OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT

4776/01

MATHEMATICS (MEI)

Numerical Methods

WEDNESDAY 20 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

Section A (36 marks)

1 Show that the equation $x^2 + \sqrt{1+x} = 3$ has a root in the interval $(1, 1.4)$.

Use the bisection method to obtain an estimate of the root with maximum possible error 0.025.

Determine how many additional iterations of the bisection process would be required to reduce the maximum possible error to less than 0.005. [8]

2 For the integral $\int_0^{0.5} \frac{1}{1+x^4}\,dx$, find the values given by the trapezium rule and the mid-point rule, taking $h = 0.5$ in each case.

Hence show that the Simpson's rule estimate with $h = 0.25$ is 0.493 801.

You are now given that the Simpson's rule estimate with $h = 0.125$ is 0.493 952. Use extrapolation to determine the value of the integral as accurately as you can. [8]

3 A triangle has sides a, 3 and 4. The angle opposite side a is $(90 + \varepsilon)^\circ$, where ε is small. See Fig. 3.

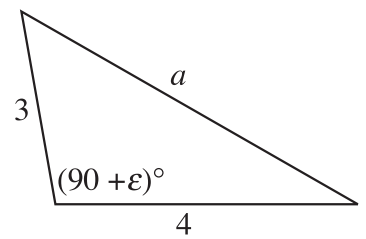

Fig. 3

Use the cosine rule to calculate a when $\varepsilon = 5$.

The approximation

$$\cos\,(90 + \varepsilon)^\circ \approx -\frac{\pi\varepsilon}{180}$$

with $\varepsilon = 5$ is now used in the cosine rule to find an approximate value for a.

Find the absolute and relative errors in this approximate value of a. [5]

4 The number x is represented in a computer program by the approximation X. You are given that $X = x(1 + r)$ where r is small.

(i) State what r represents. [1]

(ii) Use the first two terms in a binomial expansion to show that the relative error in X^n as an approximation to x^n is approximately nr. [2]

(iii) A lazy programmer has approximated π by $\frac{22}{7}$. Find the relative error in this approximation.

Use the result in part **(ii)** to write down the approximate relative errors in the values of π^2 and $\sqrt{\pi}$ when π is taken as $\frac{22}{7}$. [5]

5 The function $f(x)$ has the values shown in the table.

x	–1	0	4
$f(x)$	3	2	9

Use Lagrange's interpolation method to obtain the quadratic function that fits the three data points.

Hence estimate the value of x for which $f(x)$ takes its minimum value. [7]

Section B (36 marks)

6 **(i)** Explain, with the aid of a sketch, the principle underlying the Newton-Raphson method for the solution of the equation $f(x) = 0$. [3]

(ii) Draw a sketch of the function $f(x) = \tan x - 2x$ for $0 \leqslant x < \frac{1}{2}\pi$ (x in radians). Mark on your sketch the non-zero root, α, of the equation $\tan x - 2x = 0$. Show by means of your sketch that, for some starting values, the Newton-Raphson method will fail to converge to α. Identify two distinct cases that can arise. [6]

(iii) Given that the derivative of $\tan x$ is $1 + \tan^2 x$, show that the Newton-Raphson iteration for the solution of the equation $\tan x - 2x = 0$ is

$$x_{r+1} = x_r - \frac{(\tan x_r - 2x_r)}{(\tan^2 x_r - 1)}.$$

Use this iteration with $x_0 = 1.2$ to determine α correct to 4 decimal places.

Show carefully that this iteration is faster than first order. [9]

[Question 7 is printed overleaf.]

7 The function $g(x)$ has the values shown in the table.

x	$g(x)$
1	2.87
2	4.73
3	6.23
4	7.36
5	8.05

(i) Draw up a difference table for $g(x)$ as far as second differences. State with a reason whether or not $g(x)$ is quadratic. [5]

(ii) Draw up another difference table, based this time on $x = 1, 3, 5$. Use Newton's forward difference formula to find the quadratic approximation to $g(x)$ based on these three points. Simplify the coefficients of this quadratic. [8]

(iii) Find the absolute and relative errors when this quadratic is used to estimate $g(2)$ and $g(4)$. [5]