OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

4761/01

MATHEMATICS (MEI)

Mechanics 1

THURSDAY 17 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 A cyclist starts from rest and takes 10 seconds to accelerate at a constant rate up to a speed of $15\,\mathrm{m\,s^{-1}}$. After travelling at this speed for 20 seconds, the cyclist then decelerates to rest at a constant rate over the next 5 seconds.

(i) Sketch a velocity-time graph for the motion. [3]

(ii) Calculate the distance travelled by the cyclist. [3]

2 The force acting on a particle of mass 1.5 kg is given by the vector $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$ N.

(i) Give the acceleration of the particle as a vector. [2]

(ii) Calculate the angle that the acceleration makes with the direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. [2]

(iii) At a certain point of its motion, the particle has a velocity of $\begin{pmatrix} -2 \\ 3 \end{pmatrix}\,\mathrm{m\,s^{-1}}$. Calculate the displacement of the particle over the next two seconds. [3]

3

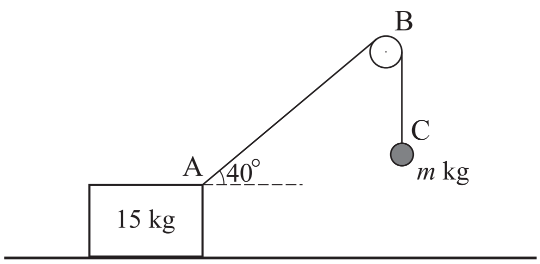

Fig. 3

Fig. 3 shows a block of mass 15 kg on a rough, horizontal plane. A light string is fixed to the block at A, passes over a smooth, fixed pulley B and is attached at C to a sphere. The section of the string between the block and the pulley is inclined at 40° to the horizontal and the section between the pulley and the sphere is vertical.

The system is in equilibrium and the tension in the string is 58.8 N.

(i) The sphere has a mass of m kg. Calculate the value of m. [2]

(ii) Calculate the frictional force acting on the block. [3]

(iii) Calculate the normal reaction of the plane on the block. [3]

4 Force **F** is $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ N and force **G** is $\begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix}$ N.

(i) Find the resultant of **F** and **G** and calculate its magnitude. [4]

(ii) Forces **F**, 2**G** and **H** act on a particle which is in equilibrium. Find **H**. [3]

5

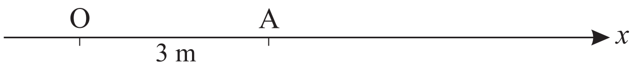

Fig. 5

A toy car is moving along the straight line Ox, where O is the origin. The time t is in seconds. At time $t = 0$ the car is at A, 3 m from O as shown in Fig. 5. The velocity of the car, $v\,\mathrm{m\,s^{-1}}$, is given by

$$v = 2 + 12t - 3t^2.$$

Calculate the distance of the car from O when its acceleration is zero. [8]

Section B (36 marks)

6 A helicopter rescue activity at sea is modelled as follows. The helicopter is stationary and a man is suspended from it by means of a vertical, light, inextensible wire that may be raised or lowered, as shown in Fig. 6.1.

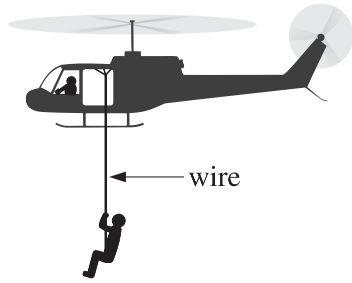

Fig. 6.1

(i) When the man is descending with an acceleration $1.5\,\mathrm{m\,s^{-2}}$ downwards, how much time does it take for his speed to increase from $0.5\,\mathrm{m\,s^{-1}}$ downwards to $3.5\,\mathrm{m\,s^{-1}}$ downwards?

How far does he descend in this time? [4]

The man has a mass of 80 kg. All resistances to motion may be neglected.

