OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

4755

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 **(i)** Find the roots of the quadratic equation $z^2 - 6z + 10 = 0$ in the form $a + b\mathrm{j}$. **[2]**

(ii) Express these roots in modulus-argument form. **[3]**

2 Find the values of A, B and C in the identity $2x^2 - 13x + 25 \equiv A(x-3)^2 - B(x-2) + C$. **[4]**

3 Fig. 3 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

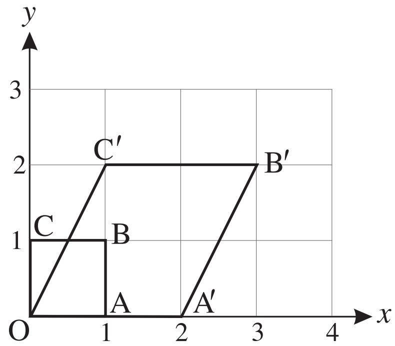


Fig. 3

(i) Write down the matrix **P** representing this transformation. **[1]**

(ii) The parallelogram OA′B′C′ is transformed by the matrix $\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$. Find the coordinates of the vertices of its image, OA″B″C″, following this transformation. **[2]**

(iii) Describe fully the transformation represented by **QP**. **[2]**

4 Write down the equation of the locus represented in the Argand diagram shown in Fig. 4. **[3]**

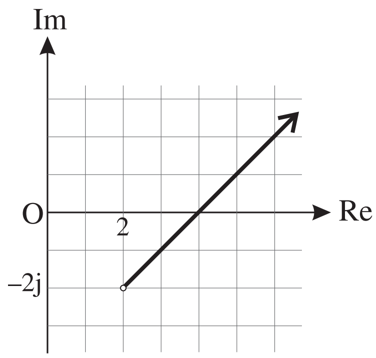


Fig. 4

5 The cubic equation $x^3 - 5x^2 + px + q = 0$ has roots α, -3α and $\alpha + 3$. Find the values of α, p and q. **[6]**

6 Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \tfrac{1}{4}n(n+1)(n+3)(n-2).$$ **[6]**

7 Prove by induction that $12 + 36 + 108 + \ldots + 4 \times 3^n = 6(3^n - 1)$ for all positive integers n. **[7]**

Section B (36 marks)

8 Fig. 8 shows part of the graph of $y = \dfrac{x^2 - 3}{(x-4)(x+2)}$. Two sections of the graph have been omitted.

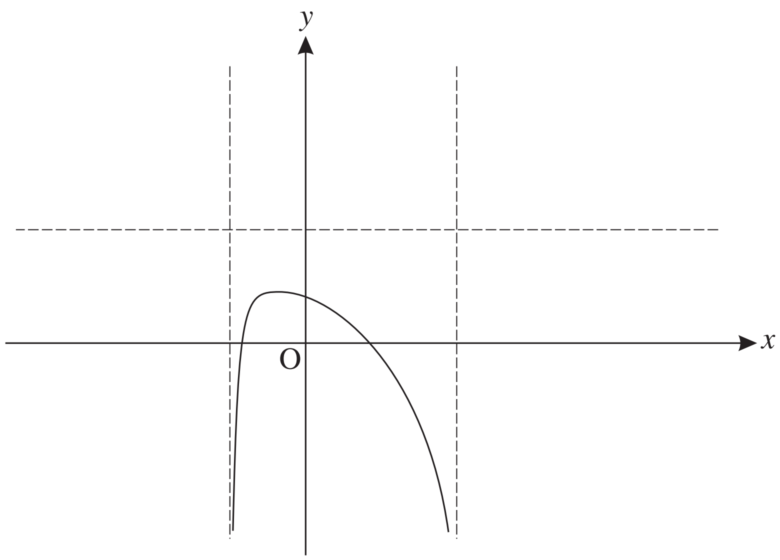


Fig. 8

(i) Write down the coordinates of the points where the curve crosses the axes. **[2]**

(ii) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

(iii) Copy Fig. 8 and draw in the two missing sections. **[4]**

(iv) Solve the inequality $\dfrac{x^2 - 3}{(x-4)(x+2)} \leqslant 0$. **[3]**

[Questions 9 and 10 are printed overleaf.]

