OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4772/01**

MATHEMATICS (MEI)

Decision Mathematics 2

MONDAY 16 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

CUP/T44381

Additional materials (enclosed): None

Additional materials (required):
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

[Turn over

1 **(a)** The Plain English Society presents an annual "Foot in Mouth" award for "a truly baffling comment". In 2004 it was presented to Boris Johnson MP for a comment on the 12th December 2003 edition of "Have I Got News For You":

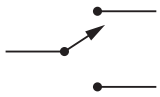
"I could not fail to disagree with you less."

(i) Explain why this can be rewritten as:

"I could succeed in agreeing with you more." [2]

(ii) Rewrite the comment more simply in your own words without changing its meaning. [2]

(b) Two switches are to be wired between a mains electricity supply and a light so that when the state of either switch is changed the state of the light changes (i.e. from off to on, or from on to off). Draw a switching circuit to achieve this.

The switches are both 2-way switches, thus:  [5]

(c) Construct a truth table to show the following.

$[(a \wedge b) \vee ((\sim a) \wedge (\sim b))] \Leftrightarrow [((\sim a) \vee b) \wedge (a \vee (\sim b))]$ [7]

2 Jane has a house on a Mediterranean island. She spends eight weeks a year there, either visiting twice for four weeks each trip or four times for two weeks each trip. Jane is wondering whether it is best for her to fly out and rent a car, or to drive out.
Flights cost £500 return and car rental costs £150 per week.
Driving out costs £900 for ferries, road tolls, fuel and overnight expenses.

(i) Draw a decision tree to model this situation. Advise Jane on the cheapest option. [6]

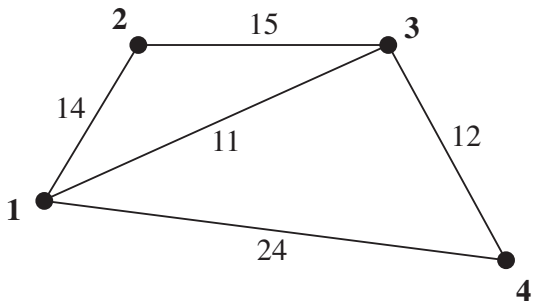
As an alternative Jane considers buying a car to keep at the house. This is a long-term alternative, and she decides to cost it over 10 years. She has to cost the purchase of the car and her flights, and compare this with the other two options.

In her costing exercise she decides that she will not be tied to two trips per year nor to four trips per year, but to model this as a random process in which she is equally likely to do either.

(ii) Draw a decision tree to model this situation. Advise Jane on how much she could spend on a car using the EMV criterion. [8]

(iii) Explain what is meant by "the EMV criterion" and state an alternative approach. [2]

3 The weights on the network represent distances.



(a) (i) Apply Floyd's algorithm to the network to find the complete network of shortest distances, showing that the final matrices are as follows.

	1	**2**	**3**	**4**
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	**2**	**3**	**4**
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3

Draw the complete network of shortest distances. [8]

(ii) Starting at vertex **1**, apply the nearest neighbour algorithm to the complete network of shortest distances to find a Hamilton cycle. Give the length of your cycle and interpret it in the original network. [4]

(iii) By temporarily deleting vertex **1** and its connecting arcs from the complete network of shortest distances, find a lower bound for the solution to the Travelling Salesperson's Problem in that network. Say what this implies in the original network. [4]

(b) Solve the route inspection problem in the **original** network, and say how you can be sure that your solution is optimal. [4]

[Question 4 is printed overleaf]

4 A factory's output includes three products. To manufacture a tonne of product A, 3 tonnes of water are needed. Product B needs 2 tonnes of water per tonne produced, and product C needs 5 tonnes of water per tonne produced.

Product A uses 5 hours of labour per tonne produced, product B uses 6 hours and product C uses 2 hours.

There are 60 tonnes of water and 50 hours of labour available for tomorrow's production.

