ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4751

Introduction to Advanced Mathematics (C1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Insert for Question 13 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 9 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Question **13**.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

No calculator can be used for this paper

Section A (36 marks)

1 State the value of each of the following.

(i) 2^{-3} **[1]**

(ii) 9^0 **[1]**

2 Find the equation of the line passing through $(-1, -9)$ and $(3, 11)$. Give your answer in the form $y = mx + c$. **[3]**

3 Solve the inequality $7 - x < 5x - 2$. **[3]**

4 You are given that $f(x) = x^4 + ax - 6$ and that $x - 2$ is a factor of $f(x)$.

Find the value of a. **[3]**

5 **(i)** Find the coefficient of x^3 in the expansion of $(x^2 - 3)(x^3 + 7x + 1)$. **[2]**

(ii) Find the coefficient of x^2 in the binomial expansion of $(1 + 2x)^7$. **[3]**

6 Solve the equation $\dfrac{3x + 1}{2x} = 4$. **[3]**

7 **(i)** Express $125\sqrt{5}$ in the form 5^k. **[2]**

(ii) Simplify $\left(4a^3b^5\right)^2$. **[2]**

8 Find the range of values of k for which the equation $2x^2 + kx + 18 = 0$ does not have real roots. **[4]**

9 Rearrange $y + 5 = x(y + 2)$ to make y the subject of the formula. **[4]**

10 **(i)** Express $\sqrt{75} + \sqrt{48}$ in the form $a\sqrt{3}$. **[2]**

(ii) Express $\dfrac{14}{3 - \sqrt{2}}$ in the form $b + c\sqrt{d}$. **[3]**

Section B (36 marks)

11

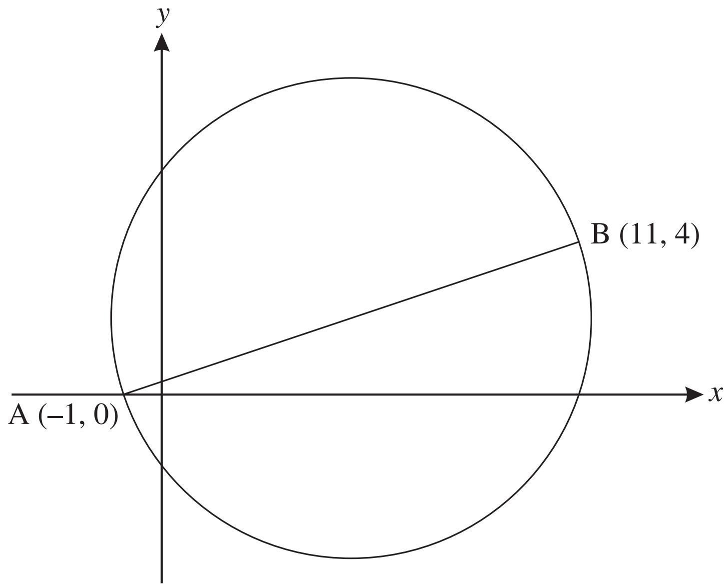


Fig. 11

Fig. 11 shows the points A and B, which have coordinates $(-1, 0)$ and $(11, 4)$ respectively.

(i) Show that the equation of the circle with AB as diameter may be written as

$$(x-5)^2 + (y-2)^2 = 40.$$ **[4]**

(ii) Find the coordinates of the points of intersection of this circle with the y-axis. Give your answer in the form $a \pm \sqrt{b}$. **[4]**

(iii) Find the equation of the tangent to the circle at B. Hence find the coordinates of the points of intersection of this tangent with the axes. **[6]**

12 **(i)** Find algebraically the coordinates of the points of intersection of the curve $y = 3x^2 + 6x + 10$ and the line $y = 2 - 4x$. **[5]**

(ii) Write $3x^2 + 6x + 10$ in the form $a(x+b)^2 + c$. **[4]**

(iii) Hence or otherwise, show that the graph of $y = 3x^2 + 6x + 10$ is always above the x-axis. **[2]**

[Question 13 is printed overleaf.]

13 Answer part (i) of this question on the insert provided.

The insert shows the graph of $y = \frac{1}{x}$.

