ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4771

Decision Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- Printed Answer Book (inserted)
- * MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 19 January 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the printed Answer Book.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- This document consists of **8** pages. Any blank pages are indicated.

SP (SC/CGW) T67882/5

Answer all questions in the printed answer book provided.

Section A (24 marks)

1 Alfred, Ben, Charles and David meet, and some handshaking takes place.

- Alfred shakes hands with David.
- Ben shakes hands with Charles and David.
- Charles shakes hands with Ben and David.

(i) Complete the bipartite graph in your answer book showing A (Alfred), B (Ben), C (Charles) and D (David), and the number of people each shakes hands with. **[4]**

(ii) Explain why, whatever handshaking takes place, the resulting bipartite graph cannot contain both an arc terminating at 0 and another arc terminating at 3. **[2]**

(iii) Explain why, whatever number of people meet, and whatever handshaking takes place, there must always be two people who shake hands with the same number of people. **[2]**

2 The following algorithm computes the number of comparisons made when Prim's algorithm is applied to a complete network on n vertices ($n > 2$).

```
Step 1   Input the value of $n$
Step 2   Let $i = 1$
Step 3   Let $j = n - 2$
Step 4   Let $k = j$
Step 5   Let $i = i + 1$
Step 6   Let $j = j - 1$
Step 7   Let $k = k + (i \times j) + (i - 1)$
Step 8   If $j > 0$ then go to Step 5
Step 9   Print $k$
Step 10  Stop
```

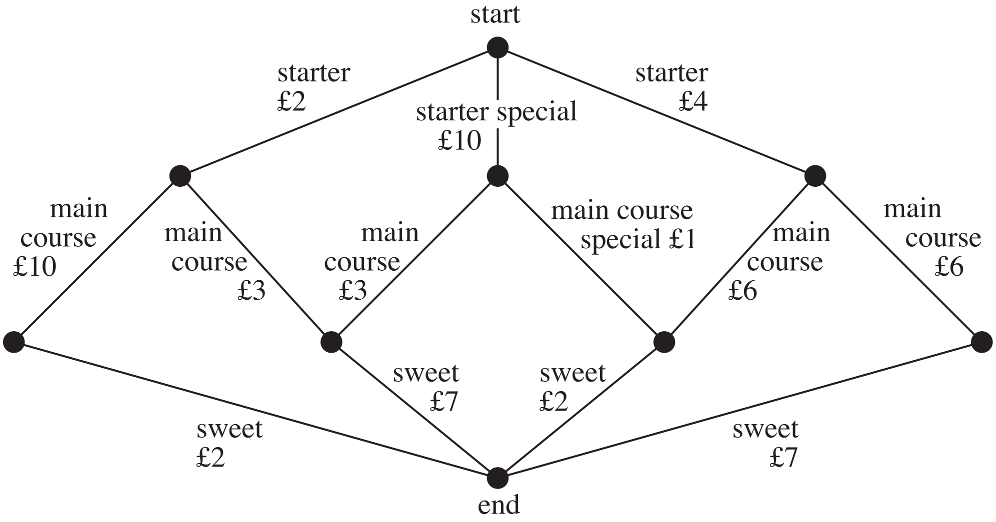
(i) Apply the algorithm when $n = 5$, showing the intermediate values of i, j and k. **[5]**

The function $f(n) = \frac{1}{6}n^3 - \frac{7}{6}n + 1$ gives the same output as the algorithm.

(ii) Showing your working, check that f(5) is the same value as your answer to part **(i)**. **[2]**

(iii) What does the structure of f(n) tell you about Prim's algorithm? **[1]**

3 Whilst waiting for her meal to be served, Alice tries to construct a network to represent the meals offered in the restaurant.



(i) Use Dijkstra's algorithm to find the cheapest route through the undirected network from "start" to "end". Give the cost and describe the route. Use the lettering given on the network in your answer book. **[6]**

(ii) Criticise the model and suggest how it might be improved. **[2]**

Section B (48 marks)

4 A ski-lift gondola can carry 4 people. The weight restriction sign in the gondola says "4 people – 325 kg".

The table models the distribution of weights of people using the gondola.

