OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4751/01**

MATHEMATICS (MEI)

Introduction to Advanced Mathematics (C1)

WEDNESDAY 9 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

WARNING

You are not allowed to use a calculator in this paper.

This document consists of **4** printed pages.

Section A (36 marks)

1 Make v the subject of the formula $E = \frac{1}{2}mv^2$. [3]

2 Factorise and hence simplify $\dfrac{3x^2 - 7x + 4}{x^2 - 1}$. [3]

3 **(i)** Write down the value of $\left(\frac{1}{4}\right)^0$. [1]

(ii) Find the value of $16^{-\frac{3}{2}}$. [3]

4 Find, algebraically, the coordinates of the point of intersection of the lines $y = 2x - 5$ and $6x + 2y = 7$. [4]

5 **(i)** Find the gradient of the line $4x + 5y = 24$. [2]

(ii) A line parallel to $4x + 5y = 24$ passes through the point $(0, 12)$. Find the coordinates of its point of intersection with the x-axis. [3]

6 When $x^3 + kx + 7$ is divided by $(x - 2)$, the remainder is 3. Find the value of k. [3]

7 **(i)** Find the value of ${}^8C_3$. [2]

(ii) Find the coefficient of x^3 in the binomial expansion of $\left(1 - \frac{1}{2}x\right)^8$. [2]

8 **(i)** Write $\sqrt{48} + \sqrt{3}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. [2]

(ii) Simplify $\dfrac{1}{5 + \sqrt{2}} + \dfrac{1}{5 - \sqrt{2}}$. [3]

9 **(i)** Prove that 12 is a factor of $3n^2 + 6n$ for all even positive integers n. [3]

(ii) Determine whether 12 is a factor of $3n^2 + 6n$ for all positive integers n. [2]

Section B (36 marks)

10 **(i)**

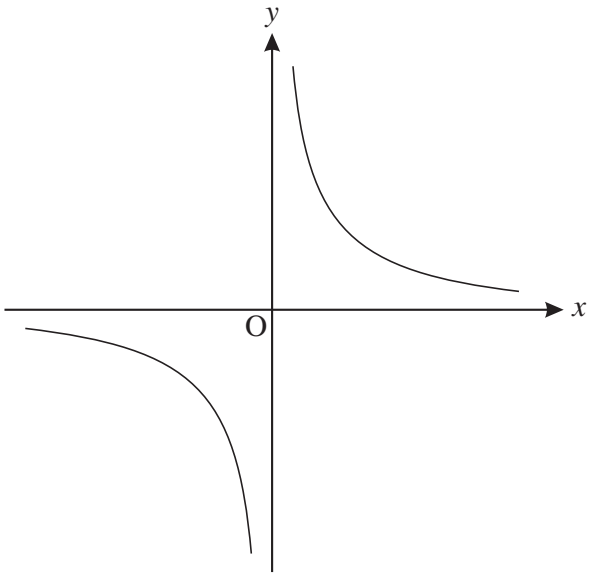

Fig. 10

Fig. 10 shows a sketch of the graph of $y = \frac{1}{x}$.

Sketch the graph of $y = \frac{1}{x-2}$, showing clearly the coordinates of any points where it crosses the axes. [3]

(ii) Find the value of x for which $\frac{1}{x-2} = 5$. [2]

(iii) Find the x-coordinates of the points of intersection of the graphs of $y = x$ and $y = \frac{1}{x-2}$. Give your answers in the form $a \pm \sqrt{b}$.

Show the position of these points on your graph in part **(i)**. [6]

11 **(i)** Write $x^2 - 5x + 8$ in the form $(x - a)^2 + b$ and hence show that $x^2 - 5x + 8 > 0$ for all values of x. [4]

(ii) Sketch the graph of $y = x^2 - 5x + 8$, showing the coordinates of the turning point. [3]

(iii) Find the set of values of x for which $x^2 - 5x + 8 > 14$. [3]

(iv) If $f(x) = x^2 - 5x + 8$, does the graph of $y = f(x) - 10$ cross the x-axis? Show how you decide. [2]

[Question 12 is printed overleaf.]