(i) On the insert, on the same axes, plot the graph of $y = x^2 - 5x + 5$ for $0 \leqslant x \leqslant 5$. **[4]**

(ii) Show algebraically that the x-coordinates of the points of intersection of the curves $y = \frac{1}{x}$ and $y = x^2 - 5x + 5$ satisfy the equation $x^3 - 5x^2 + 5x - 1 = 0$. **[2]**

(iii) Given that $x = 1$ at one of the points of intersection of the curves, factorise $x^3 - 5x^2 + 5x - 1$ into a linear and a quadratic factor.

Show that only one of the three roots of $x^3 - 5x^2 + 5x - 1 = 0$ is rational. **[5]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) **4751**

Introduction to Advanced Mathematics (C1)
INSERT for Question 13

Friday 9 January 2009
Morning

Duration: 1 hour 30 minutes

Candidate Forename		Candidate Surname				
Centre Number						Candidate Number

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- This insert should be used to answer Question **13** part **(i)**.
- Write your answers to Question **13** part **(i)** in the spaces provided in this insert, and **attach it to your Answer Booklet**.

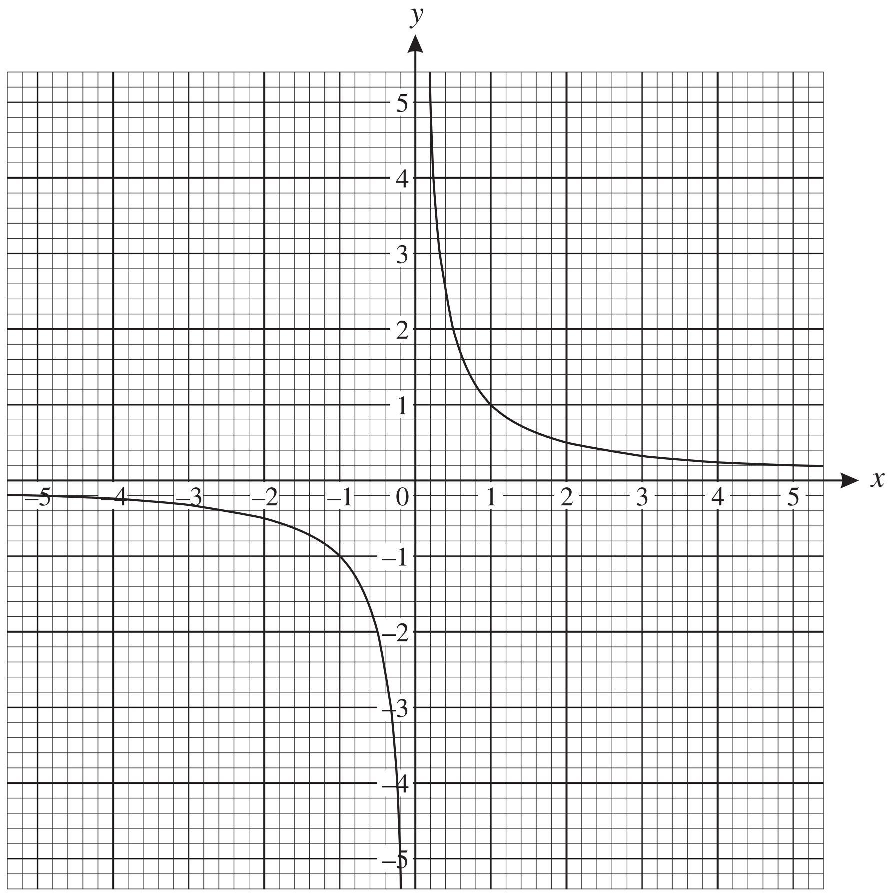
INFORMATION FOR CANDIDATES

- This document consists of **2** pages. Any blank pages are indicated.

 OCR is an exempt Charity
2R–8G04

Turn over

13 (i)