9 Two complex numbers, α and β, are given by $\alpha = 1 + \mathrm{j}$ and $\beta = 2 - \mathrm{j}$.

(i) Express $\alpha + \beta$, $\alpha\alpha^*$ and $\dfrac{\alpha + \beta}{\alpha}$ in the form $a + b\mathrm{j}$. **[5]**

(ii) Find a quadratic equation with roots α and α^*. **[2]**

(iii) α and β are roots of a quartic equation with real coefficients. Write down the two other roots and find this quartic equation in the form $z^4 + Az^3 + Bz^2 + Cz + D = 0$. **[5]**

10 You are given that $\mathbf{A} = \begin{pmatrix} 3 & 4 & -1 \\ 1 & -1 & k \\ -2 & 7 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 5+k \\ -5 & 29 & 7 \end{pmatrix}$ and that $\mathbf{AB}$ is of the form

$$\mathbf{AB} = \begin{pmatrix} 42 & \alpha & 4k-8 \\ 10-5k & -16+29k & -12+6k \\ 0 & 0 & \beta \end{pmatrix}.$$

(i) Show that $\alpha = 0$ and $\beta = 28 + 7k$. **[3]**

(ii) Find $\mathbf{AB}$ when $k = 2$. **[2]**

(iii) For the case when $k = 2$ write down the matrix $\mathbf{A}^{-1}$. **[3]**

(iv) Use the result from part **(iii)** to solve the following simultaneous equations. **[4]**

$$\begin{aligned} 3x + 4y - z &= 1 \\ x - y + 2z &= -9 \\ -2x + 7y - 3z &= 26 \end{aligned}$$

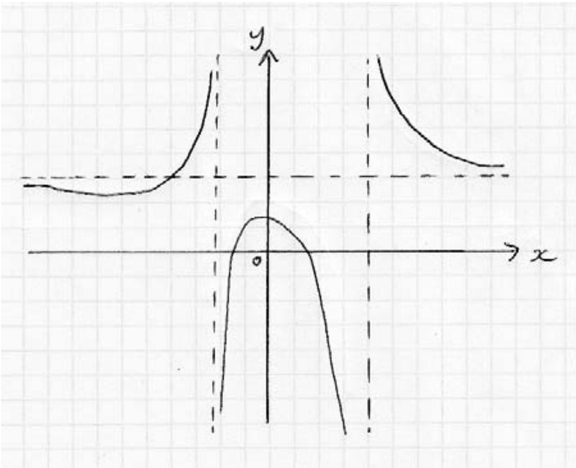
4755 (FP1) Further Concepts for Advanced Mathematics

Section A

1(i)	$z = \dfrac{6 \pm \sqrt{36-40}}{2}$ $\Rightarrow z = 3 + \mathrm{j}$ or $z = 3 - \mathrm{j}$	M1 A1 **[2]**	Use of quadratic formula/completing the square For both roots
1(ii)	$\lvert 3+\mathrm{j} \rvert = \sqrt{10} = 3.16$ (3s.f.) $\arg(3+\mathrm{j}) = \arctan\left(\dfrac{1}{3}\right) = 0.322$ (3s.f.) $\Rightarrow$ roots are $\sqrt{10}(\cos 0.322 + \mathrm{j}\sin 0.322)$ and $\sqrt{10}(\cos 0.322 - \mathrm{j}\sin 0.322)$ or √10(cos(-0.322)+jsin(-0.322))	M1 M1 A1 **[3]**	Method for modulus Method for argument (both methods must be seen following A0) One mark for both roots in modulus-argument form – accept surd and decimal equivalents and (r, θ) form. Allow $\pm 18.4^{\circ}$ for θ.
2	$2x^2 - 13x + 25 = A(x-3)^2 - B(x-2) + C$ $\Rightarrow 2x^2 - 13x + 25$ $= Ax^2 - (6A+B)x + (2B+C) + 9A$ A = 2 B = 1 C = 5	B1 M1 A1 A1 **[4]**	For A=2 Attempt to compare coefficients of x^1 or x^0, or other valid method. For B and C, cao.
3(i)	$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$	B1 **[1]**	
3(ii)	$\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 2 & 3 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 6 & 6 \end{pmatrix}$ $\Rightarrow$ A″=(4, 0), B″=(4, 6), C″=(0, 6)	M1 A1 **[2]**	Applying matrix to column vectors, with a result. All correct
3(iii)	Stretch factor 4 in x-direction. Stretch factor 6 in y-direction	B1 B1 **[2]**	Both factor and direction for each mark. SC1 for “enlargement”, not stretch.

