ADVANCED SUBSIDIARY GCE **4776/01**

MATHEMATICS (MEI)

Numerical Methods

MONDAY 16 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

CUP/T38891

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks for each question is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 The equation $f(x) = 0$ is known to have a single root in the interval (3, 3.5). Given that $f(3) = 0.5$ and $f(3.5) = -0.8$, estimate the root using linear interpolation.

State the maximum possible error in this estimate. [6]

2 The function $f(x)$ has the values shown in the table. The value of k is to be determined.

x	1	3	5	7	9
$f(x)$	2	1	5	k	2

Use a difference table to obtain the value of k, assuming that $f(x)$ is a cubic. [6]

3 The function $f(x) = \sqrt{1 + 3^x}$ is to be differentiated numerically.

Use the central difference method with $h = 0.2$ to estimate the derivative at $x = 2$. Obtain further estimates with $h = 0.1$ and $h = 0.05$.

By considering the differences between successive estimates, find the value of the derivative to an accuracy of 3 decimal places. [8]

4 Show that a Newton-Raphson iteration to find the cube root of 25 is

$$x_{r+1} = x_r - \frac{x_r^3 - 25}{3x_r^2}.$$

Perform three steps of this iteration, beginning with $x_0 = 4$. Show, by considering the differences between successive iterates, that the convergence is faster than first order. [8]

5 **(i)** Find sin 86° – sin 85° to the accuracy given by your calculator. [1]

(ii) A simple spreadsheet works to an accuracy of 6 significant figures. All intermediate answers used in calculations are rounded to 6 significant figures.

Write down the values of sin 86° and sin 85° as given by this spreadsheet. Hence find the value the spreadsheet gives for sin 86° – sin 85°. [3]

(iii) You are now *given* that sin 86° – sin 85° = 2 cos 85.5° sin 0.5°. Find the value the spreadsheet gives for this expression. [2]

(iv) Use your working from parts **(ii)** and **(iii)** to explain how two expressions that are mathematically identical can nevertheless evaluate differently. [2]

Section B (36 marks)

6 The integral $\int_1^3 \sqrt{1 + \sin x}\, dx$, where x is in radians, is to be evaluated numerically.

(i) Copy and complete the following table. [7]

h	Mid-point rule estimate	Trapezium rule estimate
2	$M_1 = 2.763\,547$	$T_1 =$
1	$M_2 =$	$T_2 =$
0.5	$M_4 =$	$T_4 =$

(ii) Show that the differences between successive mid-point rule estimates reduce by a factor of about 4.

State a result about the differences between successive trapezium rule estimates. [4]

(iii) Now let $S_1 = \frac{1}{3}(2M_1 + T_1)$, with S_2 and S_4 defined similarly.

Calculate S_1, S_2, S_4 and the differences $S_2 - S_1$, $S_4 - S_2$. By considering these differences, give the value of the integral to the accuracy that appears justified. [7]

7 The equation $x^2 = 4 + \frac{1}{x}$ has three roots.

(i) Show graphically that the equation has exactly one root for $x > 0$. Find the integer a such that this positive root lies in the interval $(a, a + 1)$.

Use the fixed-point iteration

$$x_{r+1} = \sqrt{\left(4 + \frac{1}{x_r}\right)} \qquad (*)$$

to determine the positive root correct to 4 decimal places. [7]

(ii) The equation also has two negative roots. Without doing any calculations, explain why the iteration (*) cannot be used to find these negative roots.

Use the fixed-point iteration

$$x_{r+1} = -\sqrt{\left(4 + \frac{1}{x_r}\right)} \qquad (**)$$

to find a negative root near to $x = -2$ correct to 4 decimal places. [5]

(iii) The third root of the equation lies in the interval (–1, 0). Show that the iteration (**) used in part **(ii)** will not converge to this third root. Use another fixed point iteration to find the third root correct to 4 decimal places. [6]