OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Decision Mathematics Computation

THURSDAY 12 JUNE 2008

4773/01

Morning

Time: 2 hours 30 minutes

CUP/T44379

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.

COMPUTING RESOURCES

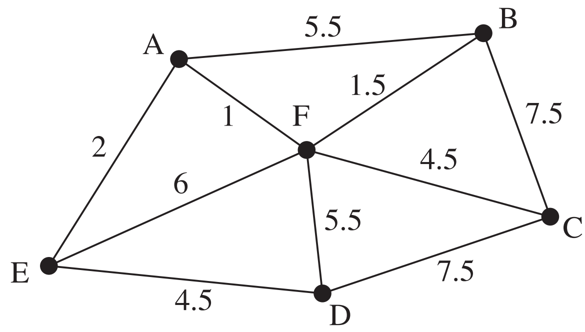
- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.

This document consists of **6** printed pages and **2** blank pages.

1 The vertices of the network represent six villages in a rural area in a developing country. The arc weights represent journey times (hours) along connecting tracks.



Health centres are to be built in some villages, so that all villages are within 5 hours journey time of a health centre.

The matrix shows the complete set of least journey times between the villages.

	A	**B**	**C**	**D**	**E**	**F**
A	0	2.5	5.5	6.5	2	1
B	2.5	0	6	7	4.5	1.5
C	5.5	6	0	7.5	7.5	4.5
D	6.5	7	7.5	0	4.5	5.5
E	2	4.5	7.5	4.5	0	3
F	1	1.5	4.5	5.5	3	0

(i) Let XA, XB, … be indicator variables representing whether or not a health centre is built in village A, B, …

Write down an inequality using these variables which, when satisfied, will ensure that village A is within 5 hours journey of a health centre. Explain your inequality by reference to the matrix above. [5]

(ii) Formulate an LP, the solution to which will give the minimum number of health centres required and their locations. [5]

(iii) Run your LP and interpret the solution. [3]

(iv) Modify your LP so as to find a second optimal solution to the problem. [2]

(v) Find a third optimal solution. [1]

(vi) Explain why it is not necessary to force your variables to be indicator variables, and why it is preferable not to do so. [2]

2 For a small population of organisms it is approximately the case that in any one-second interval either a birth occurs, or a death, or both, or neither. There is never more than one birth or one death in a one-second interval.

Let n be the number of organisms in existence at the beginning of a one-second interval.

The probability of a birth occurring in that interval is $\alpha \times n$, where α is a small positive number.

Similarly, the probability of a death occurring in that interval is $\beta \times n$, where β is a small positive number.

(i) Build a spreadsheet to simulate the development of a population of size 10 over a one-minute period.

Print out your spreadsheet and show the formulae which you use. [6]

(ii) Using $\alpha = 0.01$ and $\beta = 0.04$, run your simulation 10 times.

Tabulate your results and estimate the probability of extinction over the one-minute period. [3]

(iii) Investigate how the probability of extinction varies for different βs, given that α remains at 0.01.

Tabulate your results. [3]

You are now to investigate what happens if the population is increased by immigration. In each one-second period there may be either 0 or 1 immigrants arrive into the population. The probability of an arrival is γ.

(iv) Modify your spreadsheet to incorporate immigration.

Print out your spreadsheet and show the formulae which you use. [4]

(v) Investigate how immigration affects your results in part **(iii)**, given that $\gamma = 0.05$. [2]

3 **(a)**

	A	B	C	D	E	F	G
A	–	8	–	–	3	–	–
B	8	–	5	7	–	–	–
C	–	5	–	1	–	–	8
D	–	7	1	–	2	–	–
E	3	–	–	2	–	4	–
F	–	–	–	–	4	–	6
G	–	–	8	–	–	6	–

Table 3.1

Table 3.1 represents the capacities of arcs in a flow network.