	Men	Women	Children
Weight (kg)	90	80	40
Probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

(i) Give an efficient rule for using 2-digit random numbers to simulate the weight of a person entering the gondola. **[5]**

(ii) Give a reason for using 2-digit rather than 1-digit random numbers in these circumstances. **[1]**

(iii) Using the random numbers given in your answer book, simulate the weights of four people entering the gondola, and hence give its simulated load. **[3]**

(iv) Using the random numbers given in your answer book, repeat your simulation 9 further times. Hence estimate the probability of the load of a fully-laden gondola exceeding 325 kg. **[6]**

(v) What in reality might affect the pattern of loading of a gondola which is not modelled by your simulation? **[1]**

5 The tasks involved in turning around an "AirGB" aircraft for its return flight are listed in the table. The table gives the durations of the tasks and their immediate predecessors.

Activity		Duration (mins)	Immediate Predecessors
A	Refuel	30	–
B	Clean cabin	25	–
C	Pre-flight technical check	15	A
D	Load luggage	20	–
E	Load passengers	25	A, B
F	Safety demonstration	5	E
G	Obtain air traffic clearance	10	C
H	Taxi to runway	5	G, D

(i) Draw an activity on arc network for these activities. **[4]**

(ii) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. **[6]**

Because of delays on the outbound flight the aircraft has to be turned around within 50 minutes, so as not to lose its air traffic slot for the return journey. There are four tasks on which time can be saved. These, together with associated costs, are listed below.

Task	A	B	D	E
New time (mins)	20	20	15	15
Extra cost	250	50	50	100

(iii) List the activities which need to be speeded up in order to turn the aircraft around within 50 minutes at minimum extra cost. Give the extra cost and the new set of critical activities. **[6]**

6 A company is planning its production of "MPowder" for the next three months.

- Over the next 3 months 20 tonnes must be produced.
- Production quantities must not be decreasing. The amount produced in month 2 cannot be less than the amount produced in month 1, and the amount produced in month 3 cannot be less than the amount produced in month 2.
- No more than 12 tonnes can be produced in total in months 1 and 2.
- Production costs are £2000 per tonne in month 1, £2200 per tonne in month 2 and £2500 per tonne in month 3.

The company planner starts to formulate an LP to find a production plan which minimises the cost of production:

Minimise $2000x_1 + 2200x_2 + 2500x_3$
subject to $x_1 \geq 0 \;\; x_2 \geq 0 \;\; x_3 \geq 0$
$x_1 + x_2 + x_3 = 20$
$x_1 \leq x_2$
• • •

(i) Explain what the variables x_1, x_2 and x_3 represent, and write down two more constraints to complete the formulation. **[4]**

(ii) Explain how the LP can be reformulated to:

Maximise $500x_1 + 300x_2$
subject to $x_1 \geq 0 \;\; x_2 \geq 0$
$x_1 \leq x_2$
$x_1 + 2x_2 \leq 20$
$x_1 + x_2 \leq 12$ **[3]**

(iii) Use a graphical approach to solve the LP in part **(ii)**. Interpret your solution in terms of the company's production plan, and give the minimum cost. **[9]**

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RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) **4771**

Decision Mathematics 1

PRINTED ANSWER BOOK

Monday 19 January 2009
Afternoon

Duration: 1 hour 30 minutes

Candidate Forename		Candidate Surname	

Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

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- Use black ink. Pencil may be used for graphs and diagrams only.
- Write your answers in the spaces provided on the answer book. If extra paper is required use a 4 page answer booklet making sure that you label your work clearly. Attach any extra answer booklets to this Printed Answer Book.