(ii) Calculate the tension in the wire when the man is being lowered

(A) with an acceleration of $1.5\,\mathrm{m\,s^{-2}}$ downwards,

(B) with an acceleration of $1.5\,\mathrm{m\,s^{-2}}$ upwards. [5]

Subsequently, the man is raised and this situation is modelled with a constant resistance of 116 N to his upward motion.

(iii) For safety reasons, the tension in the wire should not exceed 2500 N. What is the maximum acceleration allowed when the man is being raised? [4]

At another stage of the rescue, the man has equipment of mass 10 kg at the bottom of a vertical rope which is hanging from his waist, as shown in Fig. 6.2. The man and his equipment are being raised; the rope is light and inextensible and the tension in it is 80 N.

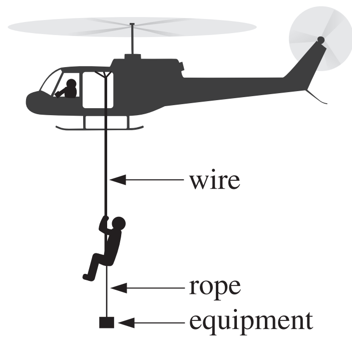

Fig. 6.2

(iv) Assuming that the resistance to the upward motion of the man is still 116 N and that there is negligible resistance to the motion of the equipment, calculate the tension in the wire. [4]

7 A small firework is fired from a point O at ground level over horizontal ground. The highest point reached by the firework is a horizontal distance of 60 m from O and a vertical distance of 40 m from O, as shown in Fig. 7. Air resistance is negligible.

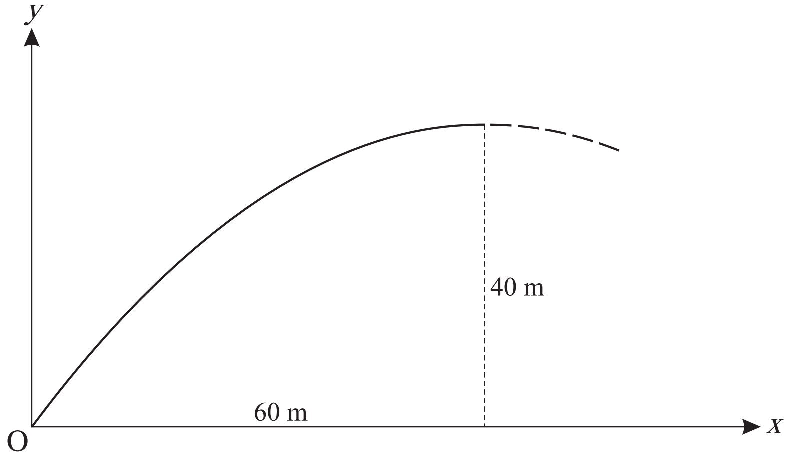

Fig. 7

The initial horizontal component of the velocity of the firework is $21\,\mathrm{m\,s^{-1}}$.

(i) Calculate the time for the firework to reach its highest point and show that the initial vertical component of its velocity is $28\,\mathrm{m\,s^{-1}}$. [4]

(ii) Show that the firework is $(28t - 4.9t^2)$ m above the ground t seconds after its projection. [1]

When the firework is at its highest point it explodes into several parts. Two of the parts initially continue to travel horizontally in the original direction, one with the original horizontal speed of $21\,\mathrm{m\,s^{-1}}$ and the other with a quarter of this speed.

(iii) State why the two parts are always at the same height as one another above the ground and hence find an expression in terms of t for the distance between the parts t seconds after the explosion. [3]

(iv) Find the distance between these parts of the firework

(A) when they reach the ground, [2]

(B) when they are 10 m above the ground. [5]

(v) Show that the cartesian equation of the trajectory of the firework before it explodes is $y = \frac{1}{90}(120x - x^2)$, referred to the coordinate axes shown in Fig. 7. [4]