4	$\arg\left(z-\left(2-2\mathrm{j}\right)\right)=\frac{\pi}{4}$	B1 B1 B1 **[3]**	Equation involving arg(complex variable). Argument (complex expression) = $\frac{\pi}{4}$ All correct
5	Sum of roots = $\alpha+\left(-3\alpha\right)+\alpha+3=3-\alpha=5$ $\Rightarrow \alpha=-2$ Product of roots $=\ -2\times6\times1=-12$ Product of roots in pairs $=-2\times6+\left(-2\right)\times1+6\times1=-8$ $\Rightarrow\ p=-8$ and $q=12$ Alternative solution $(x-\alpha)(x+3\alpha)(x-\alpha-3)$ $=x^3+(\alpha-3)x^2+(-5\alpha^2-6\alpha)x+3\alpha^3+9\alpha^2$ => α = -2, p = -8 and q = 12	M1 A1 M1 M1 A1 A1 **[6]** M1 M1A1 M1 A1A1 **[6]**	Use of sum of roots Attempt to use product of roots Attempt to use sum of products of roots in pairs One mark for each, ft if α incorrect Attempt to multiply factors Matching coefficient of x^2 ,cao. Matching other coefficients One mark for each, ft incorrect α.
6	$\sum_{r=1}^{n}\left[r\left(r^2-3\right)\right]=\sum_{r=1}^{n}r^3-3\sum_{r=1}^{n}r$ $=\frac{1}{4}n^2\left(n+1\right)^2-\frac{3}{2}n\left(n+1\right)$ $=\frac{1}{4}n\left(n+1\right)\left(n\left(n+1\right)-6\right)$ $=\frac{1}{4}n\left(n+1\right)\left(n^2+n-6\right)=\frac{1}{4}n\left(n+1\right)\left(n+3\right)\left(n-2\right)$	M1 M1 A2 M1 A1 **[6]**	Separate into separate sums. (may be implied) Substitution of standard result in terms of n. For two correct terms (indivisible) Attempt to factorise with $n(n+1)$. Correctly factorised to give fully factorised form

7	When $n = 1$, $6\left(3^n - 1\right) = 12$, so true for $n = 1$ Assume true for $n = k$ $12 + 36 + 108 + ... + (4 \times 3^k) = 6\left(3^k - 1\right)$ $\Rightarrow 12 + 36 + 108 + ... + (4 \times 3^{k+1})$ $= 6\left(3^k - 1\right) + (4 \times 3^{k+1})$ $= 6\left[\left(3^k - 1\right) + \frac{2}{3} \times 3^{k+1}\right]$ $= 6\left[3^k - 1 + 2 \times 3^k\right]$ $= 6\left(3^{k+1} - 1\right)$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$, it is true for $n = k + 1$. Since it is true for $n = 1$, it is true for $n = 1, 2, 3$... and so true for all positive integers.	B1 E1 M1 M1 A1 E1 E1 **[7]**	 Assume true for k Add correct next term to both sides Attempt to factorise with a factor 6 c.a.o. with correct simplification Dependent on A1 and first E1 Dependent on B1 and second E1
			Section A Total: 36