12 A circle has equation $x^2 + y^2 - 8x - 4y = 9$.

(i) Show that the centre of this circle is C (4, 2) and find the radius of the circle. [3]

(ii) Show that the origin lies inside the circle. [2]

(iii) Show that AB is a diameter of the circle, where A has coordinates (2, 7) and B has coordinates (6, −3). [4]

(iv) Find the equation of the tangent to the circle at A. Give your answer in the form $y = mx + c$. [4]

4751 (C1) Introduction to Advanced Mathematics

Section A

1	$[v =][\pm]\sqrt{\frac{2E}{m}}$ www	3	M2 for $v^2 = \frac{2E}{m}$ or for $[v =][\pm]\sqrt{\frac{E}{\frac{1}{2}m}}$ or M1 for a correct constructive first step and M1 for $v = [\pm]\sqrt{k}$ ft their $v^2 = k$; if M0 then SC1 for $\sqrt{E}/\ ½\ m$ or $\sqrt{2E}/m$ etc	3
2	$\frac{3x-4}{x+1}$ or $3-\frac{7}{x+1}$ www as final answer	3	M1 for $(3x - 4)(x - 1)$ and M1 for $(x + 1)(x - 1)$	3
3	(i) 1 (ii) 1/64 www	1 3	 M1 for dealing correctly with each of reciprocal, square root and cubing (allow 3 only for 1/64) eg M2 for 64 or −64 or $1/\sqrt{4096}$ or $¼^3$ or M1 for $1/16^{3/2}$ or 4^3 or -4^3 or 4^{-3} etc	4
4	$6x + 2(2x - 5) = 7$ $10x = 17$ $x = 1.7$ o.e. isw $y = -1.6$ o.e .isw	M1 M1 A1 A1	for subst or multn of eqns so one pair of coeffts equal (condone one error) simplification (condone one error) or appropriate addn/subtn to eliminate variable allow as separate or coordinates as requested graphical soln: M0	4
5	(i) −4/5 or −0.8 o.e. (ii) (15, 0) or 15 found www	2 3	M1 for 4/5 or 4/−5 or 0.8 or −4.8/6 or correct method using two points on the line (at least one correct) (may be graphical) or for $-0.8x$ o.e. M1 for y = their (i) $x + 12$ o.e. or $4x + 5y = k$ and (0, 12) subst and M1 for using $y = 0$ eg $-12 = -0.8x$ or ft their eqn <u>or</u> M1 for given line goes through (0, 4.8) and (6, 0) and M1 for $6 \times 12/4.8$ graphical soln: allow M1 for correct required line drawn and M1 for answer within 2mm of (15, 0)	5

19

6	f(2) used $2^3 + 2k + 7 = 3$ $k = -6$	M1 M1 A1	or division by $x - 2$ as far as $x^2 + 2x$ obtained correctly or remainder 3 = 2(4 + k) + 7 o.e. 2nd M1 dep on first	3
7	(i) 56 (ii) −7 or ft from −their (i)/8	2 2	M1 for $\frac{8\times7\times6}{3\times2\times1}$ or more simplified M1 for 7 or ft their (i)/8 or for 56 × (−1/2)^3 o.e. or ft; condone x^3 in answer or in M1 expression; 0 in qn for just Pascal's triangle seen	4
8	(i) $5\sqrt{3}$ (ii) common denominator = $(5 - \sqrt{2})(5 + \sqrt{2})$ =23 numerator = 10	2 M1 A1 B1	M1 for $\sqrt{48} = 4\sqrt{3}$ allow M1A1 for $\frac{5-\sqrt{2}}{23} + \frac{5+\sqrt{2}}{23}$ allow 3 only for 10/23	5
9	(i) $n = 2m$ $3n^2 + 6n = 12m^2 + 12m$ or $= 12m(m + 1)$ (ii) showing false when n is odd e.g. $3n^2 + 6n$ = odd + even = odd	M1 M2 B2	or any attempt at generalising; M0 for just trying numbers or M1 for $3n^2 + 6n = 3n(n + 2) = 3 \times$ even × even and M1 for explaining that 4 is a factor of even × even or M1 for 12 is a factor of $6n$ when n is even and M1 for 4 is a factor of n^2 so 12 is a factor of $3n^2$ or $3n(n + 2)$ =3 × odd × odd = odd or counterexample showing not always true; M1 for false with partial explanation or incorrect calculation	5