INFORMATION FOR CANDIDATES

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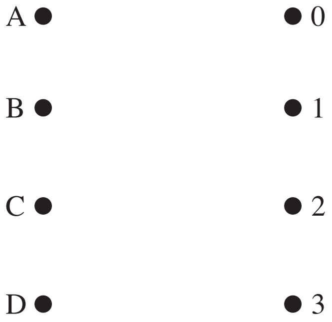
FOR EXAMINER'S USE	
1	
2	
3	
4	
5	
6	
TOTAL	

Turn over

1 **(i)**



(ii)

(iii)

2 **(i)**

n	*i*	*j*	*k*

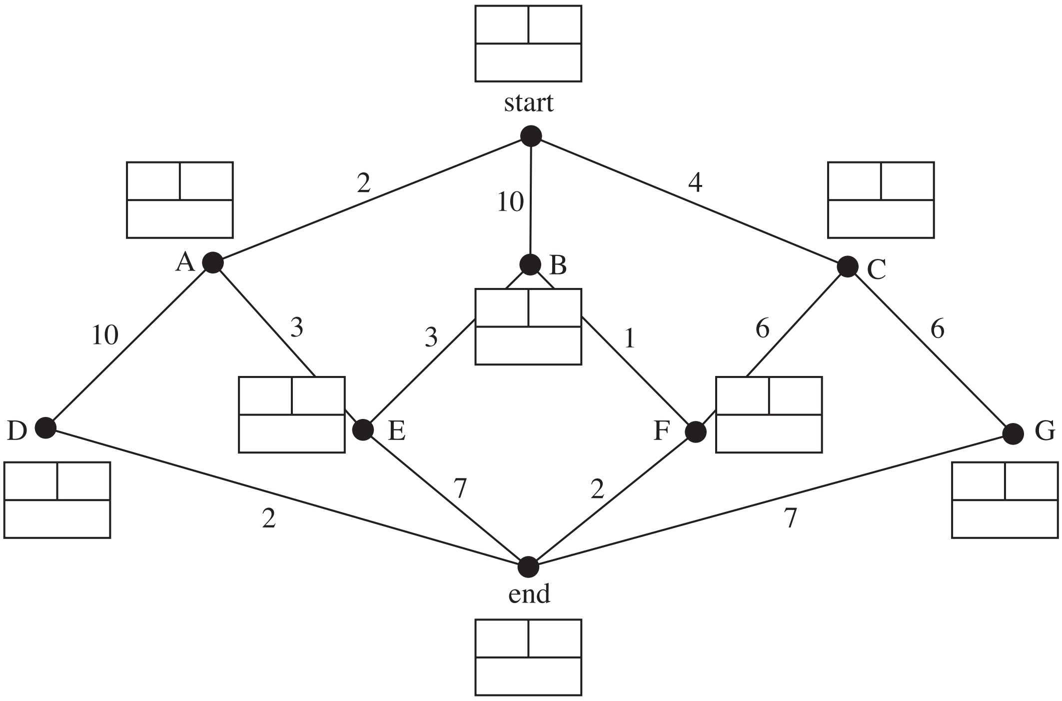
$k =$ ________

(ii)

(iii)

Turn over

3 **(i)**



Cheapest cost and route:

(ii)

There is a spare copy of the network on page 8

4 **(i)**

(ii)

(iii) Random numbers

Run 1	32	98	00	17	56	76

(iv) Random numbers

Run 2	37	97	53	24	63	06
Run 3	69	04	78	58	67	82
Run 4	07	81	67	05	35	15
Run 5	83	34	24	36	99	29
Run 6	96	16	34	14	02	99
Run 7	71	61	94	23	13	85
Run 8	60	77	73	23	92	82
Run 9	43	59	21	27	67	26
Run 10	08	90	89	53	47	89

(v)

