RECOGNISING ACHIEVEMENT

ADVANCED GCE **4762/01**

MATHEMATICS (MEI)

Mechanics 2

THURSDAY 17 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

1 **(a)** A battering-ram consists of a wooden beam fixed to a trolley. The battering-ram runs along horizontal ground and collides directly with a vertical wall, as shown in Fig. 1.1. The battering-ram has a mass of 4000 kg.

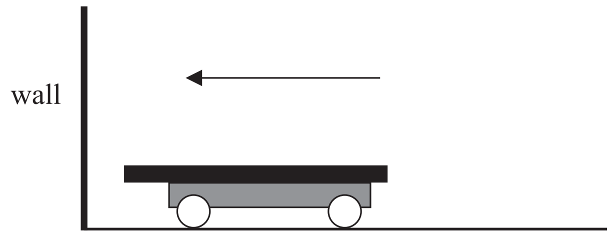

Fig. 1.1

Initially the battering-ram is at rest. Some men push it for 8 seconds and let go just as it is about to hit the wall. While the battering-ram is being pushed, the constant overall force on it in the direction of its motion is 1500 N.

(i) At what speed does the battering-ram hit the wall? [3]

The battering-ram hits a loose stone block of mass 500 kg in the wall. Linear momentum is conserved and the coefficient of restitution in the impact is 0.2.

(ii) Calculate the speeds of the stone block and of the battering-ram immediately after the impact. [6]

(iii) Calculate the energy lost in the impact. [3]

(b) Small objects A and B are sliding on smooth, horizontal ice. Object A has mass 4 kg and speed 18 m s^{-1} in the $\mathbf{i}$ direction. B has mass 8 kg and speed 9 m s^{-1} in the direction shown in Fig. 1.2, where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors.

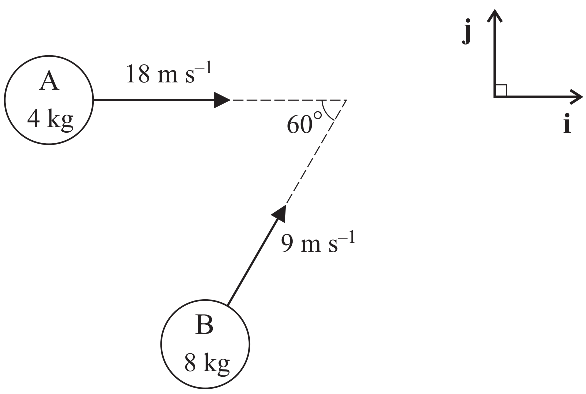

Fig. 1.2

(i) Write down the linear momentum of A and show that the linear momentum of B is $(36\mathbf{i} + 36\sqrt{3}\,\mathbf{j})$ N s. [2]

After the objects meet they stick together (coalesce) and move with a common velocity of $(u\mathbf{i} + v\mathbf{j})\text{ m s}^{-1}$.

(ii) Calculate u and v. [3]

(iii) Find the angle between the direction of motion of the combined object and the $\mathbf{i}$ direction. Make your method clear. [2]

2 A cyclist and her bicycle have a combined mass of 80 kg.

(i) Initially, the cyclist accelerates from rest to $3\,\text{m}\,\text{s}^{-1}$ against negligible resistances along a horizontal road.

(A) How much energy is gained by the cyclist and bicycle? [2]

(B) The cyclist travels 12 m during this acceleration. What is the average driving force on the bicycle? [2]

(ii) While exerting no driving force, the cyclist free-wheels down a hill. Her speed increases from $4\,\text{m}\,\text{s}^{-1}$ to $10\,\text{m}\,\text{s}^{-1}$. During this motion, the total work done against friction is 1600 J and the drop in vertical height is h m.

Without assuming that the hill is uniform in either its angle or roughness, calculate h. [5]

(iii) The cyclist reaches another horizontal stretch of road and there is now a constant resistance to motion of 40 N.