Formulate and solve an LP to find the maximum flow from A to G. [9]

(b)

	S1	S2	S3	S4	S5	S6	S7
W1	–	8	–	–	3	–	–
W2	8	–	5	7	–	–	–
W3	–	5	–	1	–	–	8
W4	–	7	1	–	2	–	–
W5	3	–	–	2	–	4	–
W6	–	–	–	–	4	–	6
W7	–	–	8	–	–	6	–

Table 3.2

Table 3.2 has the same numerical entries as Table 3.1, but its rows are labelled W1 to W7, representing warehouses, and its columns are labelled S1 to S7, representing shops supplied from those warehouses. Each numerical entry represents the cost of moving a single unit from the corresponding warehouse to the corresponding shop.

Each warehouse has 10 units in stock. Each shop requires 10 units.

Formulate and solve an LP to find a minimum cost distribution pattern. [9]

4 **(a)** Solve the recurrence relation $u_{n+2} = \frac{3}{2}u_{n+1} - \frac{1}{2}u_n$, given that $u_0 = 5$ and $u_1 = 3$.

Give the values of u_2, u_3, u_{10} and $u_{1\,000\,000}$. [13]

(b) Successive waves arriving on a beach are found to have heights which follow approximately the recurrence relation $w_{n+2} = \frac{3}{2}w_{n+1} - w_n + 5$, with $w_0 = 5$ and $w_1 = 3$.

(i) Construct a spreadsheet to model wave height. [2]

(ii) Give the values of w_2, w_3, w_{10} and w_{20}. [2]

(iii) State the limitation of using a spreadsheet to model a recurrence relation. [1]

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1.

(i) XA + XB +XE + XF >= 1	M1 A1 ">" OK
Indicator variables correspond to matrix column A (or row A) entries which are less than or equal to 5.	B1 indicator vars B1 <= 5 B1
Ensures that at least one such indicator is 1.	B1
(ii) Min XA+XB+XC+XD+XE+XF st XA+XB+XE+XF >= 1 XA+XB+XE+XF >= 1 XC+XF >= 1 XD+XE >= 1 XA+XB+XD+XE+XF >= 1 XA+XB+XC+XE+XF >= 1	M1 A3 (–1 each error/ omission) allow (correct) reduced set of inequalities
(iii) 2 centres, at F&D or E&C or E&F	M1 A1 A1
(iv) e.g. add XF=0 to force solution E and C	M1 A1
(v) Three solutions are F & D, E & C, E & F.	B1
(vi) Problem is unimodular (or convincing argument).	B1
In the interests of efficiency (and parsimony).	B1

2.

(i) e.g. (candidates should show formulae)	
<table><tr><td>α =</td><td>0.01</td><td>10</td><td>P(birth)</td><td>P(death)</td><td>rand</td><td>rand</td><td>birth</td><td>death</td></tr><tr><td>β =</td><td>0.04</td><td>9</td><td>0.1</td><td>0.4</td><td>0.4261</td><td>0.3537</td><td>0</td><td>1</td></tr><tr><td></td><td></td><td>8</td><td>0.09</td><td>0.36</td><td>0.257</td><td>0.1405</td><td>0</td><td>1</td></tr><tr><td></td><td></td><td>8</td><td>0.08</td><td>0.32</td><td>0.8854</td><td>0.8632</td><td>0</td><td>0</td></tr></table>	B1 handling parameters B1 births B1 deaths B1 use of "rand" B1 use of "if" B1 updating population
(ii) e.g. 1 1 3 2 0 2 2 3 3 0 – 0.2	M1 A1 B1
(iii) e.g. <table><tr><td>β</td><td>0.01</td><td>0.02</td><td>0.03</td><td>0.04</td><td>0.05</td><td>0.06</td><td>0.07</td></tr><tr><td>prob extinction</td><td>0</td><td>0</td><td>0.1</td><td>0.2</td><td>0.5</td><td>0.7</td><td>0.8</td></tr></table>	M1 A1 decent range A1 reasonable outcomes
(iv) Addition of another rand + another if + extra add-on	B1 M1 A1 B1
(v) e.g. <table><tr><td>β</td><td>0.01</td><td>0.02</td><td>0.03</td><td>0.04</td><td>0.05</td><td>0.06</td><td>0.07</td></tr><tr><td>prob. extinction</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0.2</td><td>0.3</td></tr></table>	M1 A1

3.

