OCR
RECOGNISING ACHIEVEMENT

FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

6993/01

ADDITIONAL MATHEMATICS

FRIDAY 6 JUNE 2008

Afternoon

Time: 2 hours

Additional materials: Answer Booklet (16 pages)
Graph paper

You are not allowed a formulae booklet in this paper.

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 100.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

[Turn over

Section A

1 A driver of a car, initially moving at $30\,\text{m}\,\text{s}^{-1}$, applies the brakes so that the car comes to rest with constant deceleration in 10 seconds.

(i) Find the value of the deceleration. [2]

(ii) Find the distance travelled in this time. [2]

2 The points A and B have coordinates (0, 8) and (6, 0) respectively.

(i) Find the equation of the line AB. [3]

(ii) Find the equation of the line perpendicular to AB through its midpoint. [4]

3 Find the points of intersection of the line $y = 5x + 13$ with the circle $x^2 + y^2 = 13$. [5]

4 Glass marbles are produced in two colours, red and green, in the proportion 7 : 3 respectively. From a large stock of the marbles, 5 are taken at random.

Find the probability that

(i) all 5 are red, [2]

(ii) exactly 3 are red. [3]

5 **(i)** Use calculus to find the stationary points on the curve $y = x^3 - 3x + 1$, identifying which is a maximum and which is a minimum. [6]

(ii) Sketch the curve. [1]

6 A speedboat accelerates from rest so that t seconds after starting its velocity, in $\text{m}\,\text{s}^{-1}$, is given by the formula $v = 0.36t^2 - 0.024t^3$.

(i) Find the acceleration at time t. [3]

(ii) Find the distance travelled in the first 10 seconds. [4]

7 A pyramid stands on a horizontal triangular base, ABC, as shown in Fig. 7.
The angles CAB and ABC are 50° and 60° respectively.
The vertex, V, is directly above C with VC = 10 m.
The angle which the edge VA makes with the vertical is 40°.

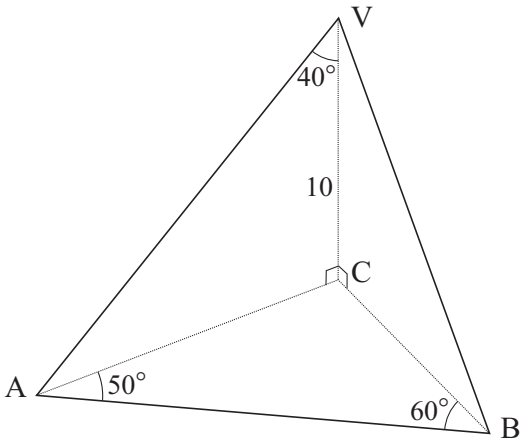

Fig. 7

(i) Calculate AC. [2]

(ii) Hence calculate AB. [4]

8 It is required to solve the equation $2\cos^2 x = 5\sin x - 1$.

(i) Show that this equation may be written as $2\sin^2 x + 5\sin x - 3 = 0$. [2]

(ii) Hence solve the equation $2\cos^2 x = 5\sin x - 1$ for values of x in the range $0° \leqslant x \leqslant 360°$. [4]

9 The cubic equation $x^3 + ax^2 + bx - 26 = 0$ has 3 positive, distinct, integer roots.

Find the values of a and b. [5]

[Turn over

Section B

10 Simon and Gavin drive a distance of 140 km along a motorway, both at constant speed. Simon drives at 5 km per hour faster than Gavin.

Let Gavin's speed be v km per hour.

(i) Write down expressions in terms of v for the times, in hours, taken by Gavin and Simon. [2]

Simon completes the journey in 15 minutes less than Gavin.