4751 (C1) Introduction to Advanced Mathematics

Section A

1	(i) 0.125 or 1/8 (ii) 1	1 1	as final answer	2
2	$y = 5x - 4$ www	3	M2 for $\frac{y-11}{-9-11} = \frac{x-3}{-1-3}$ o.e. or M1 for grad = $\frac{11-(-9)}{3-(-1)}$ or 5 eg in $y = 5x + k$ and M1 for $y - 11$ = their $m(x - 3)$ o.e. or subst (3, 11) or (−1, −9) in y = their $mx + c$ or M1 for $y = kx - 4$ (eg may be found by drawing)	3
3	$x > 9/6$ o.e. or $9/6 < x$ o.e. www isw	3	M2 for $9 < 6x$ or M1 for $-6x < -9$ or $k < 6x$ or $9 < kx$ or $7 + 2 < 5x + x$ [condone $\le$ for Ms]; if 0, allow SC1 for 9/6 o.e found	3
4	$a = -5$ www	3	M1 for f(2) = 0 used and M1 for $10 + 2a = 0$ or better long division used: M1 for reaching $(8 + a)x - 6$ in working and M1 for $8 + a = 3$ equating coeffts method: M2 for obtaining $x^3 + 2x^2 + 4x + 3$ as other factor	3
5	(i) $4[x^3]$ (ii) $84[x^2]$ www	2 3	ignore any other terms in expansion M1 for $-3[x^3]$ and $7[x^3]$ soi; M1 for $\frac{7 \times 6}{2}$ or 21 or for Pascal's triangle seen with 1 7 21 … row and M1 for 2^2 or 4 or $\{2x\}^2$	5

16

6	1/5 or 0.2 o.e. www	3	M1 for $3x + 1 = 2x \times 4$ and M1 for $5x = 1$ o.e. <u>or</u> M1 for $1.5 + \frac{1}{2x} = 4$ and M1 for $\frac{1}{2x} = 2.5$ o.e.	3
7	(i) $5^{3.5}$ or $k = 3.5$ or 7/2 o.e. (ii) $16a^6b^{10}$	2 2	M1 for $125 = 5^3$ or $\sqrt{5} = 5^{\frac{1}{2}}$ SC1 for $5^{\frac{3}{2}}$ o.e. as answer without working M1 for two 'terms' correct and multiplied; mark final answer only	4
8	$b^2 - 4ac$ soi $k^2 - 4 \times 2 \times 18 < 0$ o.e. $-12 < k < 12$	M1 M1 A2	allow in quadratic formula or clearly looking for perfect square condone $\leq$; or M1 for 12 identified as boundary may be two separate inequalities; A1 for $\leq$ used or for one 'end' correct if two separate correct inequalities seen, isw for then wrongly combining them into one statement; condone b instead of k; if no working, SC2 for $k < 12$ and SC2 for $k > -12$ (ie SC2 for each 'end' correct)	4
9	$y + 5 = xy + 2x$ $y - xy = 2x - 5$ oe or ft $y(1 - x) = 2x - 5$ oe or ft $[y =]\frac{2x-5}{1-x}$ oe or ft as final answer	M1 M1 M1 M1	for expansion for collecting terms for taking out y factor; dep on xy term for division and no wrong work after ft earlier errors for equivalent steps if error does not simplify problem	4
10	(i) $9\sqrt{3}$ (ii) $6 + 2\sqrt{2}$ www	2 3	M1 for $5\sqrt{3}$ or $4\sqrt{3}$ seen M1 for attempt to multiply num. and denom. by $3 + \sqrt{2}$ and M1 for denom. 7 or $9 - 2$ soi from denom. mult by $3 + \sqrt{2}$	5

20

Section B

11	**i**	C, mid pt of AB = $\left(\frac{11+(-1)}{2}, \frac{4}{2}\right)$ = (5, 2)	B1	evidence of method required – may be on diagram, showing equal steps, or start at A or B and go half the difference towards the other	
		[AB^2 =] $12^2 + 4^2$ [= 160] oe or [CB^2 =] $6^2 + 2^2$ [=40] oe with AC	B1	or square root of these; accept unsimplified	
		quote of $(x - a)^2 + (y - b)^2 = r^2$ o.e with different letters	B1	or (5, 2) clearly identified as centre and $\sqrt{40}$ as r (or 40 as r^2) www or quote of *gfc* formula and finding c = −11	
		completion (ans given)	B1	dependent on centre (or midpt) and radius (or radius^2) found independently and correctly	4
	ii	correct subst of $x = 0$ in circle eqn soi	M1		
		$(y-2)^2 = 15$ or $y^2 - 4y - 11$ [= 0]	M1	condone one error	
		$y - 2 = \pm\sqrt{15}$ or ft	M1	or use of quad formula (condone one error in formula); ft only for 3 term quadratic in y	
		$[y =] 2 \pm \sqrt{15}$ cao	A1	if $y = 0$ subst, allow SC1 for (11, 0) found alt method: M1 for y values are $2 \pm a$ M1 for $a^2 + 5^2 = 40$ soi M1 for $a^2 = 40 - 5^2$ soi A1 for $[y =] 2 \pm \sqrt{15}$ cao	4
	iii	grad AB = $\frac{4}{11-(-1)}$ or 1/3 o.e.	M1	or grad AC (or BC)	
		so grad tgt = – 3	M1	or ft −1/their gradient of AB	
		eqn of tgt is $y - 4 = -3(x - 11)$	M1	or subst (11, 4) in $y = -3x + c$ or ft (no ft for their grad AB used)	
		$y = -3x + 37$ or $3x + y = 37$	A1	accept other simplified versions	
		(0, 37) and (37/3, 0) o.e. ft isw	B2	B1 each, ft their tgt for grad ≠ 1 or 1/3; accept $x = 0$, $y = 37$ etc NB alt method: intercepts may be found first by proportion then used to find eqn	6

