OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4757/01**

MATHEMATICS (MEI)

Further Applications of Advanced Mathematics (FP3)

THURSDAY 14 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

Option 1: Vectors

1 Three planes P, Q and R have the following equations.

$$\begin{aligned} \text{Plane } P\text{:} \quad & 8x - y - 14z = 20 \\ \text{Plane } Q\text{:} \quad & 6x + 2y - 5z = 26 \\ \text{Plane } R\text{:} \quad & 2x + y - z = 40 \end{aligned}$$

The line of intersection of the planes P and Q is K.

The line of intersection of the planes P and R is L.

(i) Show that K and L are parallel lines, and find the shortest distance between them. [9]

(ii) Show that the shortest distance between the line K and the plane R is $5\sqrt{6}$. [3]

The line M has equation $\mathbf{r} = (\mathbf{i} - 4\mathbf{j}) + \lambda(5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$.

(iii) Show that the lines K and M intersect, and find the coordinates of the point of intersection. [7]

(iv) Find the shortest distance between the lines L and M. [5]

Option 2: Multi-variable calculus

2 A surface has equation $z = xy^2 - 4x^2y - 2x^3 + 27x^2 - 36x + 20$.

(i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. [3]

(ii) Find the coordinates of the four stationary points on the surface, showing that one of them is $(2, 4, 8)$. [8]

(iii) Sketch, on separate diagrams, the sections of the surface defined by $x = 2$ and by $y = 4$. Indicate the point $(2, 4, 8)$ on these sections, and deduce that it is neither a maximum nor a minimum. [6]

(iv) Show that there are just two points on the surface where the normal line is parallel to the vector $36\mathbf{i} + \mathbf{k}$, and find the coordinates of these points. [7]

Option 3: Differential geometry

3 The curve C has equation $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$, and a is a constant with $a \geqslant 1$.

(i) Show that the length of the arc of C for which $1 \leqslant x \leqslant a$ is $\frac{1}{2}a^2 + \frac{1}{4}\ln a - \frac{1}{2}$. [5]

(ii) Find the area of the surface generated when the arc of C for which $1 \leqslant x \leqslant 4$ is rotated through 2π radians **about the *y*-axis**. [5]

(iii) Show that the radius of curvature of C at the point where $x = a$ is $a\left(a + \dfrac{1}{4a}\right)^2$. [5]

(iv) Find the centre of curvature corresponding to the point $(1, \frac{1}{2})$ on C. [5]

C is one member of the family of curves defined by $y = px^2 - p^2 \ln x$, where p is a parameter.

(v) Find the envelope of this family of curves. [4]

[Questions 4 and 5 are printed overleaf.]

Option 4: Groups

4 **(i)** Prove that, for a group of order 10, every proper subgroup must be cyclic. [4]

The set $M = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is a group under the binary operation of multiplication modulo 11.

(ii) Show that M is cyclic. [4]

(iii) List all the proper subgroups of M. [3]

The group P of symmetries of a regular pentagon consists of 10 transformations

$$\{A, B, C, D, E, F, G, H, I, J\}$$

and the binary operation is composition of transformations. The composition table for P is given below.

	A	B	C	D	E	F	G	H	I	J
A	C	J	G	H	A	B	I	F	E	D
B	F	E	H	G	B	A	D	C	J	I
C	G	D	I	F	C	J	E	B	A	H
D	J	C	B	E	D	G	F	I	H	A
E	A	B	C	D	E	F	G	H	I	J
F	H	I	D	C	F	E	J	A	B	G
G	I	H	E	B	G	D	A	J	C	F
H	D	G	J	A	H	I	B	E	F	C
I	E	F	A	J	I	H	C	D	G	B
J	B	A	F	I	J	C	H	G	D	E

One of these transformations is the identity transformation, some are rotations and the rest are reflections.

(iv) Identify which transformation is the identity, which are rotations and which are reflections. [4]

(v) State, giving a reason, whether P is isomorphic to M. [2]

(vi) Find the order of each element of P. [3]

(vii) List all the proper subgroups of P. [4]

Option 5: Markov chains

5 A computer is programmed to generate a sequence of letters. The process is represented by a Markov chain with four states, as follows.

The first letter is A, B, C or D, with probabilities 0.4, 0.3, 0.2 and 0.1 respectively.

After A, the next letter is either C or D, with probabilities 0.8 and 0.2 respectively.

After B, the next letter is either C or D, with probabilities 0.1 and 0.9 respectively.

After C, the next letter is either A or B, with probabilities 0.4 and 0.6 respectively.

After D, the next letter is either A or B, with probabilities 0.3 and 0.7 respectively.

(i) Write down the transition matrix $\mathbf{P}$. [2]

(ii) Use your calculator to find $\mathbf{P}^4$ and $\mathbf{P}^7$. (Give elements correct to 4 decimal places.) [4]

(iii) Find the probability that the 8th letter is C. [2]

(iv) Find the probability that the 12th letter is the same as the 8th letter. [4]

(v) By investigating the behaviour of $\mathbf{P}^n$ when n is large, find the probability that the $(n+1)$th letter is A when

(A) n is a large even number,

(B) n is a large odd number. [4]

The program is now changed. The initial probabilities and the transition probabilities are the same as before, except for the following.

After D, the next letter is A, B or D, with probabilities 0.3, 0.6 and 0.1 respectively.

(vi) Write down the new transition matrix $\mathbf{Q}$. [1]

(vii) Verify that $\mathbf{Q}^n$ approaches a limit as n becomes large, and hence write down the equilibrium probabilities for A, B, C and D. [4]

(viii) When n is large, find the probability that the $(n+1)$th, $(n+2)$th and $(n+3)$th letters are DDD. [3]

BLANK PAGE

BLANK PAGE