OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4761/01**

MATHEMATICS (MEI)

Mechanics 1

FRIDAY 23 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 Fig. 1.1 shows a circular cylinder of mass 100 kg being raised by a light, inextensible vertical wire AB. There is negligible air resistance.

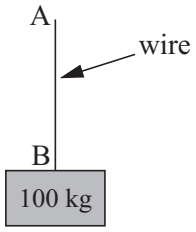

Fig. 1.1

(i) Calculate the acceleration of the cylinder when the tension in the wire is 1000 N. [3]

(ii) Calculate the tension in the wire when the cylinder has an upward acceleration of $0.8\,\text{m}\,\text{s}^{-2}$. [2]

The cylinder is now raised inside a fixed smooth vertical tube that prevents horizontal motion but provides negligible resistance to the upward motion of the cylinder. When the wire is inclined at 30° to the vertical, as shown in Fig. 1.2, the cylinder again has an upward acceleration of $0.8\,\text{m}\,\text{s}^{-2}$.

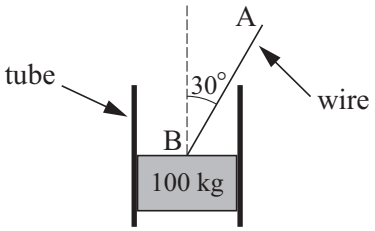

Fig. 1.2

(iii) Calculate the new tension in the wire. [3]

2 A particle has a position vector **r**, where $\mathbf{r} = 4\mathbf{i} - 5\mathbf{j}$ and **i** and **j** are unit vectors in the directions east and north respectively.

(i) Sketch **r** on a diagram showing **i** and **j** and the origin O. [1]

(ii) Calculate the magnitude of **r** and its direction as a bearing. [4]

(iii) Write down the vector that has the same direction as **r** and three times its magnitude. [1]

3 An object of mass 5 kg has a constant acceleration of $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ m s^{-2} for $0 \leqslant t \leqslant 4$, where t is the time in seconds.

(i) Calculate the force acting on the object. [2]

When $t = 0$, the object has position vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ m and velocity $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ m s^{-1}.

(ii) Find the position vector of the object when $t = 4$. [3]

4

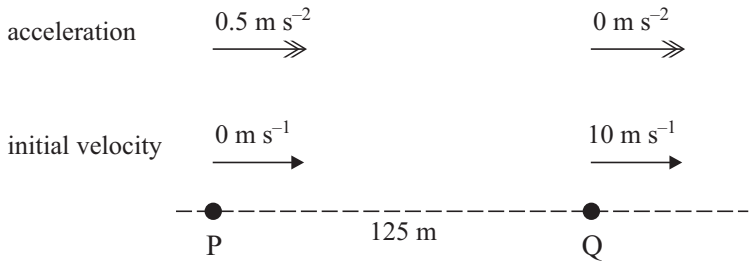

Fig. 4

Particles P and Q move in the same straight line. Particle P starts from rest and has a constant acceleration towards Q of 0.5 m s^{-2}. Particle Q starts 125 m from P at the same time and has a constant speed of 10 m s^{-1} away from P. The initial values are shown in Fig. 4.

(i) Write down expressions for the distances travelled by P and by Q at time t seconds after the start of the motion. [2]

(ii) How much time does it take for P to catch up with Q and how far does P travel in this time? [5]

5 Boxes A and B slide on a smooth, horizontal plane. Box A has a mass of 4 kg and box B a mass of 5 kg. They are connected by a light, inextensible, horizontal wire. Horizontal forces of 9 N and 135 N act on A and B in the directions shown in Fig. 5.

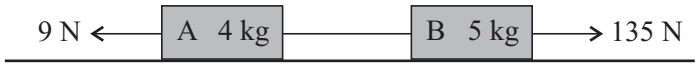

Fig. 5

Calculate the tension in the wire joining the boxes. [4]

6 In this question take $g = 10$.

A golf ball is hit from ground level over horizontal ground. The initial velocity of the ball is 40 m s^{-1} at an angle α to the horizontal, where $\sin \alpha = 0.6$ and $\cos \alpha = 0.8$. Air resistance may be neglected.