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12	**i**	$3x^2 + 6x + 10 = 2 - 4x$ $3x^2 + 10x + 8$ [= 0] $(3x + 4)(x + 2)$ [=0] $x = -2$ or $-4/3$ o.e. $y = 10$ or $22/3$ o.e.	M1 M1 M1 A1 A1	for subst for x or y or subtraction attempted or $3y^2 - 52y + 220$ [=0]; for rearranging to zero (condone one error) or $(3y - 22)(y - 10)$; for sensible attempt at factorising or formula or completing square or A1 for each of $(-2, 10)$ and $(-4/3, 22/3)$ o.e.	5
	ii	$3(x + 1)^2 + 7$	4	1 for $a = 3$, 1 for $b = 1$, 2 for $c = 7$ or M1 for $10 - 3 \times$ their b^2 soi or for 7/3 or for $10/3 -$ their b^2 soi	4
	iii	min at $y = 7$ or ft from (ii) for positive c (ft for (ii) only if in correct form)	B2	may be obtained from (ii) or from good symmetrical graph or identified from table of values showing symmetry condone error in x value in stated min ft from (iii) [getting confused with 3 factor] B1 if say turning pt at $y = 7$ or ft without identifying min <u>or</u> M1 for min at $x = -1$ [e.g. may start again and use calculus to obtain $x = -1$] or min when $(x + 1)^{[2]} = 0$; and A1 for showing y positive at min <u>or</u> M1 for showing discriminant neg. so no real roots and A1 for showing above axis not below eg positive x^2 term or goes though (0, 10) <u>or</u> M1 for stating bracket squared must be positive [or zero] and A1 for saying other term is positive	2

11

13	**i**	any correct y value calculated from quadratic seen or implied by plots	B1	for $x \neq 0$ or 1; may be for neg x or eg min.at (2.5, −1.25)	
		(0, 5)(1, 1)(2, −1)(3, −1)(4,1) and (5,5) plotted	P2	tol 1 mm; P1 for 4 correct [including (2.5, −1.25) if plotted]; plots may be implied by curve within 1 mm of correct position	
		good quality smooth parabola within 1mm of their points	C1	allow for correct points only [accept graph on graph paper, not insert]	4
	ii	$x^2 - 5x + 5 = \frac{1}{x}$	M1		
		$x^3 - 5x^2 + 5x = 1$ and completion to given answer	M1		2
	iii	divn of $x^3 - 5x^2 + 5x - 1$ by $x - 1$ as far as $x^3 - x^2$ used in working	M1	or inspection eg $(x - 1)(x^2.....+ 1)$ or equating coeffts with two correct coeffts found	
		$x^2 - 4x + 1$ obtained	A1		
		use of $b^2 - 4ac$ or formula with quadratic factor	M1	or $(x - 2)^2 = 3$; may be implied by correct roots or $\sqrt{12}$ obtained	
		$\sqrt{12}$ obtained and comment re shows other roots (real and) irrational <u>or</u> for $2 \pm \sqrt{3}$ or $\frac{4 \pm \sqrt{12}}{2}$ obtained isw	A2	[A1 for $\sqrt{12}$ and A1 for comment] NB A2 is available only for correct quadratic factor used; if wrong factor used, allow A1 ft for obtaining two irrational roots or for their discriminant and comment re irrational [no ft if their discriminant is negative]	5

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4751 Introduction to Advanced Mathematics (C1)

General Comments

This paper proved to be more accessible than the last two January papers, with many candidates gaining high marks on section A, particularly in the first five questions. In general, the majority of candidates seemed well prepared, with fewer who had little idea of what was required being entered than in past years. For instance, there were fewer poor attempts at using the quadratic formula than there used to be. However, some were thrown by anything that required them to think beyond questions that they had done in practice, with question 7(i) in particular gaining a surprisingly poor response.