(ii) Explain why $\frac{140}{v} - \frac{140}{v+5} = \frac{1}{4}$ and show that this equation reduces to the equation

$$v^2 + 5v - 2800 = 0.$$ [5]

(iii) Solve this equation to find v and hence find the times taken by Simon and Gavin. Give your answers correct to the nearest minute. [5]

11 The side of a fairground slide is in the shaded shape as shown in Fig. 11. Units are metres.

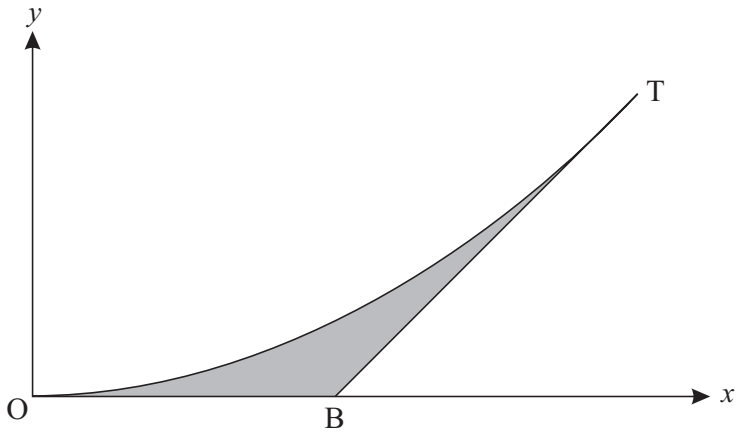

Fig. 11

The curve has equation $y = \lambda x^2$.

T has coordinates (4, 2). The line BT is a tangent to the curve at T. It meets the x-axis at the point B.

(i) Find the value of λ. [1]

(ii) Find the equation of the tangent BT and hence find the coordinates of the point B. [6]

(iii) Find the area of the shaded portion of the graph. [5]

12 A furniture manufacturer produces tables and chairs.

In each week the following constraints apply.

- There are 24 workers, each working for 40 hours (i.e. there are 960 worker-hours available).
- There is a maximum of £1800 available for the purchase of materials.
- Each table requires £30 worth of materials and 12 worker-hours.
- Each chair requires £10 worth of materials and 6 worker-hours.
- It is necessary to make at least 3 times as many chairs as tables.

Let x be the number of tables produced each week and y be the number of chairs produced each week.

(i) Show that the worker-hour constraint reduces to the inequality $2x + y \leqslant 160$. [2]

(ii) Find the inequality relating to the cost of materials constraint and the inequality relating to the numbers of tables and chairs. [3]

(iii) Plot these three inequalities on a graph, using 1 cm to represent 10 tables on the x-axis and 1 cm to represent 10 chairs on the y-axis. Indicate the region for which these inequalities hold. You should shade the region which is **not** required. [4]

When finished, each table is sold for a profit of £20 and each chair is sold for a profit of £5.

(iv) The manufacturer wishes to maximise the profit. Explain why the objective function is given by $P = 20x + 5y$. [1]

(v) Find the number of tables and chairs that should be made in order to maximise the profit. [2]

[Question 13 is printed overleaf.]

13 In the triangle shown in Fig. 13, M is the midpoint of BC.

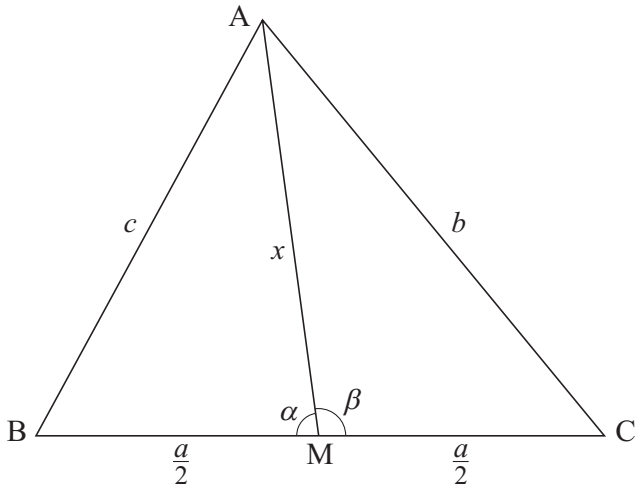

Fig. 13

(i) Explain why $\cos\alpha = -\cos\beta$. [2]