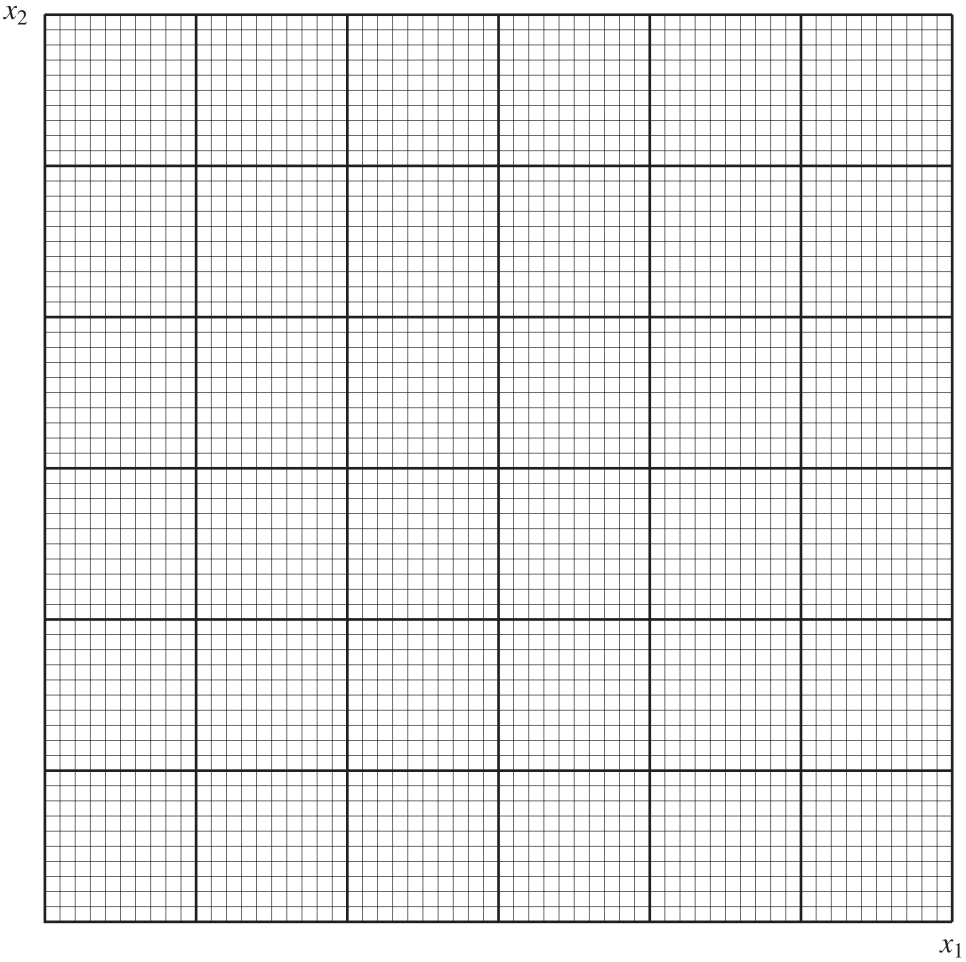
5 (i)&(ii)

(iii)

6 (i)

(ii)

(iii)