(i) Formulate a linear programming problem to maximise production within the given constraints. [5]

(ii) Use the simplex algorithm to solve your LP, **pivoting first on your column relating to product C**. [7]

(iii) An extra constraint is imposed by a contract to supply at least 8 tonnes of A. Use either two stage simplex or the big M method to solve this revised problem. [8]

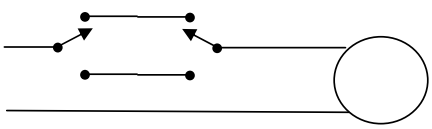
4772 Decision Mathematics 2

1.

(a)(i) "not fail" → "succeed" B1
"disagree less" → "agree more" B1

(ii) e.g. "I don't entirely agree with you". M1 same meaning
A1 simpler

(b) e.g.



M1 2 switches + light in a circuit
A4 one for each correct setting

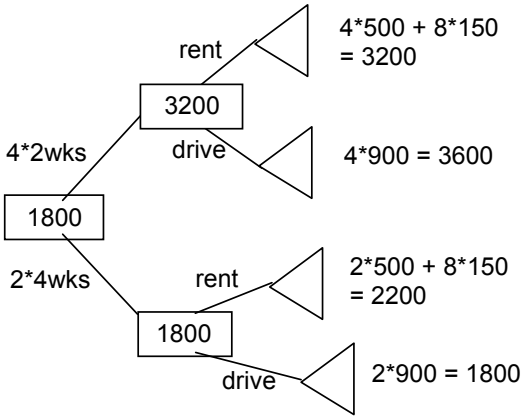
(c)

((a	∧	b)	∨	(~a	∧	~b))	⇔	((~a	∨	b)	∧	(a	∨	~b))
1	1	1	**1**	0	0	0	**1**	0	1	1	**1**	1	1	0
1	0	0	**0**	0	0	1	**1**	0	0	0	**0**	1	1	1
0	0	1	**0**	1	0	0	**1**	1	1	1	**0**	0	0	0
0	0	0	**1**	1	1	1	**1**	1	1	0	**1**	0	1	1

M1 4 lines
A1 a's and b's
A1 negations
A1 level 1 and's
A1 level 1 or's
A1 level 2
A1 result

2.

(i) e.g. (Decisions could be in other order.)



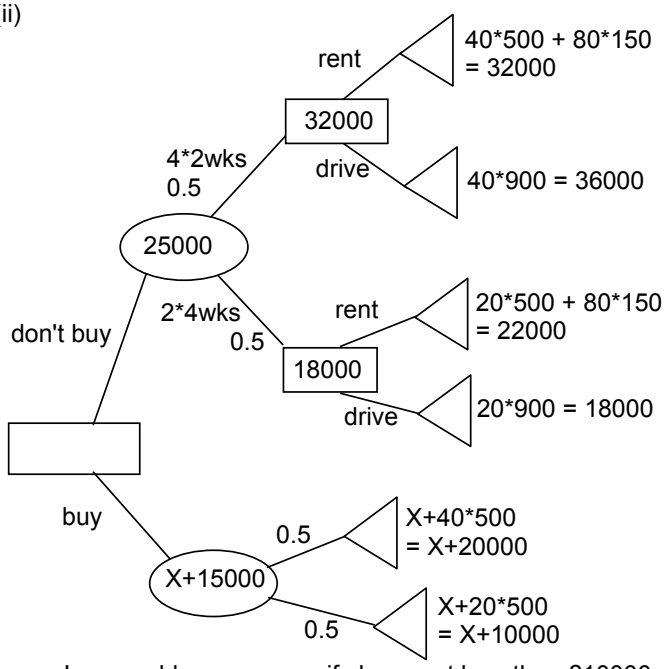
Drive down for 2 lots of 4 weeks

M1 4*2/2*4
M1 rent/drive
A1

M1 costs
A1

B1 advice

(ii)



Jane could save money if she spent less than £10000 on a car

B1 buy/don't buy

M1 don't buy analysis
A1 costings

M1 chance node
A1 buy analysis

M1 buy costings
A1

B1

B1

(iii) EMV – expected monetary value – probabilistically weighted cash values
Utility measure is an alternative.