(A) When the power of the driving force on the bicycle is a constant 200 W, what constant speed can the cyclist maintain? [3]

(B) Find the power of the driving force on the bicycle when travelling at a speed of $0.5\,\text{m}\,\text{s}^{-1}$ with an acceleration of $2\,\text{m}\,\text{s}^{-2}$. [5]

3

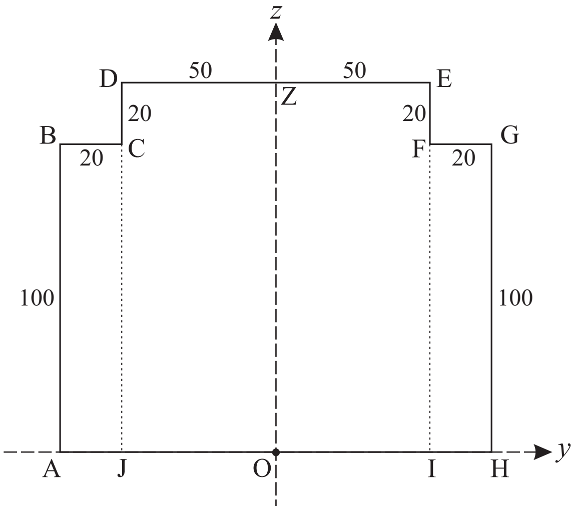

Fig. 3.1

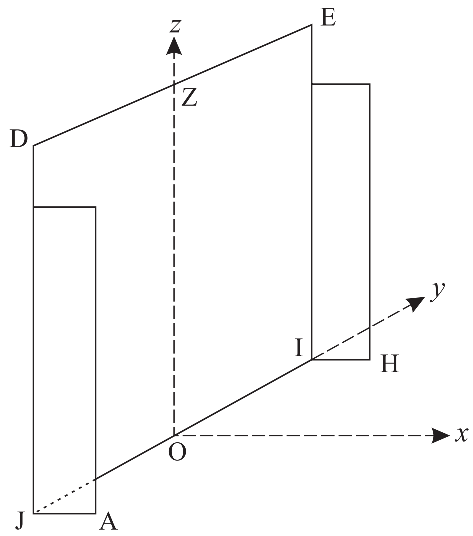

Fig. 3.2

A lamina is made from uniform material in the shape shown in Fig. 3.1. BCJA, DZOJ, ZEIO and FGHI are all rectangles. The lengths of the sides are shown in centimetres.

(i) Find the coordinates of the centre of mass of the lamina, referred to the axes shown in Fig. 3.1. [5]

The rectangles BCJA and FGHI are folded through $90°$ about the lines CJ and FI respectively to give the fire-screen shown in Fig. 3.2.

(ii) Show that the coordinates of the centre of mass of the fire-screen, referred to the axes shown in Fig. 3.2, are (2.5, 0, 57.5). [4]

The x- and y-axes are in a horizontal floor. The fire-screen has a weight of 72 N. A horizontal force P N is applied to the fire-screen at the point Z. This force is perpendicular to the line DE in the **positive** x direction. The fire-screen is on the point of tipping about the line AH.

(iii) Calculate the value of P. [5]

The coefficient of friction between the fire-screen and the floor is μ.

(iv) For what values of μ does the fire-screen slide before it tips? [4]

4 Fig. 4.1 shows a uniform beam, CE, of weight 2200 N and length 4.5 m. The beam is freely pivoted on a fixed support at D and is supported at C. The distance CD is 2.75 m.

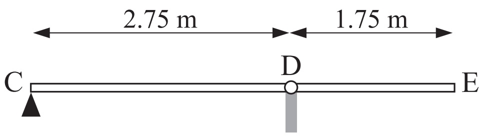

Fig. 4.1

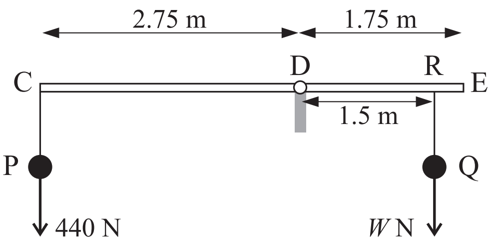

Fig. 4.2

The beam is horizontal and in equilibrium.

(i) Show that the anticlockwise moment of the weight of the beam about D is 1100 N m.

Find the value of the normal reaction on the beam of the support at C. [6]

The support at C is removed and spheres at P and Q are suspended from the beam by light strings attached to the points C and R. The sphere at P has weight 440 N and the sphere at Q has weight W N. The point R of the beam is 1.5 m from D. This situation is shown in Fig. 4.2.