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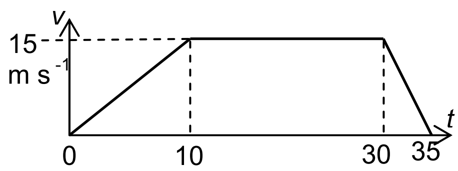
Q 1		Mark	Comment	Sub
(i)		B1 B1 B1	Acc and dec shown as straight lines Horizontal straight section All correct with *v* and times marked and at least one axis labelled. Accept (*t*, *v*) or (*v*, *t*) used.	3
(ii)	Distance is found from the area area is $\frac{1}{2}\times10\times15+20\times15+\frac{1}{2}\times5\times15$ (or $\frac{1}{2}\times(20+35)\times15$) = 412.5 so distance is 412.5 m	M1 A1 A1	At least one area attempted or equivalent *uvast* attempted over one appropriate interval. Award for at least two areas (or equivalent) correct Allow if a trapezium used and only 1 substitution error. FT **their** diagram. cao (Accept 410 or better accuracy)	3
		6		
2 (i)	$\begin{pmatrix}6\\9\end{pmatrix}=1.5\mathbf{a}$ giving $\mathbf{a}=\begin{pmatrix}4\\6\end{pmatrix}$ so $\begin{pmatrix}4\\6\end{pmatrix}$ m s^{-2}	M1 A1	Use of N2L with an attempt to find **a**. Condone spurious notation. Must be a vector in proper form. Penalise only once in paper.	2
(ii)	Angle is $\arctan\left(\frac{6}{4}\right)$ = 56.309… so 56.3° (3 s. f.)	M1 F1	Use of arctan with **their** $\frac{6}{4}$ or $\frac{4}{6}$ or equiv. May use **F**. FT **their a** provided both cpts are +ve and non-zero.	2
(iii)	Using $\mathbf{s}=t\mathbf{u}+0.5t^2\mathbf{a}$ we have $\mathbf{s}=2\begin{pmatrix}-2\\3\end{pmatrix}+0.5\times4\begin{pmatrix}4\\6\end{pmatrix}$ so $\begin{pmatrix}4\\18\end{pmatrix}$ m	M1 A1 A1 7	Appropriate single ***uvast*** (or equivalent sequence of ***uvast***). If integration used twice condone omission of **r**(0) but not **v**(0). FT **their a** only cao. isw for magnitude subsequently found. Vector must be in proper form (penalise only once in paper).	3

Q 3		Mark	Comment	Sub
(i)	$m \times 9.8 = 58.8$ so $m = 6$	M1 A1	$T = mg$. Condone sign error. cao. CWO.	2
(ii)	Resolve $\rightarrow$ $58.8\cos 40 - F = 0$ $F = 45.043\ldots$ so 45.0 N (3 s. f.)	M1 B1 A1	Resolving **their** tension. Accept $s \leftrightarrow c$. Condone sign errors but not extra forces. (**their** T) × $\cos 40$ (or equivalent) seen Accept $\pm$ 45 only.	3
(iii)	Resolve $\uparrow$ $R + 58.8\sin 40 - 15 \times 9.8 = 0$ $R = 109.204\ldots$ so 109 N (3 s. f.)	M1 A1 A1	Resolving **their** tension. All forces present. No extra forces. Accept $s \leftrightarrow c$. Condone errors in sign. All correct cao	3
		8		
Q 4		**Mark**	**Comment**	**Sub**
(i)	Resultant is $\begin{pmatrix}4\\1\\2\end{pmatrix} + \begin{pmatrix}-6\\2\\4\end{pmatrix} = \begin{pmatrix}-2\\3\\6\end{pmatrix}$ Magnitude is $\sqrt{(-2)^2 + 3^2 + 6^2} = \sqrt{49} = 7$ N	M1 A1 M1 F1	Adding the vectors. Condone spurious notation. Vector must be in proper form (penalise only once in the paper). Accept clear components. Pythagoras on **their** 3 component vector. Allow e.g. – 2² for (– 2)² even if evaluated as – 4. FT **their** resultant.	4
(ii)	**F** + 2**G** + **H** = **0** So **H** = - 2**G** - **F** = $-\begin{pmatrix}-12\\4\\8\end{pmatrix} - \begin{pmatrix}4\\1\\2\end{pmatrix}$ = $\begin{pmatrix}8\\-5\\-10\end{pmatrix}$	M1 A1 A1	Either **F** + 2**G** + **H** = **0** or **F** + 2**G** = **H** Must see attempt at **H** = – 2 **G** – **F** cao. Vector must be in proper form (penalise only once in the paper).	3
		7		