Section B			
8(i)	$(\sqrt{3},\ 0),(-\sqrt{3},\ 0)\ \left(0,\ \frac{3}{8}\right)$	B1 B1 [2]	Intercepts with x axis (both) Intercept with y axis SC1 if seen on graph or if $x = \pm\sqrt{3}$, $y = 3/8$ seen without $y = 0$, $x = 0$ specified.
8(ii)	$x=4,\ x=-2,\ y=1$	B3 [3]	Minus 1 for each error. Accept equations written on the graph.
8(iii)		B1 B1B1 B1 [4]	Correct approaches to vertical asymptotes, LH and RH branches LH and RH branches approaching horizontal asymptote On LH branch 0<y<1 as x→-∞.
8(iv)	$-2 < x \le -\sqrt{3}$ and $4 > x \ge \sqrt{3}$	B1 B2 [3]	LH interval and RH interval correct (Award this mark even if errors in inclusive/exclusive inequality signs) All inequality signs correct, minus 1 each error

9(i)	$\alpha+\beta=3$ $\alpha\alpha^*=(1+\mathrm{j})(1-\mathrm{j})=2$ $\frac{\alpha+\beta}{\alpha}=\frac{3}{1+\mathrm{j}}=\frac{3(1-\mathrm{j})}{(1+\mathrm{j})(1-\mathrm{j})}=\frac{3}{2}-\frac{3}{2}\mathrm{j}$	B1 M1 A1 M1 A1 **[5]**	 Attempt to multiply $(1+\mathrm{j})(1-\mathrm{j})$ Multiply top and bottom by $1-\mathrm{j}$
9(ii)	$(z-(1+\mathrm{j}))(z-(1-\mathrm{j}))$ $=z^2-2z+2$	M1 A1 **[2]**	Or alternative valid methods (Condone no "=0" here)
9(iii)	$1-\mathrm{j}$ and $2+\mathrm{j}$ Either $(z-(2-\mathrm{j}))(z-(2+\mathrm{j}))$ $=z^2-4z+5$ $(z^2-2z+2)(z^2-4z+5)$ $=z^4-6z^3+15z^2-18z+10$ So equation is $z^4-6z^3+15z^2-18z+10=0$ Or alternative solution Use of $\sum\alpha=6$, $\sum\alpha\beta=15$, $\sum\alpha\beta\gamma=18$ and $\alpha\beta\gamma\delta=10$ to obtain the above equation.	B1 M1 M1 A2 **[5]** M1 A3 **[5]**	For both For attempt to obtain an equation using the product of linear factors involving complex conjugates Using the correct four factors All correct, -1 each error (including omission of "=0") to min of 0 Use of relationships between roots and coefficients. All correct, -1 each error, to min of 0

10(i)	$\alpha = 3\times -5 + 4\times 11 + -1\times 29 = 0$ $\beta = -2\times -7 + 7\times(5+k) + -3\times 7 = 28 + 7k$	B1 M1 A1 **[3]**	 Attempt at row 3 x column 3
10(ii)	$\mathbf{AB} = \begin{pmatrix} 42 & 0 & 0 \\ 0 & 42 & 0 \\ 0 & 0 & 42 \end{pmatrix}$	B2 **[2]**	Minus 1 each error to min of 0
10(iii)	$\mathbf{A}^{-1} = \frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}$	M1 B1 A1 **[3]**	Use of **B** $\frac{1}{42}$ Correct inverse, allow decimals to 3 sf
10(iv)	$\frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}\begin{pmatrix} 1 \\ -9 \\ 26 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $= \frac{1}{42}\begin{pmatrix} -126 \\ 84 \\ -84 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -2 \end{pmatrix}$ $x = -3,\ y = 2,\ z = -2$	M1 A3 **[4]**	Attempt to pre-multiply by $\mathbf{A}^{-1}$ SC B2 for Gaussian elimination with 3 correct solutions, -1 each error to min of 0 Minus 1 each error
			Section B Total: 36
			Total: 72

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments

Most candidates were well prepared for the examination and were able to score highly. Although Section A was found more difficult by some candidates the questions in Section B were accessible and very well answered by many. Section A revealed some algebraic errors. It did not appear that candidates suffered from lack of time to complete the paper except in a few cases.

Comments on Individual Questions

1) (i) This was usually attempted by using the formula, although substituting a+jb and equating real and imaginary parts to zero was also used by several candidates. It was extraordinary how many went from $\sqrt{(-4)}/2$ to 2j instead of j.