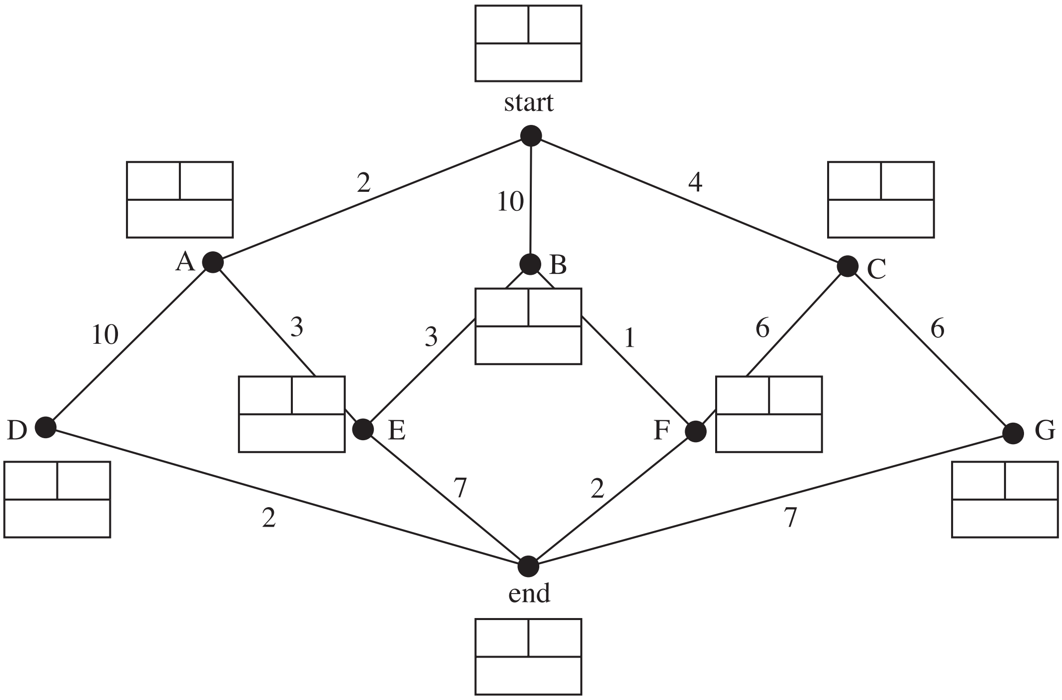


Optimal production plan:

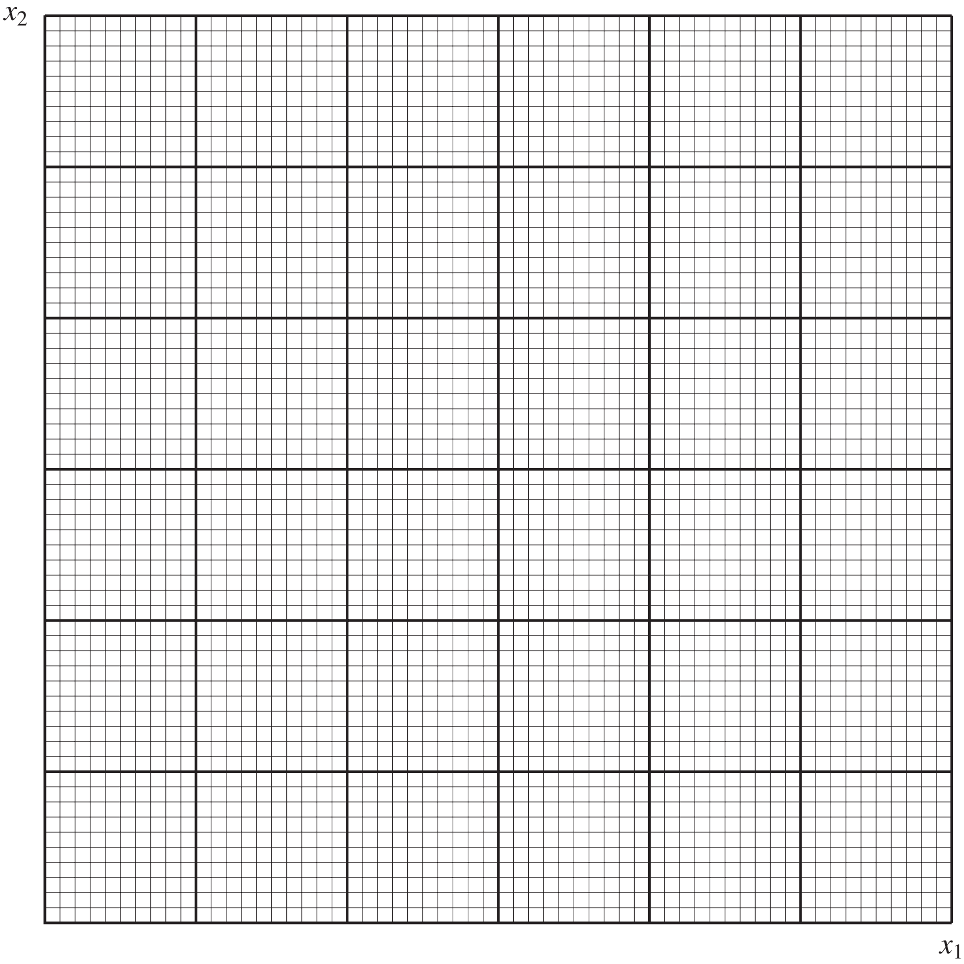
Cost:

