OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 **(i)** Write down the matrix for reflection in the y-axis. [1]

(ii) Write down the matrix for enlargement, scale factor 3, centred on the origin. [1]

(iii) Find the matrix for reflection in the y-axis, followed by enlargement, scale factor 3, centred on the origin. [2]

2 Indicate on a single Argand diagram

(i) the set of points for which $|z - (-3 + 2\mathrm{j})| = 2$, [3]

(ii) the set of points for which $\arg(z - 2\mathrm{j}) = \pi$, [3]

(iii) the two points for which $|z - (-3 + 2\mathrm{j})| = 2$ and $\arg(z - 2\mathrm{j}) = \pi$. [1]

3 Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}$. [3]

4 Find the values of A, B, C and D in the identity $3x^3 - x^2 + 2 \equiv A(x-1)^3 + (x^3 + Bx^2 + Cx + D)$. [5]

5 You are given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 2 & 5 \\ 4 & 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & 0 & 2 \\ 14 & -14 & 7 \\ -5 & 7 & -4 \end{pmatrix}$.

(i) Calculate $\mathbf{AB}$. [3]

(ii) Write down $\mathbf{A}^{-1}$. [2]

6 The roots of the cubic equation $2x^3 + x^2 - 3x + 1 = 0$ are α, β and γ. Find the cubic equation whose roots are 2α, 2β and 2γ, expressing your answer in a form with integer coefficients. [5]

7 **(i)** Show that $\dfrac{1}{3r-1} - \dfrac{1}{3r+2} \equiv \dfrac{3}{(3r-1)(3r+2)}$ for all integers r. [2]

(ii) Hence use the method of differences to find $\displaystyle\sum_{r=1}^{n} \frac{1}{(3r-1)(3r+2)}$. [5]

Section B (36 marks)

8 A curve has equation $y = \dfrac{2x^2}{(x-3)(x+2)}$.

(i) Write down the equations of the three asymptotes. [3]

(ii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. [3]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\dfrac{2x^2}{(x-3)(x+2)} < 0$. [3]

9 Two complex numbers, α and β, are given by $\alpha = 2 - 2\mathrm{j}$ and $\beta = -1 + \mathrm{j}$.

α and β are both roots of a quartic equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$, where A, B, C and D are real numbers.

(i) Write down the other two roots. [2]

(ii) Represent these four roots on an Argand diagram. [2]

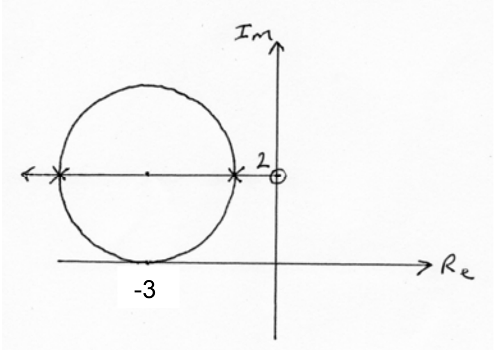
(iii) Find the values of A, B, C and D. [7]

10 **(i)** Using the standard formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, prove that

$$\sum_{r=1}^{n} r^2(r+1) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]

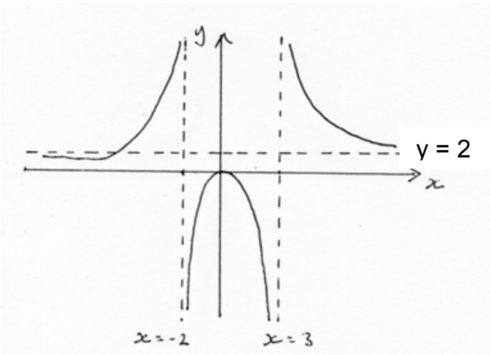
(ii) Prove the same result by mathematical induction. [8]

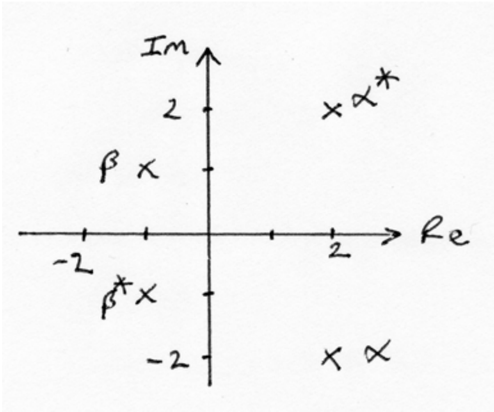
4755 (FP1) Further Concepts for Advanced Mathematics