Q 5		Mark	Comment	Sub
	$a = 12 - 6t$	M1 A1	Differentiation, at least one term correct.	
	a = 0 gives t = 2	F1	Follow **their** a	
	$x = \int (2 + 12t - 3t^2)\mathrm{d}x$	M1	Integration indefinite or definite, at least one term correct.	
	$2t + 6t^2 - t^3 + C$	A1	Correct. Need not be simplified. Allow as definite integral. Ignore C or limits	
	x = 3 when t = 0	M1	Allow x = ± 3 or argue it is $\int_0^2$ from A then ± 3	
	so $3 = C$ and			
	$x = 2t + 6t^2 - t^3 + 3$	A1	Award if seen WWW or $x = 2t + 6t^2 - t^3$ seen with +3 added later.	
	$x(2) = 4 + 24 - 8 + 3 = 23$ m	B1	FT **their** t and **their** x if obtained by integration but not if -3 obtained instead of +3. [If 20 m seen WWW for displacement award SC6] [Award SC1 for position if constant acceleration used for displacement and then +3 applied]	8
		8		

Q 6		Mark	Comment	Sub
(i)	$3.5 = 0.5 + 1.5T$ so T = 2 so 2 s	M1 A1	Suitable *uvast*, condone sign errors. cao	
	$s = \frac{3.5+0.5}{2} \times 2$ so s = 4 so 4 m	M1 F1	Suitable *uvast*, condone sign errors. FT **their** T. [If s found first then it is cao. In this case when finding T, FT **their** s, if used.]	4
(ii) (A)	N2L ↓ : $80 \times 9.8 - T = 80 \times 1.5$	M1	Use of N2L. Allow weight omitted and use of $F = mga$ Condone errors in sign but do not allow extra forces.	
		B1	weight correct (seen in (A) or (B))	
	T = 664 so 664 N	A1	cao	
(B)	N2L ↓ : $80 \times 9.8 - T = 80 \times (-1.5)$	M1	N2L with all forces and using $F = ma$. Condone errors in sign but do not allow extra forces.	
	T = 904 so 904 N	A1	cao [Accept 904 N seen for M1 A1]	5
(iii)	N2L ↑ : $2500 - 80 \times 9.8 - 116 = 80a$	M1	Use of N2L with $F = ma$. Allow 1 force missing. No extra forces. Condone errors in sign.	
		A1		
	a = 20 so 20 m s^{-2} upwards.	A1	± 20, accept direction wrong or omitted	
		A1	upwards made clear (accept diagram)	4
(iv)	N2L ↑ on equipment: $80 - 10 \times 9.8 = 10a$	M1	Use of N2L on equipment. All forces. $F = ma$. No extra forces. Allow sign errors.	
	$a = -1.8$	A1	Allow ± 1.8	
	N2L ↑	M1	N2L for system or for man alone. Forces correct (with no extras); accept sign errors; **their** ± 1.8 used	
	either all: $T - (80+10) \times 9.8 - 116 = 90 \times (-1.8)$ **or** on man: T – (80×9.8) – 116 – 80 = 80×(– 1.8)			
	T = 836 so 836 N	A1	cao [NB The answer 836 N is independent of the value taken for g and hence may be obtained if all weights are omitted.]	4
		17		