(a) max AB+AE	B1 objective
st AB-BC-BD+CB+DB=0	
BC+DC-CB-CD-CG=0	M1 flow balance
BD+ED+CD-DB-DC-DE=0	A2 constraints
AE-ED-EF+DE+FE=0	
EF-FE-FG=0	
AB<8	
AE<3	
BC<5	M1 capacity
CB<5	A1 constraints
BD<7	
DB<7	
CD<1	
DC<1	
DE<2	
ED<2	
EF<4	
FE<4	
CG<8	
FG<6	
END	
OBJECTIVE FUNCTION VALUE	
	M1 run
1) 10.00000	

VARIABLE	VALUE	REDUCED COST
AB	7.000000	0.000000
AE	3.000000	0.000000
BC	5.000000	0.000000
BD	2.000000	0.000000
CB	0.000000	1.000000
DB	0.000000	0.000000
DC	1.000000	0.000000
CD	0.000000	1.000000
CG	6.000000	0.000000
ED	0.000000	0.000000
DE	1.000000	0.000000
EF	4.000000	0.000000
FE	0.000000	1.000000
FG	4.000000	0.000000

A1 results

Max flow of 10 with flows of 7 from A to B, … etc. — B1 interpretation

(b)

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min    8X12+3X15+8X21+5X23+7X24+5X32+X34+8X37+7X42
       +X43+2X45+3X51+2X54+4X56+4X65+6X67+8X73+6X76
st     X12+X15=10
       X21+X23+X24=10
       X32+X34+X37=10
       X42+X43+X45=10
       X51+X54+X56=10
       X65+X67=10
       X73+X76=10
       X21+X51=10
       X12+X32+X42=10
       X23+X43+X73=10
       X24+X34+X54=10
       X15+X45+X65=10
       X56+X76=10
       X37+X67=10
END
```

M1 A1 objective

M1 A1 supply constraints

M1 A1 demand constraints

OBJECTIVE FUNCTION VALUE

1) 310.0000

M1 run

VARIABLE	VALUE	REDUCED COST
X12	0.000000	0.000000
X15	10.000000	0.000000
X21	0.000000	0.000000
X23	10.000000	0.000000
X24	0.000000	0.000000
X32	0.000000	0.000000
X34	10.000000	0.000000
X37	0.000000	6.000000
X42	10.000000	0.000000
X43	0.000000	0.000000
X45	0.000000	0.000000
X51	10.000000	0.000000
X54	0.000000	0.000000
X56	0.000000	6.000000
X65	0.000000	0.000000
X67	10.000000	0.000000
X73	0.000000	0.000000
X76	10.000000	0.000000

A1 results

Cost = 310 by sending 10 from W1 to S5, … etc.

B1 interpretation

4.

(a) Auxiliary equation: $2\lambda^2 - 3\lambda + 1 = 0$ $(2\lambda - 1)(\lambda - 1) = 0$ $\lambda = 1$ or ½	 M1 A1 M1 A1
$u_n = A + B(½)^n$	B1 B1
$5 = A + B$ $3 = A + ½B$	B1 B1
$u_n = 1 + 4(½)^n$	M1 A1
$u_2 = 2$, $u_3 = 1.5$, $u_{10} = 1.003906$ $u_{1000000} \approx 1$	B1 B1 B1
(b)(i) & (ii)	
0 5 1 3 2 4.5 3 8.75 4 13.625 5 16.6875	 M1 A1
6 16.40625 7 12.92188 8 7.976563 9 4.042969 10 3.087891	 A1 3.087891
11 5.588867 12 10.29541 13 14.85425 14 16.98596 15 15.62469 16 11.45108 17 6.551926 18 3.376808 19 3.513287 20 6.893122	 A1 6.893122
(iii) Limited wrt to (very) long-term	B1