There is spare graph paper on page 8

3 **(i)** Spare copy of network



6 Spare graph paper



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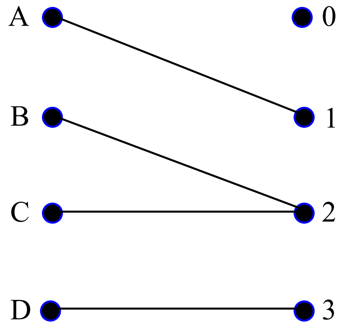
1.

(i)		M1 bipartite A1 one arc from each letter A1 David A1 rest
(ii)	Can't both have someone shaking hands with everyone and someone not shaking hands at all.	B1 0 ⇒ ~3 B1 3 ⇒ ~0
(iii)	n arcs leaving By (ii) only n–1 destinations	B1 B1

2.

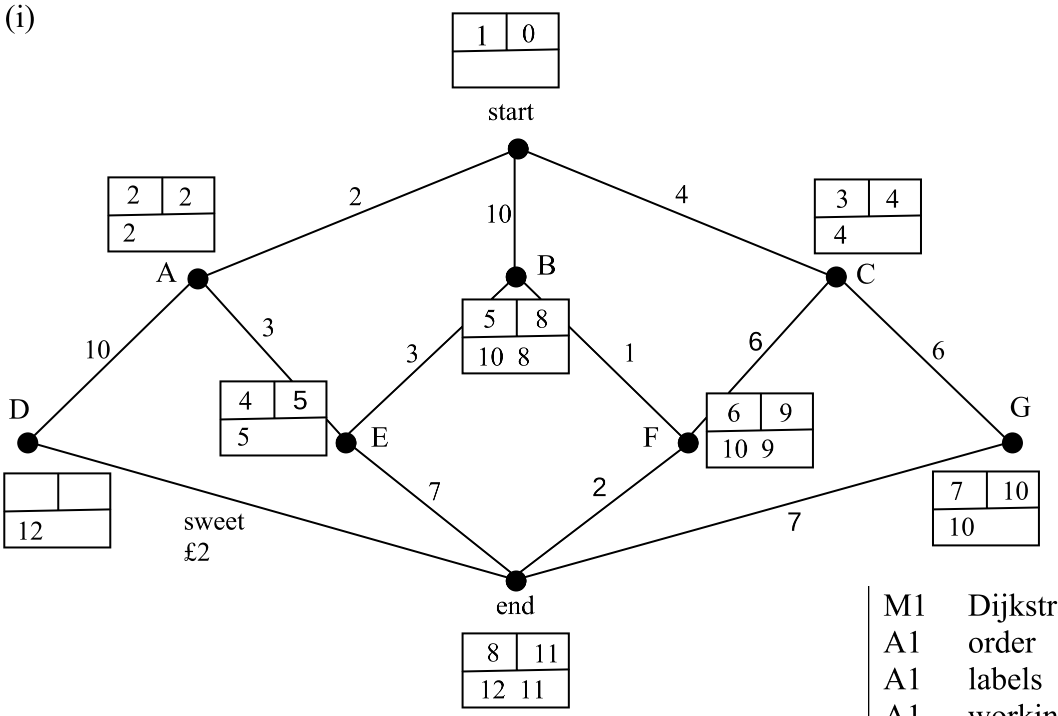
(i)

n	i	j	k	
5	1	3	3	B1
	2	2	8	B1
	3	1	13	B1
	4	0	16	B1

	k = 16	B1
(ii)	f(5) = 125/6 – 35/6 + 1 = 90/6 + 1 = 16	M1 substituting
	(Need to see 125 or 20.8$\dot{3}$ for A1)	A1
(iii)	cubic complexity	B1

3.