(ii) Using the cosine rule in the triangle BMA, show that

$$\cos\alpha = \frac{4x^2 + a^2 - 4c^2}{4ax}.$$ [2]

(iii) Find a similar expression for $\cos\beta$. [1]

(iv) Using the results in parts **(i)**, **(ii)** and **(iii)**, show that $4x^2 + a^2 = 2(c^2 + b^2)$. [5]

(v) A triangular lawn has sides 46 m, 29 m and 27 m. Find the distance from the midpoint of the longest side to the opposite corner. [2]

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OCR
RECOGNISING ACHIEVEMENT

FSMQ

Additional Mathematics

ADVANCED FSMQ **6993**

Mark Scheme for the Unit

June 2008

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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CONTENTS

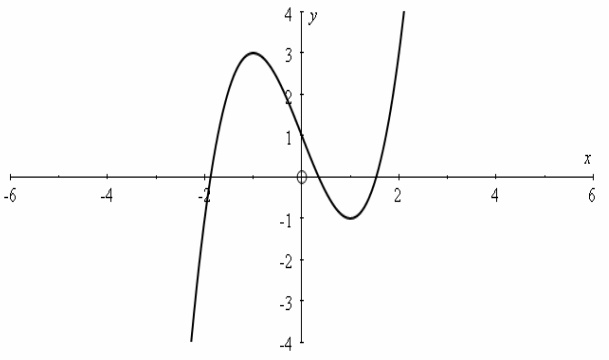
Additional Mathematics FSMQ (6993)

MARK SCHEME FOR THE UNIT

6993 Additional Mathematics

Section A

Q.		Answer	Marks	Notes
1	(i)	$v = u + at$ with $v = 0, u = 30, t = 10$ $\Rightarrow 10a = -30$ $\Rightarrow a = -3$ Deceleration is 3 ms^{-2}	M1 A1 **2**	Must be used a = 3 or decel = −3 are wrong
	(ii)	E.g. $v^2 = u^2 + 2as$ with $v = 0, u = 30, a = -3$ $\Rightarrow 6s = 900$ $\Rightarrow s = 150$ Distance is 150 m Alternatives: $s = \left(\frac{u+v}{2}\right)t$ with $v = 0, u = 30, t = 10$ $\Rightarrow s = 15 \times 10 = 150$ Or: $s = ut + \frac{1}{2}at^2$ with $u = 30, t = 10, a = -3$ $\Rightarrow s = 300 - 150 = 150$ Or: $s = vt - \frac{1}{2}at^2$ with $v = 0, t = 10, a = -3$ $\Rightarrow s = 0 - (-150) = 150$	M1 A1 **2**	 Allow alternatives
2	(i)	$\frac{x}{6} + \frac{y}{8} = 1$ $\Rightarrow 4x + 3y = 24$ Any correct equation will do. Usual answer $y = -\frac{4}{3}x + 8$ **SC.** Omission of y = : give M1 A0	B1 soi M1 A1 isw **3**	Gradient Any valid method In form $ax + by = c$ N.B. Drawing of graph is 0.
	(ii)	Midpoint is (3, 4) Gradient is $\frac{3}{4}$ $\Rightarrow$ equation is $y - 4 = \frac{3}{4}(x - 3)$ $\Rightarrow 4y = 3x + 7$ **SC.** Omission of y = : give M1 A0	B1 soi E1 M1 A1 **4**	 -ve reciprocal of their gradient Use *their* gradient plus *their* midpoint In form $ax + by = c$ N.B. Drawing of graph is 0.