Qu	Answer	Mark	Comment
Section A			
1(i)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	B1	
1(ii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1	
1(iii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$	M1 A1 **[4]**	Multiplication, or other valid method (may be implied) c.a.o.
2		B3 B3 B1 **[7]**	Circle, B1; centre $-3+2\mathrm{j}$, B1; radius = 2, B1 Line parallel to real axis, B1; through (0, 2), B1; correct half line, B1 Points $-1+2\mathrm{j}$ and $-5+2\mathrm{j}$ indicated c.a.o.
3	$\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow -x-y=x,\ 2x+2y=y$ $\Rightarrow y=-2x$	M1 M1 B1 **[3]**	For $\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$
4	$3x^3-x^2+2\equiv A(x-1)^3+(x^3+Bx^2+Cx+D)$ $\equiv Ax^3-3Ax^2+3Ax-A+x^3+Bx^2+Cx+D$ $\equiv (A+1)x^3+(B-3A)x^2+(3A+C)x+(D-A)$ $\Rightarrow A=2,\ B=5,\ C=-6,\ D=4$	M1 B4 **[5]**	Attempt to compare coefficients One for each correct value

5(i)	$\mathbf{AB} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$	B3 **[3]**	Minus 1 each error to minimum of 0
5(ii)	$\mathbf{A}^{-1} = \frac{1}{7}\begin{pmatrix} -1 & 0 & 2 \\ 14 & -14 & 7 \\ -5 & 7 & -4 \end{pmatrix}$	M1 A1 **[2]**	Use of B c.a.o.
6	$w = 2x \Rightarrow x = \frac{w}{2}$ $\Rightarrow 2\left(\frac{w}{2}\right)^3 + \left(\frac{w}{2}\right)^2 - 3\left(\frac{w}{2}\right) + 1 = 0$ $\Rightarrow w^3 + w^2 - 6w + 4 = 0$	B1 M1 A1 A2 **[5]**	Substitution. For substitution $x = 2w$ give B0 but then follow through for a maximum of 3 marks Substitute into cubic Correct substitution Minus 1 for each error (including '= 0' missing), to a minimum of 0 Give full credit for integer multiple of equation
6	**OR** $\alpha + \beta + \gamma = -\frac{1}{2}$ $\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{3}{2}$ $\alpha\beta\gamma = -\frac{1}{2}$ Let new roots be *k*, *l*, *m* then $k + l + m = 2(\alpha + \beta + \gamma) = -1 = \frac{-B}{A}$ $kl + km + lm = 4(\alpha\beta + \alpha\gamma + \beta\gamma) = -6 = \frac{C}{A}$ $klm = 8\alpha\beta\gamma = -4 = \frac{-D}{A}$ $\Rightarrow \omega^3 + \omega^2 - 6\omega + 4 = 0$	B1 M1 A1 A2 **[5]**	All three Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation Sums and products all correct ft their coefficients; minus one for each error (including '= 0' missing), to minimum of 0 Give full credit for integer multiple of equation

7(i)	$\frac{1}{3r-1}-\frac{1}{3r+2}\equiv\frac{3r+2-(3r-1)}{(3r-1)(3r+2)}$	M1	Attempt at correct method
	$\equiv\frac{3}{(3r-1)(3r+2)}$	A1	Correct, without fudging
		[2]	
7(ii)	$\sum_{r=1}^{n}\frac{1}{(3r-1)(3r+2)}=\frac{1}{3}\sum_{r=1}^{n}\left[\frac{1}{3r-1}-\frac{1}{3r+2}\right]$	M1	Attempt to use identity
	$=\frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+\ldots\ldots+\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)\right]$	A1 M1	Terms in full (at least two) Attempt at cancelling
	$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{3n+2}\right]$	A2	A1 if factor of $\frac{1}{3}$ missing,
		[5]	A1 max if answer not in terms of n
			Section A Total: 36