(i) Find an expression for the height of the ball above the ground t seconds after projection. [2]

(ii) Calculate the horizontal range of the ball. [4]

[Turn over

Section B (36 marks)

7

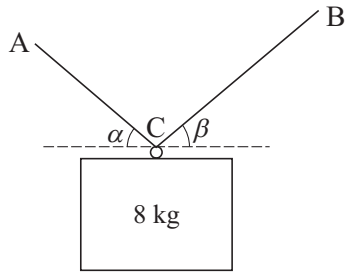

Fig. 7.1

A box of mass 8 kg is supported by a continuous light string ACB that is fixed at A and at B and passes through a smooth ring on the box at C, as shown in Fig. 7.1. The box is in equilibrium and the tension in the string section AC is 60 N.

(i) What information in the question indicates that the tension in the string section CB is also 60 N? [2]

(ii) Show that the string sections AC and CB are equally inclined to the horizontal (so that $\alpha = \beta$ in Fig. 7.1). [2]

(iii) Calculate the angle of the string sections AC and CB to the horizontal. [5]

In a different situation the same box is supported by two separate light strings, PC and QC, that are tied to the box at C. There is also a horizontal force of 10 N acting at C. This force and the angles between these strings and the horizontal are shown in Fig. 7.2. The box is in equilibrium.

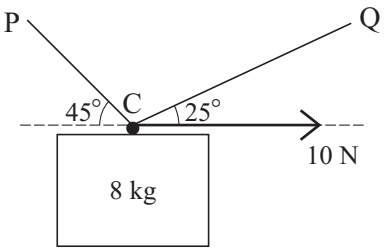

Fig. 7.2

(iv) Calculate the tensions in the two strings. [8]

8 The displacement, x m, from the origin O of a particle on the x-axis is given by

$$x = 10 + 36t + 3t^2 - 2t^3,$$

where t is the time in seconds and $-4 \leqslant t \leqslant 6$.

(i) Write down the displacement of the particle when $t = 0$. [1]

(ii) Find an expression in terms of t for the velocity, v m s^{-1}, of the particle. [2]

(iii) Find an expression in terms of t for the acceleration of the particle. [2]

(iv) Find the maximum value of v in the interval $-4 \leqslant t \leqslant 6$. [3]

(v) Show that $v = 0$ only when $t = -2$ and when $t = 3$. Find the values of x at these times. [5]

(vi) Calculate the *distance* travelled by the particle from $t = 0$ to $t = 4$. [3]

(vii) Determine how many times the particle passes through O in the interval $-4 \leqslant t \leqslant 6$. [3]

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Q 1		mark	comment	sub
(i)	N2L ↑ $1000-100\times9.8=100a$ a = 0.2 so 0.2 m s^{-2} upwards	M1 B1 A1	N2L. Accept $F = mga$ and no weight Weight correct (including sign). Allow if seen. Accept ±0.2. Ignore units and direction	3
(ii)	$T_{BA}-980=100\times0.8$ so tension is 1060 N	M1 A1	N2L. $F = ma$. Weight present, no extras. Accept sign errors.	2
(iii)	$T_{BA}\cos30=1060$ $T_{BA}=1223.98...$ so 1220 N (3 s. f.)	M1 A1 A1	Attempt to resolve their (ii). Do not award for **their** 1060 resolved unless all forces present and all resolutions needed are attempted. If start again allow no weight. Allow $\sin\leftrightarrow\cos$. No extra forces. Condone sign errors FT **their** 1060 only cao	3
		8		

Q 2		mark	comment	sub
(i)		B1	Sketch. O, **i**, **j** and **r** (only require correct quadrant.) Vectors must have arrows. Need not label **r**.	1
(ii)	$\sqrt{4^2+(-5)^2}$ = $\sqrt{41}$ or 6.4031… so 6.40 (3 s. f.) Need $180-\arctan\left(\frac{4}{5}\right)$ 141.340 so 141°	M1 A1 M1 A1	Accept $\sqrt{4^2-5^2}$ Or equivalent. Award for $\arctan\left(\pm\frac{4}{5}\right)$ or $\arctan\left(\pm\frac{5}{4}\right)$ or equivalent seen without 180 or 90. cao	4
(iii)	12**i** – 15**j** or $\begin{pmatrix}12\\-15\end{pmatrix}$	B1	Do not award for magnitude given as the answer. Penalise spurious notation by 1 mark at most once in paper	1
		6		