(ii) The beam is horizontal and in equilibrium. Show that $W = 1540$. [3]

The sphere at P is changed for a lighter one with weight 400 N. The sphere at Q is unchanged. The beam is now held in equilibrium at an angle of 20° to the horizontal by means of a light rope attached to the beam at E. This situation (but without the rope at E) is shown in Fig. 4.3.

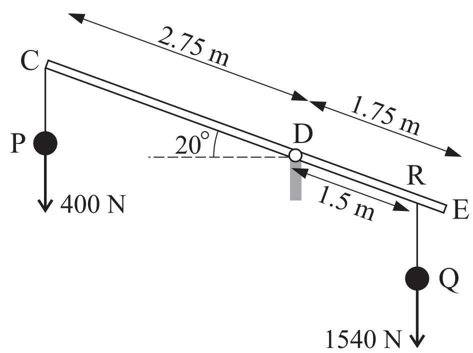

Fig. 4.3

(iii) Calculate the tension in the rope when it is

(A) at 90° to the beam, [6]

(B) horizontal. [3]

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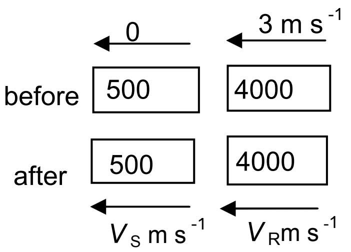
Q1		Mark	**Comment**	**Sub**
(a) (i)	**either** In direction of the force $I = Ft = mv$ so $1500 \times 8 = 4000v$ giving $v = 3$ so 3 m s^{-1} **or** N2L gives $a = \frac{1500}{4000}$ $v = 0 + \frac{1500}{4000} \times 8$ giving $v = 3$ so 3 m s^{-1}	 M1 A1 A1 M1 A1 A1	 Use of $Ft = mv$ Appropriate use of N2L **and** *uvast*	3
(ii)	 PCLM $12000 = 4000V_R + 500V_S$ so $24 = 8V_R + V_S$ NEL $\frac{V_S - V_R}{0 - 3} = -0.2$ so $V_S - V_R = 0.6$ Solving $V_R = 2.6,\ V_S = 3.2$ so ram 2.6 m s^{-1} and stone 3.2 m s^{-1}	M1 A1 M1 A1 A1 F1	Appropriate use of PCLM Any form Appropriate use of NEL Any form Either value	6
(iii)	$0.5 \times 4000 \times 3^2 - 0.5 \times 4000 \times 2.6^2 - 0.5 \times 500 \times 3.2^2$ $= 1920$ J	M1 B1 A1	Change in KE. Accept two terms Any relevant KE term correct (FT their speeds) cao	3
(b)	see over			

1		Mark	Comment	Sub
(b) (i)	72**i** N s $8(9\cos 60\mathbf{i}+9\sin 60\mathbf{j})$ = ($36\mathbf{i}+36\sqrt{3}\mathbf{j}$) N s	B1 E1	Neglect units but must include direction Evidence of use of 8 kg , 9 m s^{-1} and 60°	2
(ii)	$72\mathbf{i}+(36\mathbf{i}+36\sqrt{3}\mathbf{j})=12(u\mathbf{i}+v\mathbf{j})$ Equating components 72 + 36 = 12*u* so *u* = 9 $36\sqrt{3}=12v$ so $v=3\sqrt{3}$	M1 M1 A1	PCLM. Must be momenta both sides Both	3
(iii)	**either** $4\times 18=8\times 9$ so equal momenta so 60/2 = 30° **or** $\arctan\left(\frac{3\sqrt{3}}{9}\right)=\arctan\left(\frac{1}{\sqrt{3}}\right)=30°$	 M1 A1 M1 A1	 Must be clear statements cao FT **their** *u* and *v*. cao	2
		19		