Section B			
8(i)	$x=3$, $x=-2$, $y=2$	B1 B1 B1 **[3]**	
8(ii)	Large positive *x*, $y \to 2^+$ (e.g. consider $x=100$) Large negative *x*, $y \to 2^-$ (e.g. consider $x=-100$)	M1 B1 B1 **[3]**	Evidence of method required
8(iii)	Curve Central and RH branches correct Asymptotes correct and labelled LH branch correct, with clear minimum  	B1 B1 B1 **[3]**	
8(iv)	$-2<x<3$ $x \neq 0$	B2 B1 **[3]**	B2 max if any inclusive inequalities appear B3 for $-2<x<0$ and $0<x<3$,

9(i)	$2+2\mathrm{j}$ and $-1-\mathrm{j}$	B2 **[2]**	1 mark for each
9(ii)		B2 **[2]**	1 mark for each correct pair
9(iii)	$(x-2-2\mathrm{j})(x-2+2\mathrm{j})(x+1+\mathrm{j})(x+1-\mathrm{j})$	M1 B2	Attempt to use factor theorem Correct factors, minus 1 each error B1 if only errors are sign errors
	$=(x^2-4x+8)(x^2+2x+2)$	A1	One correct quadratic with real coefficients (may be implied)
	$=x^4+2x^3+2x^2-4x^3-8x^2-8x+8x^2+16x+16$ $=x^4-2x^3+2x^2+8x+16$	M1	Expanding
	$\Rightarrow A=-2,\ B=2,\ C=8,\ D=16$	A2	Minus 1 each error, A1 if only errors are sign errors
	OR	**[7]**	
	$\sum\alpha=2$	B1	
	$\alpha\beta\gamma\delta=16$	B1	
	$\sum\alpha\beta=\alpha\alpha^*+\alpha\beta+\alpha\beta^*+\beta\beta^*+\beta\alpha^*+\beta^*\alpha^*$	M1	
	$\sum\alpha\beta\gamma=\alpha\alpha^*\beta+\alpha\alpha^*\beta^*+\alpha\beta\beta^*+\alpha^*\beta\beta^*$	M1	
	$\sum\alpha\beta=2,\ \sum\alpha\beta\gamma=-8$	A1	Both correct
	$A=-2,\ B=2,\ C=8,\ D=16$	A2	Minus 1 each error, A1 if only errors are sign errors
	OR	**[7]**	
	Attempt to substitute in one root Attempt to substitute in a second root	M1 M1 A1	Both correct
	Equating real and imaginary parts to 0 Attempt to solve simultaneous equations	M1 M1	
	$A=-2,\ B=2,\ C=8,\ D=16$	A2 **[7]**	Minus 1 each error, A1 if only errors are sign errors

Qu	Answer	Mark	Comment
Section B (continued)			
10(i)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	M1	Separation of sums (may be implied)
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	B1	One mark for both parts
	$= \frac{1}{12}n(n+1)\left[3n(n+1)+2(2n+1)\right]$	M1	Attempt to factorise (at least two linear algebraic factors)
	$= \frac{1}{12}n(n+1)(3n^2+7n+2)$	A1	Correct
	$= \frac{1}{12}n(n+1)(n+2)(3n+1)$	E1	Complete, convincing argument
		[5]	
10(ii)	$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(3n+1)$		
	n = 1, LHS = RHS = 2	B1	2 must be seen
	Assume true for $n = k$	E1	Assuming true for k
	$\sum_{r=1}^{k} r^2(r+1) = \frac{1}{12}k(k+1)(k+2)(3k+1)$		
	$\sum_{r=1}^{k+1} r^2(r+1)$		
	$= \frac{1}{12}k(k+1)(k+2)(3k+1) + (k+1)^2(k+2)$	B1	(k + 1)th term
	$= \frac{1}{12}(k+1)(k+2)\left[k(3k+1)+12(k+1)\right]$	M1	Attempt to factorise
		A1	Correct
	$= \frac{1}{12}(k+1)(k+2)(3k^2+13k+12)$	A1	Complete convincing argument
	$= \frac{1}{12}(k+1)(k+2)(k+3)(3k+4)$		
	$= \frac{1}{12}(k+1)((k+1)+1)((k+1)+2)(3(k+1)+1)$	E1	Dependent on previous A1 and previous E1
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for k = 1, 2, 3 and so true for all positive integers.	E1 **[8]**	Dependent on first B1 and previous E1
			Section B Total: 36
			Total: 72