Arithmetic with fractions remains a problem, with $\frac{4}{12} = 3$ being quite a common error in question 11(iii) for example, and some candidates failing to simplify results such as $\frac{-1}{-5}$ (where $\frac{1}{5}$ or 0.2 was expected) or $\frac{12}{3}$ or $\sqrt{144}$ (where integer answers were expected).

Candidates need to realise that, in questions where they are required to reach a given result, they must give sufficient indication of their method to show that the result has been obtained independently. In this paper, this particularly applied to questions 11(i) and 13(ii).

Centres are asked to ensure that candidates follow the instructions on the front of the question paper and use black ink, with pencil used only for graphs and diagrams.

Comments on Individual Questions

Section A

1) Most candidates knew the facts about negative and zero powers and gained both marks here. 8 or −8 were the common errors in part (i), whilst a few zeros were seen in part (ii).

2) The methods for finding the equation of a straight line joining two points were well known. Some inverted the gradient but even they generally managed a mark for a correct method to find the constant. Arithmetic or algebraic errors in expansion sometimes spoiled the final answer.

3) Nearly all candidates obtained the 3 marks here. The only real problem occurred with those who obtained $-6x < -9$ and then divided by -6 without reversing the inequality. Those who worked towards $9 < 6x$ were usually successful. A few candidates worked only with an equation instead of an inequality or made errors in collecting the terms.

4) The majority knew how to apply the factor theorem apart from those who used $f(-2)$ or failed to equate $f(2)$ to zero. A few candidates attempted long division or equating coefficients, but such attempts were rarely successful.

5) In the first part, a number of candidates failed to get both x^3 terms in the expansion, with obtaining $7x^2$ instead of $7x^3$ being a common error. The majority expanded the product fully rather than just looking for the terms which would give x^3. A few candidates omitted this part, which although testing a basic technique had not been asked in this way on past C1 papers. The second part was done reasonably but a significant number failed to square the 2 whilst a smaller cohort used the wrong number from Pascal's triangle or made an error in finding $\binom{7}{2}$.

6) Some candidates had problems here in coping with the fraction, with common wrong first steps being $2x(3x+1) = 4 - 2x$ and $2x(3x+1) = 4 \times 2x$. The vast majority obtained the solution correctly and efficiently.

7) Part (i) caused problems for most candidates. The expected $5^3 \times 5^{\frac{1}{2}} = 5^{3\frac{1}{2}}$ was not often seen. Some gave $5^3 \times 5^{\frac{1}{2}} = 5^{\frac{3}{2}}$ but many made errors such as $125\sqrt{5} = \sqrt{125} \times \sqrt{5}$ or $\sqrt[125]{5}$ etc and worked with those.

In part (ii), most had no problems, although common errors were $16a^5b^7$ and $4a^6b10$, whilst some thought it was a binomial expansion.

8) Some candidates omitted this part or simply attempted trials of various values for k and/or x. Most candidates used $b^2 - 4ac$, but the condition for no real roots was often not clearly stated, with some candidates working with $b^2 - 4ac = 0$ or $b^2 - 4ac > 0$ or with statements such as $\sqrt{k^2 - 144} < 0$. Such statements were often followed by recovery to give a correct answer, although many omitted the condition $k > -12$, simply giving $k < 12$ or sometimes $k < \pm 12$. The arithmetic in working out $4 \times 2 \times 18$ also produced frequent errors.

9) How candidates coped with this varied from centre to centre (or group to group within a centre). Some seem well-prepared and knew how to proceed. Some others had little idea how to cope with two y terms and made several attempts, none getting very far before making an error.

10) Most candidates knew how to simplify and add the surds and did so correctly, although some thought $\sqrt{75} + \sqrt{48} = \sqrt{123}$ or, more commonly, $25\sqrt{3} + 16\sqrt{3} = 41\sqrt{3}$. Poor arithmetic such as $48 = 18 \times 3$ spoilt some answers. In the second part, most candidates knew they had to multiply numerator and denominator by $3 + \sqrt{2}$. Errors were sometimes made in doing so, with 11 as denominator instead of 7 for instance, or in failing to divide both terms by 7 for the final answer, with wrong 'cancelling' usually seen in such cases.