(ii) The argument of the root 3-j was not always given in the range $-\pi$ to $+\pi$. A few candidates forgot to include j in the expressions using r and θ and some believed that having found the modulus and the argument, they had finished. Use of the abbreviation cjsθ is not to be recommended at this stage.

2) Most candidates compared coefficients. Some used substitution. A common error (and a bad one at this stage) was to expand $-B(x-2)$ to $-Bx - 2B$, and another to use 9 instead of 9A to find C.

3) This question proved surprisingly difficult for quite a few candidates.

(i) This part was less well answered than part (ii); many did not know how to obtain P and several candidates covered a lot of paper in trying to find it.

(ii) Several candidates pre-multiplied the matrix by their column vectors, somehow. Which point was which was often left for the examiner to determine.

(iii) Enlargement was extensively used instead of stretch to describe the transformation, although the combination of one with the other, provided scale factors were appropriate, was accepted. Sometimes there was seen the curious description of a stretch “about a point”.

4) There were many candidates who did not know that this required a statement about the argument of a complex variable. They earned no marks. The argument of a modulus does not exist, but credit was earned for a variable appearing.

5) There was a variety of approaches to this question. Finding α using the sum of the roots and thence obtaining the actual roots was successfully employed by many candidates, whether they then proceeded to obtain the factors and multiply out, or continued to use the $\Sigma\alpha\beta$ and $\alpha\beta\gamma$ relationships. Using the latter often led to q=-12, confusing signs. Another popular method was to form the factors in terms of α and multiply out. Yet another method used was the substitution of roots in terms of α and the attempted solution of the resulting equations. In the latter case in particular the resulting algebra proved to be too difficult.

6) This was probably the most successfully answered question in Section A, with many candidates scoring full marks. A very few failed to remember the correct result for Σr. A few did not convincingly demonstrate the steps leading to the required factorisation, which was given in the question.

7) There were very many excellently reasoned answers to this question. Those that kept to the "standard" form of words and were able correctly to form the algebraic argument scored full marks. But there were still many candidates who cannot quite show their full understanding of the method, by lack of attention to details, perhaps trying to go too fast. "Assume n=k" is not the same as "Assume the result is true when n=k". "It is true for n=k and n=k+1" is not the same as "if it is true for n=k then it is true for n=k+1". There were also the careless omissions of summations to spoil thestatements presented. As far as the needful algebra was concerned, a few candidates were under the impression that 4x3n=12n.

8) There were many answers that scored full marks, even among the less confident candidates.

(i) Some candidates were casual about writing down co-ordinates. Some failed to realise that there were three intersections with the two axes to account for.

(ii) Most candidates gave the asymptotes correctly.

(iii) Only two branches had to be sketched here but many candidates consider that a sketch can be extremely "rough". The structure of the graph should be shown, with clear approaches to the asymptotes, not a vague indication. Several sent the left-hand branch to infinity on the wrong side of y=1. Where it was shown that the asymptote had been crossed there should have been some indication of a minimum point. Less commonly the left-hand branch was shown with $y\to-\infty$ as $x\to-2^{-0}$. In just a few cases the right-hand branch was shown crossing y=1.

(iv) Where candidates had drawn a good graph the inequalities were usually correct. Some were confused between < and > and gave the wrong intervals. Some did not give the right inclusion at $\pm\sqrt{3}$. Those that attempted an algebraic solution were unsuccessful.

9) This was also a well answered question by the majority of the candidates, part (i) in particular.

(ii) This also was well done, failure to give an equation not being penalised here.

(iiii) Following the hint in (ii) most candidates successfully multiplied two appropriate quadratic expressions. Those who tried using the relationships between the roots sometimes mixed up the signs of the coefficients in the terms. In some instances not enough terms were included in $\Sigma\alpha\beta$. There was a penalty here for failure to include "=0" in presenting the result.

10) All parts were well done by most candidates.

(i) A few candidates lost marks for failing to show sufficient working, the answer was given.
(ii) Usually correct, but in (iii) some candidates thought that $\mathbf{A}^{-1}$ was (1/42) **A**.
(iii)
(iv) Some did not show the correct sequence of pre-multiplication of the column vector by the inverse matrix. Failure to use (iii) was also a reason for loss of marks here.