17

Section B

10	**i**	correct graph with clear asymptote $x = 2$ (though need not be marked)	G2	G1 for one branch correct; condone $(0, -½)$ not shown SC1 for both sections of graph shifted two to left		
		$(0, -½)$ shown	G1	allow seen calculated	3	
	ii	11/5 or 2.2 o.e. isw	2	M1 for correct first step	2	
	iii	$x = \frac{1}{x-2}$	M1	or equivs with ys		
		$x(x-2) = 1$ o.e.	M1			
		$x^2 - 2x - 1$ [= 0]; ft their equiv eqn	M1	or $(x-1)^2 - 1 = 1$ o.e.		
		attempt at quadratic formula	M1	or $(x-1) = \pm\sqrt{2}$ (condone one error)		
		$1 \pm \sqrt{2}$ cao	A1			
		position of points shown	B1	on their curve with $y = x$ (line drawn or $y = x$ indicated by both coords); condone intent of diagonal line with gradient approx 1through origin as $y = x$ if unlabelled	6	11
11	**i**	$(x-2.5)^2$ o.e.	M1			
		$-2.5^2 + 8$	M1	for clear attempt at -2.5^2		
		$(x-2.5)^2 + 7/4$ o.e.	A1	allow M2A0 for $(x-2.5) + 7/4$ o.e. with no $(x-2.5)^2$ seen		
		min $y = 7/4$ o.e. [so above x axis] or commenting $(x-2.5)^2 \geq 0$	B1	ft, dep on $(x-a)^2 + b$ with b positive; condone starting again, showing $b^2 - 4ac < 0$ or using calculus	4	
	ii	correct symmetrical quadratic shape	G1			
		8 marked as intercept on y axis	G1	or (0, 8) seen in table		
		tp (5/2, 7/4) o.e. or ft from (i)	G1		3	
	iii	$x^2 - 5x - 6$ seen or used	M1	or $(x-2.5)^2$ [> or =] 12.25 or ft $14 - b$		
		−1 and 6 obtained	M1	also implies first M1		
		$x < -1$ and $x > 6$ isw or ft their solns	M1	if M0, allow B1 for one of $x < -1$ and $x > 6$	3	
	iv	min = (2.5, − 8.25) or ft from (i)	M1	or M1 for other clear comment re translated 10 down and A1 for referring to min in (i) or graph in (ii); or M1 for correct method for solving $x^2 - 5x - 2 = 0$ or using $b^2 - 4ac$ with this and A1 for showing real solns eg $b^2 - 4ac = 33$; allow M1A0 for valid comment but error in −8.25 ft; allow M1 for showing y can be neg eg (0, −2) found and A1 for correct conclusion		
		so yes, crosses	A1		2	12

12	**i**	$(x-4)^2 - 16 + (y-2)^2 - 4 = 9$ o.e.	M2	M1 for one completing square or for $(x-4)^2$ or $(y-2)^2$ expanded correctly <u>or</u> starting with $(x-4)^2 + (y-2)^2 = r^2$: M1 for correct expn of at least one bracket and M1 for $9 + 20 = r^2$ o.e.	
		rad = $\sqrt{29}$	B1	<u>or</u> using $x^2 - 2gx + y^2 - 2fy + c = 0$ M1 for using centre is (g, f) [must be quoted] and M1 for $r^2 = g^2 + f^2 - c$	3
	ii	$4^2 + 2^2$ o.e = 20 which is less than 29	M1 A1	allow 2 for showing circle crosses x axis at −1 and 9 or equiv for y (or showing one positive; one negative); 0 for graphical solutions (often using A and B from (iii) to draw circle)	2
	iii	showing midpt of AB = (4, 2) and showing AB = $2\sqrt{29}$ or showing AC or BC = $\sqrt{29}$ or that A or B lie on circle <u>or</u> showing both A and B lie on circle (or AC = BC = $\sqrt{29}$), and showing AB = $2\sqrt{29}$ or that C is midpt of AB or that C is on AB or that gradients of AB and AC are the same or equiv. <u>or</u> showing C is on AB and showing both A and B are on circle or AC = BC = $\sqrt{29}$	2 2 2 2 2 2 2	in each method, two things need to be established. Allow M1 for the concept of what should be shown and A1 for correct completion with method shown allow M1A0 for AB just shown as $\sqrt{116}$ not $2\sqrt{29}$ allow M1A0 for stating mid point of AB = (4,2) without working/method shown NB showing AB = $2\sqrt{29}$ and C lies on AB is not sufficient – earns 2 marks only if M0, allow SC2 for accurate graph of circle drawn with compasses and AB joined with ruled line through C.	4
	iv	grad AC or AB or BC = −5/2 o.e. grad tgt = −1/their grad AC tgt is $y - 7 =$ their $m(x-2)$ o.e. $y = 2/5\,x + 31/5$ o.e.	M1 M1 M1 A1	may be seen in (iii) but only allow this M1 if they go on to use in this part allow for $m_1m_2 = -1$ used eg $y =$ their $mx + c$ then (2, 7) subst; M0 if grad AC used condone $y = 2/5x + c$ and $c = 31/5$ o.e.	4

13