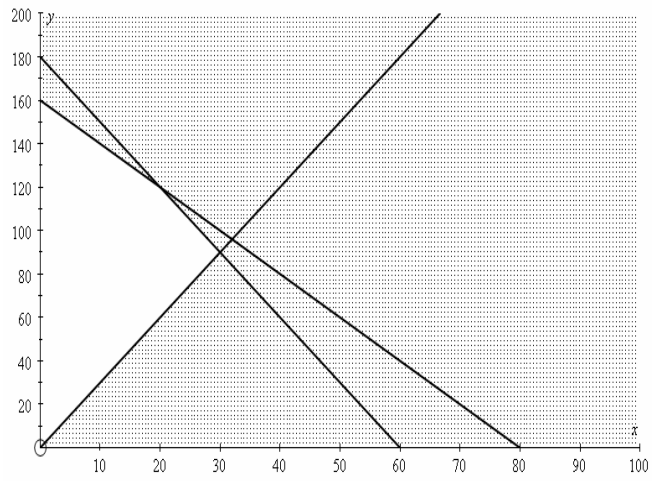
Q.		Answer	Marks	Notes
3		$x^2+(5x+13)^2=13$ $\Rightarrow x^2+25x^2+130x+169-13=0$ $\Rightarrow 26x^2+130x+156=0$ $\Rightarrow x^2+5x+6=0$ $\Rightarrow (x+2)(x+3)=0 \Rightarrow x=-2,-3$ $\Rightarrow y=3,-2$ $\Rightarrow$ Points of intersection $(-2,3),(-3,-2)$ **SC:** For each pair obtained from accurate graph or table of values, or trial, B1	M1 A1 soi M1 A1 A1 **5**	Attempt at substitution. Expansion of $(5x+13)^2$ Solve 3 term quadratic Either both x or one pair Either both y or other pair
4	(i)	$\left(\frac{7}{10}\right)^5 \approx 0.168$	B1 soi B1 **2**	p **and** power Ans
	(ii)	$\binom{5}{3}\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2 \approx 0.3087$ *Allow 3, 4 or 5 sig figs in both parts* *Apply tmsf or tfsf otherwise.* 0 if more than one term	B1 soi B1 B1 **3**	coeff powers mult (p correct) ans
5	(i)	$y=x^3-3x+1 \Rightarrow \frac{dy}{dx}=3x^2-3$ $\frac{dy}{dx}=0$ when $x=\pm 1$, giving $(1,-1)$ and $(-1,3)$ $\frac{d^2y}{dx^2}=6x$; when $x=1, \frac{d^2y}{dx^2}>0$ giving minimum at $x=1$ when $x=-1, \frac{d^2y}{dx^2}<0$ giving maximum at $x=-1$ Any alternative method OK.	B1 M1 A1 A1 M1 A1 **6**	Correct derivative Setting their derivative = 0 Both x or one pair Both y or other pair (*y values could be seen in (ii)*) Identify one turning point Both correct
	(ii)	 Curve to be consistent in (i)	E1 **1**	General shape including axes and turning points At *their* x values. (but don't worry about intercepts on the axes.) This *does* require a scale on the x axis.