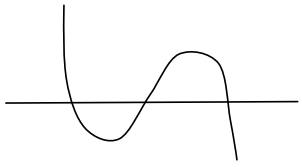
Q 3		mark	comment	sub
			Penalise spurious notation by 1 mark at most once in paper	
(i)	**F** = $5\begin{pmatrix}-1\\2\end{pmatrix}=\begin{pmatrix}-5\\10\end{pmatrix}$ so $\begin{pmatrix}-5\\10\end{pmatrix}$ N	M1	Use of N2L in vector form	
		A1	Ignore units. [Award 2 for answer seen] [SC1 for $\sqrt{125}$ or equiv seen]	2
(ii)	**s** = $\begin{pmatrix}-2\\3\end{pmatrix}+4\begin{pmatrix}4\\5\end{pmatrix}+\frac{1}{2}\times4^2\times\begin{pmatrix}-1\\2\end{pmatrix}$	M1	Use of $\mathbf{s}=t\mathbf{u}+0.5t^2\mathbf{a}$ or integration of **a**. Allow $\mathbf{s}_0$ omitted. If integrated need to consider **v** when t = 0	
		A1	Correctly evaluated; accept $\mathbf{s}_0$ omitted.	
	$\mathbf{s}=\begin{pmatrix}6\\39\end{pmatrix}$ so $\begin{pmatrix}6\\39\end{pmatrix}$ m	B1	Correctly adding $\mathbf{s}_0$ to a vector (FT). Ignore units. [NB $\begin{pmatrix}8\\36\end{pmatrix}$ seen scores M1 A1]	3
		5		

Q 4		mark	comment	sub
(i)	The distance travelled by P is $0.5\times0.5\times t^2$	B1		
	The distance travelled by Q is 10t	B1	Accept 10t + 125 if used correctly below.	2
(ii)	Meet when $0.25t^2=125+10t$	M1	Allow **their** wrong expressions for P and Q distances Allow ± 125 or 125 omitted	
		F1	Award for **their** expressions as long as one is quadratic and one linear. Must have 125 with correct sign.	
	so $t^2-40t-500=0$			
	Solving	M1	Accept any method that yields (smaller) + ve root of their 3 term quadratic	
	t = 50 (or -10)	A1	cao Allow –ve root not mentioned	
	Distance is $0.25\times50^2=625$ m	A1	cao [SC2 400 m seen]	5
		7		

Q 5		mark	comment	sub
	either Overall, N2L → 135 – 9 = (5 +4)*a* *a* = 14 so 14 m s^{-2} For A, N2L → *T* – 9 = 4×14 so 65 N **or** 135 – *T* = 5*a* *T* – 9 = 4*a* Solving *T* = 65 so 65 N	 M1 A1 M1 A1 M1 A1 M1 A1	 Use of N2L. Allow *F* = *mga* but no extra forces. Allow 9 omitted. N2L on A or B with correct mass. *F* = *ma*. All relevant forces and no extras. cao * 1 equation in *T* and *a*. Allow sign errors. Allow *F* = *mga* Both equations correct and consistent Dependent on M* solving for *T*. cao.	 4
		4		

Q 6		mark	comment	sub
(i)	$40\times0.6t-5t^2$ = $24t-5t^2$	M1 A1	Use of $s=ut+0.5at^2$ with $a=\pm9.8,\ \pm10$. Accept 40 or 40×0.8 for '*u*'. Any form	 2
(ii)	**either** Need zero vertical distance so $24t-5t^2=0$ so *t* = 0 or *t* = 4.8 **or** Time to highest point, *T* 0 = 40 × 0.6 – 10*T* so *T* = 2.4 and time of flight is 4.8 range is $40\times0.8\times4.8$ = 153.6 so 154 m (3 s. f.)	 M1 A1 M1 A1 M1 A1	 Equate **their** *y* to zero. With fresh start must have correct *y*. Accept no reference to *t* = 0 and the other root in any form. FT **their** *y* if gives *t* > 0 Allow use of *u* = 40 and 40 × 0.8. Award even if half range found. May be awarded for doubling half range later. Horiz cpt. Accept 0.6 instead of 0.8 only if consistent with expression in (i). FT **their** *t*. cao [NB Use of half range or half time to get 76.8… (g = 10) or 78.36… (g = 9.8) scores 2] [If range formula used: M1 sensible attempt at substitution; allow sin2α wrong B1 sin2α correct A1 all correct A1 cao]	 4
		6		