Q 2		Mark	Comment	Sub
(i) (A)	$0.5\times 80\times 3^2=360$ J	M1 A1	Use of KE	2
(B)	$360=F\times 12$ so *F* = 30 so 30 N	M1 F1	*W* = *Fd* attempted FT **their** WD	2
(ii)	Using the WE equation $0.5\times 80\times 10^2-0.5\times 80\times 4^2$ $=80\times 9.8\times h-1600$ *h* = 6.32653… so 6.33 (3 s. f.)	M1 M1 B1 A1 A1	Attempt to use the WE equation. Condone one missing term ΔKE attempted 1600 with correct sign All terms present and correct (neglect signs) cao	5
(iii) (A)	We have driving force *F* = 40 so 200 = 40*v* and *v* = 5 so 5 m s^{-1}	B1 M1 A1	May be implied Use of *P* = *Fv*	3
(B)	From N2L, force required to give accn is $F-40=80\times 2$ so *F* = 200 *P* = 200×0.5 = 100 so 100 W	M1 A1 A1 M1 A1	Use of N2L with all terms present (neglect signs) All terms correct correct use of P = Fv cao	5
		17		

Q 3		Mark	Comment	Sub
(i)	For $\bar{z}$ $(2\times20\times100+2\times50\times120)\bar{z}$ $=2\times2000\times50+2\times6000\times60$ so $\bar{z}$ = 57.5 and $\bar{y}$ = 0	M1 B1 B1 A1 B1	Method for c.m. Total mass of 16000 (or equivalent) At least one term correct NB This result is given below. NB This result is given below. Statement (or proof) required. N.B. If incorrect axes specified, award max 4/5	5
(ii)	$\bar{y}$ and $\bar{z}$ are not changed with the folding For $\bar{x}$ $100\times120\times0+2\times20\times100\times10=16000\bar{x}$ so $\bar{x}=\frac{40000}{16000}=2.5$	E1 M1 B1 E1	A statement, calculation or diagram required. Method for the c.m. with the folding Use of the 10 Clearly shown	4
(iii)	Moments about AH. Normal reaction acts through this line c.w. $P\times120-72\times(20-2.5)=0$ so P = 10.5	M1 B1 B1 A1 A1	May be implied by diagram or statement $20-2.5$ or equivalent All correct cao	5
(iv)	$F_{\max}=\mu R$ so $F_{\max}=72\mu$ For slipping before tipping we require $72\mu<10.5$ so $\mu<0.1458333...\ (7/48)$	M1 A1 M1 A1	Allow $F=\mu R$ Must have clear indication that this is max F Accept $\le$. Accept **their** $F_{\max}$ and *R*. cao	4
		18		

Q 4		Mark	Comment	Sub
(i)	Centre of CE is 0.5 m from D a.c. moment about D $2200 \times 0.5 = 1100$ so 1100 N m c.w moments about D $R \times 2.75 - 1100 = 0$ R = 400 so 400 N	B1 M1 E1 M1 B1 A1	Used below correctly Use of **their** 0.5 0.5 must be clearly established. Use of moments about D in an equation Use of 1100 and 2.75 or equiv	6
(ii)	c.w moments about D $W \times 1.5 - 1100 - 440 \times 2.75 = 0$ so W = 1540	M1 A1 E1	Moments of all relevant forces attempted All correct Some working shown	3
(iii) (*A*)	c.w. moments about D $1.5 \times 1540 \cos 20 - 1.75T$ $-1100 \cos 20 - 400 \times 2.75 \cos 20 = 0$ T = 59.0663… so 59.1 N (3 s. f.)	M1 M1 A1 B1 A1 A1	Moments equation. Allow one missing term; there must be some attempt at resolution. At least one res attempt with correct length Allow sin $\leftrightarrow$ cos Any two of the terms have cos 20 correctly used (or equiv) 1.75 *T* All correct cao Accept no direction given	6
(iii) (*B*)	**either** Angle required is at 70° to the normal to CE so $T_1 \cos 70 = 59.0663...$ so $T_1 = 172.698...$ so 173 N (3 s.f.) **or** $400 \cos 20 \times 2.75 + 1100 \cos 20$ $= 1540 \cos 20 \times 1.5 - T \sin 20 \times 1.75$ T = 172.698… so 173 N (3s.f.)	 B1 M1 A1 M1 A1 A1	 FT (iii) (A) Moments attempted with all terms present All correct (neglect signs) FT(iii)(A)	3
		18		