OCR
RECOGNISING ACHIEVEMENT

**ADVANCED SUBSIDIARY GCE** **4755/01**

**MATHEMATICS (MEI)**

Further Concepts for Advanced Mathematics (FP1)

**FRIDAY 11 JANUARY 2008** Morning

Time: 1 hour 30 minutes

**Additional materials:** Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

**INSTRUCTIONS TO CANDIDATES**

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

**Section A** (36 marks)

**1** You are given that matrix $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ and matrix $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$.

**(i)** Find **BA**. [2]

**(ii)** A plane shape of area 3 square units is transformed using matrix **A**. The image is transformed using matrix **B**. What is the area of the resulting shape? [3]

**2** You are given that $\alpha = -3 + 4\mathrm{j}$.

**(i)** Calculate $\alpha^2$. [2]

**(ii)** Express $\alpha$ in modulus-argument form. [3]

**3** **(i)** Show that $z = 3$ is a root of the cubic equation $z^3 + z^2 - 7z - 15 = 0$ and find the other roots. [5]

**(ii)** Show the roots on an Argand diagram. [2]

**4** Using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{n} [(r+1)(r-2)] = \frac{1}{3}n(n^2 - 7)$. [6]

**5** The equation $x^3 + px^2 + qx + r = 0$ has roots $\alpha$, $\beta$ and $\gamma$, where

$$\alpha + \beta + \gamma = 3,$$
$$\alpha\beta\gamma = -7,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 13.$$

**(i)** Write down the values of $p$ and $r$. [2]

**(ii)** Find the value of $q$. [3]

**6** A sequence is defined by $a_1 = 7$ and $a_{k+1} = 7a_k - 3$.

**(i)** Calculate the value of the third term, $a_3$. [2]

**(ii)** Prove by induction that $a_n = \dfrac{(13 \times 7^{n-1}) + 1}{2}$. [6]

**Section B** (36 marks)

**7** The sketch below shows part of the graph of $y = \dfrac{x-1}{(x-2)(x+3)(2x+3)}$. One section of the graph has been omitted.

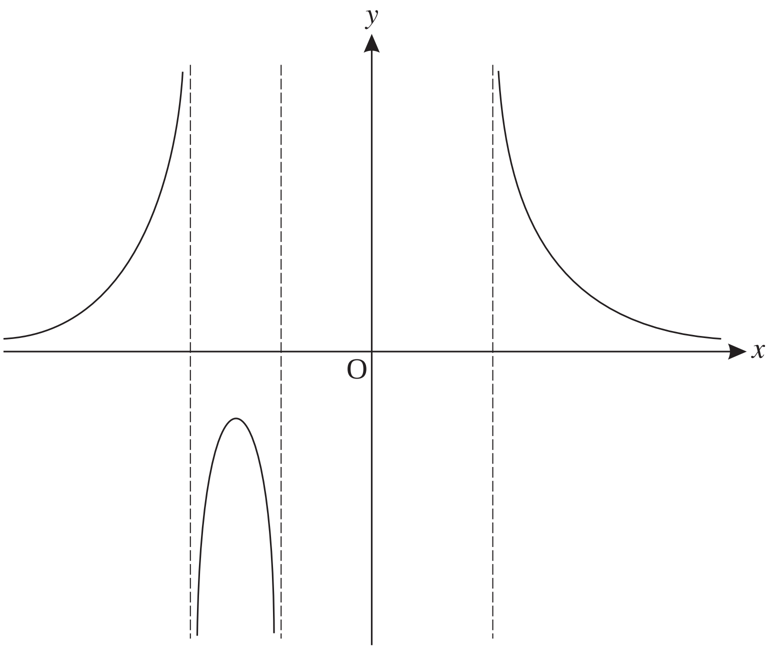


**Fig. 7**

**(i)** Find the coordinates of the points where the curve crosses the axes. [2]

**(ii)** Write down the equations of the three vertical asymptotes and the one horizontal asymptote. [4]

**(iii)** Copy the sketch and draw in the missing section. [2]

**(iv)** Solve the inequality $\dfrac{x-1}{(x-2)(x+3)(2x+3)} \geqslant 0$. [3]

**8** **(i)** On a single Argand diagram, sketch the locus of points for which

($A$) $|z - 3\mathrm{j}| = 2$, [3]

($B$) $\arg(z+1) = \frac{1}{4}\pi$. [3]

**(ii)** Indicate clearly on your Argand diagram the set of points for which

$$|z - 3\mathrm{j}| \leqslant 2 \quad \text{and} \quad \arg(z+1) \leqslant \tfrac{1}{4}\pi.$$ [2]

**(iii)** ($A$) By drawing an appropriate line through the origin, indicate on your Argand diagram the point for which $|z - 3\mathrm{j}| = 2$ and $\arg z$ has its minimum possible value. [2]

($B$) Calculate the value of $\arg z$ at this point. [2]

**9** A transformation T acts on all points in the plane. The image of a general point P is denoted by P′. P′ always lies on the line $y = x$ and has the same $x$-coordinate as P. This is illustrated in Fig. 9.

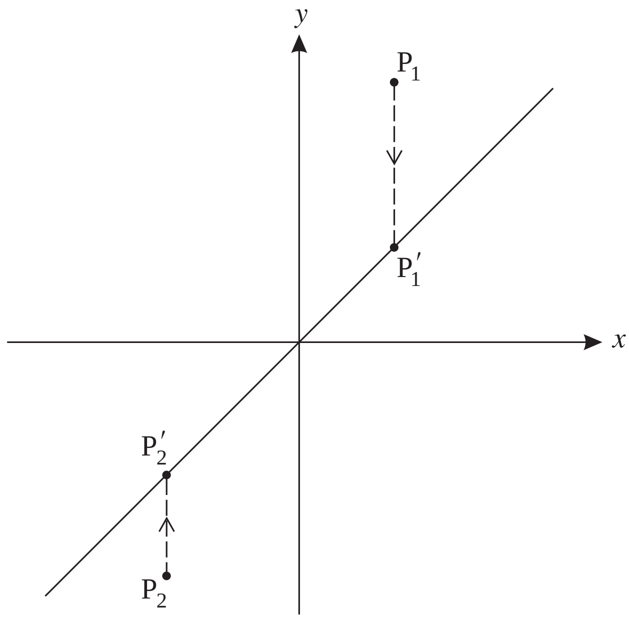


**Fig. 9**

**(i)** Write down the image of the point (−3, 7) under transformation T. [1]

**(ii)** Write down the image of the point $(x, y)$ under transformation T. [2]

**(iii)** Find the $2 \times 2$ matrix which represents the transformation. [3]

**(iv)** Describe the transformation M represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [2]

**(v)** Find the matrix representing the composite transformation of T followed by M. [2]

**(vi)** Find the image of the point $(x, y)$ under this composite transformation. State the equation of the line on which all of these images lie. [3]