(i)



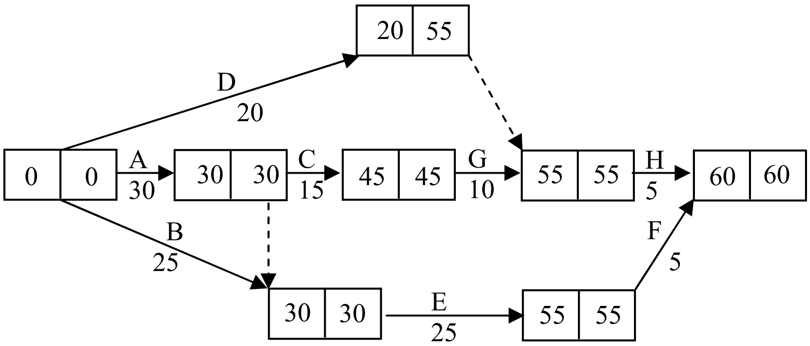
	M1 Dijkstra A1 order A1 labels A1 working values
Cheapest: £11	B1 £11
[start (£2 starter)] → A (£3 main) → E (£3 main) → B (£1 main) → F (£2 sweet) → [end]	B1 route
(ii) repeated mains !	B1
directed network	B1

4.

(i)	e.g. 00–47→90 48–79→80 80–95→40 96, 97,98, 99 ignore	M1 some rejected A3 correct proportions (–1 each error) A1 efficient
(ii)	smaller proportion rejected	B1
(iii)	e.g. 90, 90, 90, 80 350	M1 A1 A1√
(iv)	e.g. 90, 80, 90, 80 340 80, 90, 80, 80 330 90, 40, 80, 90 300 40, 90, 90, 90 310 90, 90, 90, 90 360 80, 80, 40, 90 290 80, 80, 80, 90 330 90, 80, 90, 90 350 90, 40, 40, 80 250	M1 A3 (–1 each error) √
	prob (load>325) = 0.6	M1 A1
(v)	e.g. family groups	B1

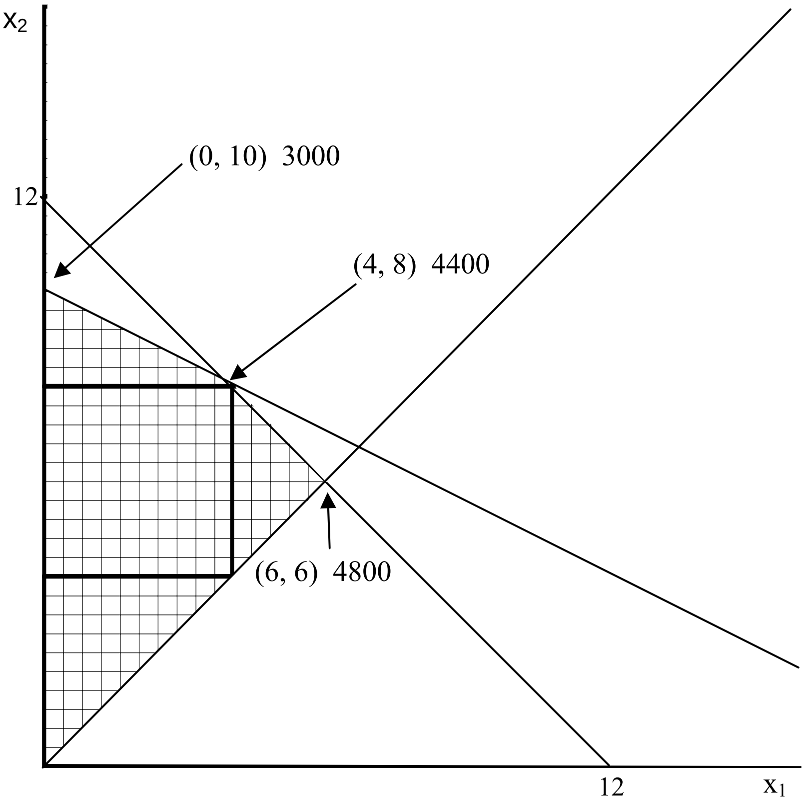
5.