Q 7		mark	comment	sub
(i)	Continuous string: smooth ring: light string	E1 E1	One reason Another reason	2
(ii)	Resolve $\leftarrow$: $60\cos\alpha - 60\cos\beta = 0$ (so $\cos\alpha = \cos\beta$) and so $\alpha = \beta$	M1 E1	[(ii) and (iii) may be argued using Lami or triangle of forces] Resolution and an equation or equivalent. Accept $s \leftrightarrow c$. Accept a *correct* equation seen without method stated. Accept the use of '*T*' instead of '60'. Shown. Must have stated method (allow $\rightarrow$ seen).	2
(iii)	Resolve $\uparrow$ $2\times 60\times\sin\alpha - 8g = 0$ so α = 40.7933… so 40.8° (3 s. f.)	M1 B1 B1 A1 A1	Resolution and an equation. Accept $s \leftrightarrow c$. Do not award for resolution that cannot give solution (e.g. horizontal) Both strings used (accept use of half weight), seen in an equation $\sin\alpha$ or equivalent seen in an equation All correct	5
(iv)	Resolve $\rightarrow$ $10 + T_{QC}\cos 25 - T_{PC}\cos 45 = 0$ Resolve $\uparrow T_{PC}\sin 45 + T_{QC}\sin 25 - 8g = 0$ Solving $T_{CQ} = 51.4701...$ so 51.5 N (3 s. f.) $T_{CP} = 80.1120...$ so 80.1 N (3 s. f.)	M1 M1 A1 M1 A1 M1 A1 F1	Recognise strings have different tensions. Resolution and an equation. Accept $s \leftrightarrow c$. No extra forces. All forces present. Allow sign errors. Correct. Any form. Resolution and an equation. Accept $s \leftrightarrow c$. No extra forces. All forces present. Allow sign errors. Correct. Any form. * A method that leads to at least one solution of a pair of simultaneous equations. cao either tension other tension. Allow FT only if M1* awarded [Scale drawing: 1st M1 then A1, A1 for answers correct to 2 s.f.]	8
		17		

Q 8		mark	comment	sub
(i)	10	B1		1
(ii)	$v = 36 + 6t - 6t^2$	M1 A1	Attempt at differentiation	2
(iii)	$a = 6 - 12t$	M1 F1	Attempt at differentiation	2
(iv)	Take a = 0 so $t = 0.5$ and v = 37.5 so 37.5 m s^{-1}	M1 A1 A1	Allow table if maximum indicated or implied FT **their** a cao Accept no justification given that this is maximum	3
(v)	**either** Solving $36 + 6t - 6t^2 = 0$ so t = -2 or t = 3 **or** Sub the values in the expression for v Both shown to be zero A quadratic so the only roots **then** x(-2) = -34 x(3) = 91	 M1 B1 E1 M1 E1 B1 B1 B1	 A method for two roots using **their** v Factorization or formula or … of **their** expression Shown Allow just 1 substitution shown Both shown Must be a clear argument cao cao	5
(vi)	$\lvert x(3) - x(0)\rvert + \lvert x(4) - x(3)\rvert$ = $\lvert 91 - 10\rvert + \lvert 74 - 91\rvert$ = 98 so 98 m	M1 A1 A1	Considering two parts Either correct cao [SC 1 for s(4) – s(0) = 64]	3
(vii)	At the SP of v x(-2) = -34 i.e. < 0 and x(3) = 91 i.e. > 0 Also x(-4) = 42 > 0 and x(6) = -98 < 0  so three times	M1 B1 B1	Or any other valid argument e.g find all the zeros, sketch, consider sign changes. Must have some working. If only a sketch, must have correct shape. Doing appropriate calculations e.g. find all 3 zeros; sketch cubic reasonably (showing 3 roots); sign changes in range 3 times seen	3
		19		