Q.		Answer	Marks	Notes
6	(i)	$a = \frac{dv}{dt} = 0.72t - 0.072t^2$	M1 A1 A1 **3**	Diffn Each term
	(ii)	$s = \int_0^{10} \left(0.36t^2 - 0.024t^3\right) dt = \left[0.12t^3 - 0.006t^4\right]_0^{10}$ $= 120 - 60 = 60\,\text{m}$ N.B. Watch $s = \left(\frac{0+12}{2}\right)10 = 60$	M1 A1 M1 A1 **4**	Int the given fn Both terms Deal with def.int
7	(i)	$\frac{AC}{VC} = \tan 40 \Rightarrow AC = 10\tan 40 = 8.39$ m **Alt forms for AC acceptable.** i.e. $AC = \frac{10\sin 40}{\sin 50} = \frac{10}{\tan 50}$	B1 B1 **2**	Tan function Correct
	(ii)	Angle $C = 180 - 50 - 60 = 70$ $\Rightarrow \frac{AB}{\sin C} = \frac{AC}{\sin B}$ $\Rightarrow AB = 8.39 \times \frac{\sin 70}{\sin 60} = 9.10$ m	B1 M1 F1 A1 **4**	 To find AB Must be 3 s.f.
8	(i)	$2(1 - \sin^2 x) = 5\sin x - 1$ $\Rightarrow 2\sin^2 x + 5\sin x - 3 = 0$	M1 A1 **2**	Use of pythag.to change $\cos^2$ **All working - answer given**
	(ii)	$(2\sin x - 1)(\sin x + 3) = 0$ $\Rightarrow \sin x = \frac{1}{2}$ $\Rightarrow x = 30^0, 150^0$ **SC.** $\sin x = -\frac{1}{2} \Rightarrow x = 210, 330$ M1 A0 A0 F1	M1 A1 A1 F1 **4**	Solve quad in sin *x* or *s* etc ½ seen 30 seen 180 – ans (only one extra angle)
9		3 roots are 1, 2, 13 – allow ±1, ±2, ±13 Equation is $(x - 1)(x - 2)(x - 13) = 0$ Giving $x^3 - 16x^2 + 41x - 26 = 0$ i.e. $a = -16$, $b = 41$ (Can be seen in cubic. **Alternative method.** $f(1) = 0 \Rightarrow a + b = 25$ B1 $f(2) = 0 \Rightarrow 4a + 2b = 18$ B1 Solve to give *a* and *b* M1 A1, A1	B1 soi B1 M1 A1 A1 isw **5**	 Factor form. Condone no = 0 Expand to give cubic

Section B

Q.		Answer	Marks	Notes
10	**(i)**	$\frac{140}{v}$, $\frac{140}{v+5}$	B1 B1 **2**	
	(ii)	Gavin's time minus Simon's time is 15 mins = $\frac{1}{4}$hr $\Rightarrow \frac{140}{v} - \frac{140}{v+5} = \frac{1}{4}$ $\Rightarrow 4\left(140(v+5) - 140v\right) = v(v+5)$ $\Rightarrow 2800 = v(v+5) \Rightarrow v^2 + 5v - 2800 = 0$	B1 B1 M1 A1 soi A1 **5**	¼ hr Subtract Clear fractions 700
	(iii)	$v = \frac{-5 \pm \sqrt{25 + 4 \times 2800}}{2} \approx 50.47$ or 50.5 $\Rightarrow$ Gavin: 2.77 hrs, Simon 2.52 hrs $\Rightarrow$ Gavin takes 2 hrs 46 mins (166 mins) Simon takes 2 hrs 31 mins (151 mins) **SC** For $v = 50 \Rightarrow$ 168, 153 give full marks but -1 tfsf	M1 A1 M1 A1 F1 **5**	Solve in decimals (ignore anything else) Convert (only one needs to be seen) Or give B1 for both in decimals This is for one 15 less than the other

Q.		Answer	Marks	Notes
11	**(i)**	$2 = 16\lambda \Rightarrow \lambda = \frac{1}{8}$	B1 **1**	
	(ii)	$\frac{dy}{dx} = \frac{1}{8}.2x = \frac{x}{4}$ When $x = 4, \frac{dy}{dx} = 1$ $\Rightarrow$ Tangent at T is $y - 2 = 1(x - 4)$ $\Rightarrow y = x - 2$ When $y = 0, x = 2$ So B is (2, 0)	E1 M1 A1 DM1 A1 A1 **6**	Correct derivative from their λ or leaving it in Sub $x = 4$ (numeric gradient to give tangent)
	(iii)	Area under curve = $\int_0^4 \frac{x^2}{8} dx = \left[\frac{x^3}{24}\right]_0^4$ Area of triangle = 2 Shaded area = $\left[\frac{x^3}{24}\right]_0^4 - 2 = 2\frac{2}{3} - 2 = \frac{2}{3}$ **N.B.** Area under (curve − line) from 0 to 4 M1 A1 only	M1 A1 B1 M1 A1 **5**	Int. Function Sub limits for int and subtract triangle