Q 7		Mark	Comment	Sub
(i)	Horiz $21t = 60$ so $\frac{20}{7}$ s (2.8571…) **either** $0 = u - 9.8 \times \frac{20}{7}$ **or** $-u = u - 9.8 \times \left(\frac{40}{7}\right)$ **or** $40 = u \times \frac{20}{7} - 4.9\left(\frac{20}{7}\right)^2$ so u = 28 so 28 m s $^{-1}$	M1 A1 M1 E1	Use of horizontal components and a = 0 or $s = vt - 0.5at^2$ with v = 0. Any form acceptable. Allow M1 A1 for answer seen WW. [If $s = ut + 0.5at^2$ and u = 0 used without justification award M1 A0] [If u = 28 *assumed* to find time then award SC1] Use of $v = u + at$ (or $v^2 = u^2 + 2as$) with v = 0. **or** Use of $v = u + at$ with $v = -u$ and appropriate t. **or** Use of $s = ut + 0.5at^2$ with s = 40 and appropriate t Condone sign errors and, where appropriate, $u \leftrightarrow v$. Accept signs not clear but not errors. Enough working must be given for 28 to be properly shown. [NB u = 28 may be found first and used to find time]	4
(ii)	$y = 28t - 0.5 \times 9.8t^2$	E1	*Clear & convincing* use of $g = -9.8$ in $s = ut + 0.5at^2$ or $s = vt - 0.5at^2$ **NB: AG**	1
(iii)	Start from same height with same (zero) vertical speed at same time, same acceleration Distance apart is $0.75 \times 21t = 15.75t$	E1 M1 A1	For two of these reasons 0.75×21t seen **or** 21t and 5.25t both seen with intention to subtract. Need simplification - LHS alone insufficient. CWO.	3
(iv) (A)	**either** Time is $\frac{20}{7}$ s by symmetry so $15.75 \times \frac{20}{7} = 45$ so 45 m **or** Hit ground at same time. By symmetry one travels 60 m so the other travels 15 m in this time ($\frac{1}{4}$ speed) so 45 m.	B1 B1 B1 B1	Symmetry or *uvast* FT their (iii) with $t = \frac{20}{7}$ [SC1 if 90 m seen]	2
(B)	see next page			

Q7	continued			
(B)			[SC1 if **either** and **or** methods mixed to give $\pm 30 = 28t - 4.9t^2$ or $\pm 10 = 4.9t^2$]	
	either Time to fall is $40 - 10 = 0.5 \times 9.8 \times t^2$	M1 A1 A1	Considering time from explosion with u = 0. Condone sign errors. LHS. Allow ± 30 All correct	
	t = 2.47435…	A1	cao	
	need $15.75 \times 2.47435.. = 38.971..$ so 39.0 (3sf)	F1	FT **their** (iii) only.	
	or Need time so $10 = 28t - 4.9t^2$	M1	Equating $28t - 4.9t^2 = \pm 10$	
	$4.9t^2 - 28t + 10 = 0$ so $t = \frac{28 \pm \sqrt{28^2 - 4 \times 4.9 \times 10}}{9.8}$	M1*	Dep. Attempt to solve quadratic by a method that could give two roots.	
	so 0.382784… or 5.33150…	A1	Larger root correct to at least 2 s. f. Both method marks may be implied from two correct roots alone (to at least 1 s. f.). [SC1 for either root seen WW]	
	Time required is 5.33150… $-\frac{20}{7}$ = 2.47435..	M1		
	need $15.75 \times 2.47435.. = 38.971..$ so 39.0 (3sf)	F1	FT **their** (iii) only.	5
(v)	Horiz $(x =)\ 21t$	B1		
	Elim t between $x = 21t$ and $y = 28t - 4.9t^2$	M1	Intention must be clear, with some attempt made.	
	so $y = 28\left(\frac{x}{21}\right) - 4.9\left(\frac{x}{21}\right)^2$	A1	t completely and correctly eliminated from their expression for x and correct y. Only accept wrong notation if subsequently explicitly given correct value e.g. $\frac{x^2}{21}$ *seen* as $\frac{x^2}{441}$.	
	so $y = \frac{4x}{3} - \frac{0.1x^2}{9} = \frac{1}{90}\left(120x - x^2\right)$	E1	Some simplification must be shown. [SC2 for 3 points shown to be on the curve. Award more only if it is made clear that (a) trajectory is a parabola (b) 3 points define a parabola]	4
		19		