripts bordered on the illegible, and there were a few candidates who appeared to delight in separating parts of questions and scattering them around the paper. Some candidates drew every sketch graph on a separate piece of graph paper. This is not necessary: sketches on the lined paper were expected, and quite acceptable.

Comments on Individual Questions

1) Maclaurin series, integration
The mean mark on this question was about 12 out of 19.

(a) Most candidates managed to produce the Maclaurin series for cos x successfully, although there were some sign errors. Some candidates worked from first principles, writing out the series as $a_0 + a_1x + a_2x^2 + ...$ and repeatedly differentiating.

The second part was less well done. Many candidates explained that cos x × sec x = 1, but many others concentrated on the fact that, if x were small, both series tended to 1, and 1 × 1 = 1. Then many candidates ignored the product, and resorted to repeated differentiation of sec x. This was often done successfully as far as the second derivative, but correct fourth derivatives, required for the coefficient of x^4, were extremely rare, and took a great deal of time. Despite the question asking candidates to work from part (i), some credit was given for this method. Those who did multiply the two expansions were often successful, but there were many algebraic slips, and candidates then sometimes produced $b = 0$ by equating the wrong coefficients to zero.

(b) The standard result in part (i) was often derived correctly, although a substantial number of candidates could not proceed beyond $\sec^2 y \frac{dy}{dx} = 1$. Some candidates used the derivative of arctan x from the formula book, and included the a via the Chain Rule: this attracted some credit. Others just stated that the result was the reverse of an integral expression to be found in the formula book: no credit was given for this. A few candidates did not treat a as a constant, and differentiated $\frac{x}{a}$ by using the quotient rule.

The integrals in (ii) were done extremely well, and most candidates gained full marks here. Errors included failing to divide by a, failing to write the expression in (B) in a form suitable for applying the standard result, and failing to evaluate arctan 1. One or two candidates worked in degrees. Those who tried explicit substitutions were usually very successful.

2) Complex numbers

This question was a good source of marks for most candidates: the mean mark was about 13 out of 18.

(i) This part caused little trouble to the vast majority of candidates, although some produced the right answers after three-quarters of a page of working, including extensive trigonometric manipulations.

(ii) There were very many fully correct solutions.
A substantial minority of candidates produced an incorrect diagram, having read the question as "ABB′ is equilateral" where B and B′ were the two possible positions of b. Others seemed to ignore the word "equilateral" and produced triangles in which B and B′ were on the axes. The modulus of a, and hence of b, was frequently given as $\sqrt{2}$.

(iii) Candidates did not always show that $z_1^6 = 8$, which was given, in sufficient detail: quite a few just produced $\sqrt{2}e^{\frac{j\pi}{3}}$ as one of the sixth roots of 8, without further comment. Because of the appearance of $\frac{\pi}{3}$ in the question, some candidates produced the sixth roots of $8e^{\frac{j\pi}{3}}$ rather than 8, displacing their regular hexagon by $\frac{\pi}{18}$. A few thought the roots did not all have the same modulus, and another group found the fifth roots instead of the sixth roots. But again, there were very many fully correct and efficiently-managed solutions.

(iv) This was frequently fully correct. However, a substantial number of candidates forgot to plot $1 + j$ anywhere, or plotted it with an obviously different modulus to their roots in (iii): it is just a rotation of z_1.

(v) Although many candidates arrived at $8e^{-\frac{j\pi}{2}}$ or equivalent, this was often left as the final answer: evaluation to the simpler $-8j$ was expected. When this was attempted, it was often -8. A few applied the Binomial Theorem to $(1 + j)^6$, usually with complete success.

3) Polar curves, eigenvalues and eigenvectors

Part (b) of this question was better done than part (a). The mean mark was about 12 out of 17.

(a) The curve was frequently correct although it was often tiny. Some candidates marked 0 and $\frac{\pi}{3}$ on their horizontal axis, which did not inspire confidence.
The area of the region caused a great deal of trouble, and many candidates spent far too much time here, usually on futile trigonometry. Most could write a correct integral expression for the area although the limits were sometimes wrong. Then comparatively few knew how to integrate $\tan^2\theta$: they often entered a "comfort

zone" by writing it as $\frac{\sin^2\theta}{\cos^2\theta}$, and then employed double angle formulae and the like, although some hit on the right method by writing $\sin^2\theta$ as $1 - \cos^2\theta$ and dividing out. Those who obtained the correct answer were sometimes not sure which region they were working with, as their sketches showed: they subtracted areas of triangles or other integrals, which lost them the final mark. Another group, having obtained $1-\frac{\pi}{4}$ in their answer, "simplified" it to $\frac{3\pi}{4}$, although this was condoned if the correct answer could be seen and there was no other error.

(b) The methods here were well known and were often carried out completely correctly. Some candidates, having obtained the correct eigenvalues and reached $0.3x + 0.8y = 0$ to find one of their eigenvectors, wrote this eigenvector down as $\begin{pmatrix}3\\8\end{pmatrix}$ or $\begin{pmatrix}3\\-8\end{pmatrix}$ rather than the correct $\begin{pmatrix}8\\-3\end{pmatrix}$. Fortunately for those candidates, the other eigenvector was $\begin{pmatrix}1\\1\end{pmatrix}$! A number of candidates decided that the decimals were not to their liking, and worked with 10**M** rather than **M**, which produced the correct eigenvectors, but not the correct eigenvalues. The characteristic equation, $\lambda^2 - 0.9\lambda - 0.1 = 0$, caused a few problems: those who used the quadratic formula to solve it sometimes made slips, and produced horrible surd eigenvalues with which they then had to work to obtain the eigenvectors. In part (ii), most knew exactly what they had to do: some went on to recalculate **M** by finding the product $\mathbf{QDQ}^{-1}$, which wasted a lot of time.

4) Hyperbolic functions

This was the choice of the vast majority of the candidates in Section B, although most found it a stiff test: the mean mark was just under 9 out of 18.

(a) Most candidates approached part (i) confidently and produced a convincing proof. Slips included losing the ¼ and mismanaging the 2s, which occasionally did not appear at all.
The responses to part (ii) were very variable, with (B) better done than (A), although many candidates left this part out altogether. Others asserted that sin and sinh, cos and cosh, and tan and tanh were interchangeable "by Osborn's rule" and produced "proofs" of one or two lines, which attracted no credit. But some very elegant and resourceful work was seen, especially in part (B). A few very able candidates, not wishing to be defeated, proved (B) first, and then used the exponential form of tanh *x*, the log laws and a great deal of paper to produce the result in (A).

(b) Most candidates scored full marks in (i): a proof that artanh is an odd function was not expected, but some candidates produced one anyway. The partial fractions in (ii) caused more problems: it was worrying, at this level, to see $\frac{1}{1-x^2}=\frac{1}{1}-\frac{1}{x^2}$ asserted so frequently, and very many candidates attempted to proceed without factorising the denominator. Some linked parts (i) and (ii) at this stage, looked up the logarithmic form of artanh *x* in the formula book, and gave this as the answer: this attracted some credit. Completely correct answers to part (iii) were rare: candidates often fudged the result, obtaining 2 artanh ½ = ½ ln 3 by only using one limit, and then ignoring the inconvenient 2. A few able candidates used the oddness of artanh *x* again here, using limits 0 and ½ in a very elegant solution.

5) Investigations of Curves

About ten candidates attempted this question, and the mean mark was about 6 out of 18. Virtually all candidates produced a correct curve in (i), and then answered several of the parts of (ii) correctly, but none scored more than 1 mark in (iii).