Section B

11) (i) Candidates from some centres showed a good understanding of what is required by 'Show that ...' and obtained all 4 marks. However, many candidates produced incomplete or incorrect answers. A common mistake by those who obtained the length of AB was to say that the square root of 160 is 40. Also frequently seen was subtracting the x and y coordinates to obtain incorrectly (5,2). The words centre and radius were absent from many answers, as was the general formula of a circle.

(ii) Most candidates attempted to substitute x = 0 in the equation of the circle. A common mistake was to omit the 25 which results from squaring $x - 5$. A few candidates omitted $-4y$ when they attempted to square $y - 2$. Some candidates failed to arrange the resulting quadratic so that it could be solved, and their solutions petered out in wrong algebra. Many candidates had a correct method but did not gain the final mark because they were unable to simplify their answer correctly to the form required.

(iii) Some candidates did this part well after obtaining few marks in parts (i) and (ii). Most knew that the tangent of the circle was perpendicular to the radius and how to obtain a perpendicular gradient. Most know how to substitute this and the coordinates of a point in the equation of a straight line. Those who made error(s) such as 4/12 = 3 were often able to obtain the final two marks for the intersections with the axes, since follow-through was applied here, but some omitted to attempt this, and some only obtained the point of intersection with one axis.

12) (i) Most candidates made a reasonable attempt at the first part of this question. The favoured method was to equate the expressions for y as shown on the mark scheme. Most of these attempts yielded the correct quadratic equation though it was not uncommon to see $3x^2 + 10x - 8 = 0$. Most candidates successfully factorised this quadratic and went on to get the correct values for x. Some candidates used the formula to get these roots. However not many candidates successfully determined both y values because they could not cope with the fraction work and negative signs involved when using $x = -\frac{4}{3}$. A few candidates used the subtraction method successfully to arrive at the quadratic in x. One or two attempts were made at substituting for x, but they rarely coped with the manipulation required to yield the correct quadratic in y.

(ii) Most candidates realised the need to take out a factor of 3 and so gained the marks for $a = 3$. However only the really good candidates got much further. The value for b was often seen as 2 or 3, but again only the best candidates had much idea about how to determine the value of c. The presence of the factor 3 caused the problem. A few candidates who checked their work by expanding their answer realised their error and were able to correct it.

(iii) Some candidates did realise that completing the square was a good lead-in to answering this part and clearly identified that there was a minimum at $y = 7$. However many candidates chose to base their argument on the basis of the value of the discriminant of the equation. Most of these showed it to be negative but rarely gave an adequate explanation as to the significance of this. Some concluded that it showed no real roots, sometimes going on to say that the graph would not cross the axis, but hardly any attempts went on to determine whether the graph was above or below the x-axis. There were also some arguments about turning points and some attempts to use a sketch, but these were rarely complete.

13) (i) As I have commented in the past, many candidates do not appreciate properly the distinction between plotting and sketching a curve – some have wasted time in past papers by plotting when a sketch was all that was required. This time, when a plot was requested, some did a sketch, calculating the minimum and joining this with a sketch to (0, 5) and (5, 5) and failing to pass correctly though points such as (1, 1) and (2, −1). Some candidates produced poor curves, with feathering or doubling or missing plotted points, with some not helping themselves since they had used pen rather than pencil for drawing. However, many good graphs were seen.

(ii) This was handled well by a very high proportion of candidates, although some were careless with "= 0", or did not show an intermediate step before the given answer and therefore lost the second mark. The most common incorrect solution seen was for candidates to simply verify the result by substituting values.

(iii) A pleasing number of candidates successfully found the correct quadratic factor – usually by inspection or long division, but occasionally by comparing coefficients. Candidates who found the quadratic factor, or those who made a simple error in finding it, most often went on to complete the question. A few candidates who had made an error arrived at complex roots and confused irrational with complex – genuinely (or so it would seem) thinking that their negative discriminant implied irrational roots. Not many candidates realised that it is sufficient to find the value of the discriminant for the quadratic equation to show that the other two roots are irrational, but most did obtain full marks by finding the roots of the quadratic in surd form. A few candidates just stated that the quadratic could not be factorised without justifying their comment at all.