(i)&(ii) e.g.



(i)&(ii)	time – 60 minutes critical – A; C; E; F; G; H	M1 sca (activity on arc) A1 single start & end A1 dummy A1 rest M1 forward pass A1 M1 backward pass A1 B1 √ B1 cao
(iii)	A and B at £300 A; C; G; H B; E; F	B1 2 out of A, B, E B1 A B1 B B1 300 from A and B B1 B1

6.

(i)	x_i represents the number of tonnes produced in month i $x_2 \le x_3$ $x_1 + x_2 \le 12$	M1 A1 B1 B1	quantities tonnes
(ii)	Substitute $x_3 = 20 - x_1 - x_2$ $x_2 \le x_3 \rightarrow x_1 + 2x_2 \le 20$ Min $2000x_1 + 2200x_2 + 2500x_3 \rightarrow$ Max $500x_1 + 300x_2$	M1 A1 A1	
(iii)		M1 A3 A1 M1 A1	sca lines shading >1 evaluated point or profit line (6, 6) or 4800
	Production plan: 6 tonnes in month 1 6 tonnes in month 2 8 tonnes in month 3	M1	√ all 3
	Cost = £45200	A1	cao

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General Comments

This unit was marked on-screen for the first time. The process was facilitated by the answer book, which imposes a structure on candidates' work. Most candidates regard this as a support rather than as an imposition.

Comments on Individual Questions

1) Graphs
Most candidates scored well on the bipartite graph, but some failed to obtain full marks. Many of those "some" were unable to sort out the number of hands shaken by David.

In part (ii) there were many who scored on one of the deductions (e.g. If one person shakes 3 hands then nobody shakes zero), but not the converse
(e.g. If one person shakes no hands then nobody shakes 3).
Few were able to make any useful progress with part (iii). Many comments were seen relating to the total of the node orders being even. That is proved by the handshaking lemma, but it has nothing to do with this question.

2) Algorithms
Most were able to run the algorithm.
Many gave incomplete justifications in part (ii), often failing to show the computation after substituting 5 for x. Decimal approximations were often seen, but exact computations were required. (Computations involving recurring decimals were accepted.)
A few candidates were unable to identify the cubic complexity in part (iii).

3) Networks
Most candidates were able to apply Dijkstra's algorithm, but there were too many who simply calculated assorted shortest paths. Those latter candidates did not gain credit.
Many failed to answer the question in part (ii), often criticising the menu rather than the mathematical modelling.

4) Simulation
This question was answered well.

The simulations asked both for simulated individual weights and for the simulated gondola loading. Some candidates failed to provide one or the other.

The phrasing in part (v) ("the pattern of loading") required the candidates to identify the issue of the grouping of users, rather than the variability of weights.

5) CPA
Candidates found the activity network easy to construct. They also did well on the forward and backward passes. These looked easy, but were intended to deliver extra difficulty through multiple critical paths – hence the care with which reference is always made to "critical activities". Either the candidates did not notice this, or they just took it in their stride.

The "crashing" in part (iii) was much more difficult, but a sizeable minority of candidates managed it well.

6) LP

As is traditional, many candidates were unable to identify the variables in part (i), “months” being the favourite answer.

Only a very few candidates were able to score anything in part (ii). However, the structure of the question allowed candidates to continue into part (iii), and many did so – with varying degrees of success.

A sizeable minority failed to draw the y = x line. Another sizeable minority drew the y = x line at 45 degrees when the scales on their axes were not equal. Others only drew the two regions with negatively sloped boundaries.

Many gave no indication at all of how they obtained the optimum from their graph, and so gained no credit for obtaining (6, 6). Mirroring the difficulties with part (ii), few were able to interpret their solution to part (ii) in terms of the original three variables.