B1

3.

(a) (i)

	1	2	3	4
1	∞	14	11	24
2	14	∞	15	∞
3	11	15	∞	12
4	24	∞	12	∞

	1	2	3	4
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

M1 sca Floyd
A1 distance
A1 route

	1	2	3	4
1	∞	14	11	24
2	14	28	15	38
3	11	15	22	12
4	24	38	12	48

	1	2	3	4
1	1	2	3	4
2	1	1	3	1
3	1	2	1	4
4	1	1	3	1

A1

	1	2	3	4
1	28	14	11	24
2	14	28	15	38
3	11	15	22	12
4	24	38	12	48

	1	2	3	4
1	2	2	3	4
2	1	1	3	1
3	1	2	1	4
4	1	1	3	1

A1

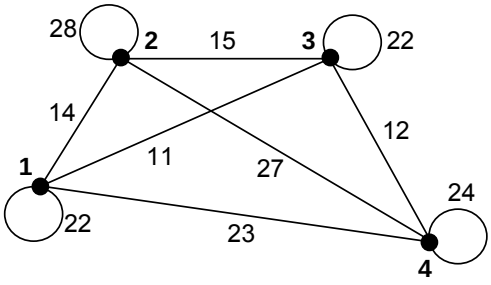
	1	2	3	4
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	2	3	4
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3

A1

	1	2	3	4
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	2	3	4
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3



B1 loops
B1 rest

(ii) **1 3 4 2 1**
64
⇒ **1 3 4 3 2 1**

M1 A1
B1
B1

(iii) 27 + 11 + 14 = 52 TSP solution has length between 52 and 64	M1 A1 M1 A1
(b) e.g. **1 3 1 2 3 4 1** length = 87 One repeated arc → Eulerian	M1 A1 A1 B1

4.

(i) Let a be the number of tonnes of A produced … M1 A1

Max a+b+c B1
st 3a+2b+5c<60 B1
5a+6b+2c<50 B1

(ii) e.g.

P	a	b	c	s_1	s_2	RHS
1	–1	–1	–1	0	0	0
0	3	2	5	1	0	60
0	5	6	2	0	1	50
1	–0.4	–0.6	0	0.2	0	12
0	0.6	0.4	1	0.2	0	12
0	3.8	5.2	0	–0.4	1	26
1	>0	0	0	>0	>0	15
0		0	1			10
0	19/26	1	0	–2/26	5/26	5

M1 initial tableau
A1

M1 pivot
A1

M1
A1

Make 5 tonnes of B and 10 tonnes of C B1 interpretation

(iii) & (iv) e.g.

A	P	a	b	c	s_1	s_2	s_3	art	RHS
1	0	1	0	0	0	0	–1	0	8
0	1	–1	–1	–1	0	0	0	0	0
0	0	3	2	5	1	0	0	0	60
0	0	5	6	2	0	1	0	0	50
0	0	1	0	0	0	0	–1	1	8
1	0	0	0	0	0	0	0	–1	0
0	1	0	–1	–1	0	0	–1	1	8
0	0	0	2	5	1	0	3	–3	36
0	0	0	6	2	0	1	5	–5	10
0	0	1	0	0	0	0	–1	1	8
	1	0	2	0	0	0.5	1.5		13
	0	0	–13	0	1	–2.5	–4.5		11
	0	0	3	1	0	0.5	2.5		5
	0	1	0	0	0	0	–1		8

B1 new constraint

M1 surplus +
A1 artificial

B1 new objective

M1
A1

B1

Make 8 tonnes of A and 5 tonnes of C B1 interpretation