Q.		Answer	Marks	Notes
12	(i)	Worker hours for tables = $12x$ Worker hours for chairs = $6y$ $\Rightarrow 12x + 6y \leq 24 \times 40 = 960 \Rightarrow 2x + y \leq 160$	M1 A1 **2**	Must see $12x$ and $6y$
	(ii)	$30x + 10y \leq 1800$ $(\Rightarrow 3x + y \leq 180)$ $y \geq 3x$	M1 A1 B1 **3**	Does not have to be simplified
	(iii)	 N.B. Intercepts on axis must be seen N.B. Ignore < instead of ≤	B1 B1 E1 E1 **4**	Each line For $y \geq 3x$ Must be a region including the y axis as boundary
	(iv)	We wish to maximise the profit. Profit per table = 20, profit per chair = 5 i.e. $P = 20x + 5y$	B1 **1**	Something that connects 20 with x
	(v)	Greatest profit will occur where the lines $y = 3x$ and $3x + y = 180$ intersect. This is at (30, 90). Allow even if shading for $y \geq 3x$ is wrong. **SC:** Trying all corners without the corect answers B1 **SC:** Drawing an O.F. line without the right answer B1	B1 B1 **2**	30 ± 2 90 ± 2 But answers must be integers.

13	**(i)**	Angles on straight line means $\alpha = 180 - \beta$ And $\cos(180 - \beta) = -\cos\beta$	B1 B1 **2**	Must make reference to the figure of the question
	(ii)	$\cos\alpha = \dfrac{x^2 + \left(a/2\right)^2 - c^2}{2.\left(a/2\right)x}$ $= \dfrac{x^2 + \frac{1}{4}a^2 - c^2}{ax} = \dfrac{4x^2 + a^2 - 4c^2}{4ax}$	M1 A1 **2**	Correct cosine formula. Condone missing brackets.
	(iii)	$\cos\beta = \dfrac{4x^2 + a^2 - 4b^2}{4ax}$ N.B. also $-\dfrac{4x^2 + a^2 - 4c^2}{4ax}$	B1 **1**	
	(iv)	$\dfrac{4x^2 + a^2 - 4b^2}{4ax} = -\dfrac{4x^2 + a^2 - 4c^2}{4ax}$ $\Rightarrow 4x^2 + a^2 - 4b^2 = -\left(4x^2 + a^2 - 4c^2\right)$ $\Rightarrow 4x^2 + a^2 - 4b^2 = -4x^2 - a^2 + 4c^2$ $\Rightarrow 8x^2 + 2a^2 = 4\left(b^2 + c^2\right)$ $\Rightarrow 4x^2 + a^2 = 2\left(b^2 + c^2\right)$	M1 M1 A1 M1 A1 **5**	Use of (i), (ii) and (iii) Clear fractions Simplify
	(v)	$a = 46,\ b = 29,\ c = 27$ gives $4x^2 + 46^2 = 2(29^2 + 27^2)$ gives $x^2 = 256$ i.e. $x = 16$ **S.C.** Use of cosine formula in large triangle to get an angle (C = 36.2, B = 33.4) Then use of cosine formula in small triangle to get $x = 16$ M1, A1 only if the answer is 16. **SC:** Scale drawing gets 0.	M1 A1 **2**	Can be substituted in any order

FSMQ Advanced Mathematics 6993

June 2008 Assessment Series

Unit Threshold Marks

Unit	Maximum Mark	A	B	C	D	E	U
6993	100	68	58	48	38	29	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
6993	26.4	36.7	46.5	56.0	64.7	100	7261

Statistics are correct at the time of publication

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