RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

MONDAY 11 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

Section A (36 marks)

1 You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$.

(i) Find the inverse of $\mathbf{M}$. [2]

(ii) A triangle of area 2 square units undergoes the transformation represented by the matrix $\mathbf{M}$. Find the area of the image of the triangle following this transformation. [1]

2 Write down the equation of the locus represented by the circle in the Argand diagram shown in Fig. 2. [3]

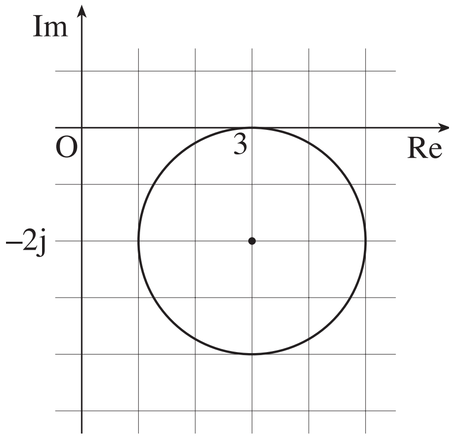

Fig. 2

3 Find the values of the constants A, B, C and D in the identity

$$x^3 - 4 \equiv (x-1)(Ax^2 + Bx + C) + D.$$ [5]

4 Two complex numbers, α and β, are given by $\alpha = 1 - 2\mathrm{j}$ and $\beta = -2 - \mathrm{j}$.

(i) Represent β and its complex conjugate β^* on an Argand diagram. [2]

(ii) Express $\alpha\beta$ in the form $a + b\mathrm{j}$. [2]

(iii) Express $\dfrac{\alpha + \beta}{\beta}$ in the form $a + b\mathrm{j}$. [3]

5 The roots of the cubic equation $x^3 + 3x^2 - 7x + 1 = 0$ are α, β and γ. Find the cubic equation whose roots are 3α, 3β and 3γ, expressing your answer in a form with integer coefficients. [6]

6 **(i)** Show that $\dfrac{1}{r+2} - \dfrac{1}{r+3} = \dfrac{1}{(r+2)(r+3)}$. [2]

(ii) Hence use the method of differences to find $\dfrac{1}{3 \times 4} + \dfrac{1}{4 \times 5} + \dfrac{1}{5 \times 6} + \ldots + \dfrac{1}{52 \times 53}$. [4]

7 Prove by induction that $\displaystyle\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$. [6]

Section B (36 marks)

8 A curve has equation $y = \dfrac{x^2 - 4}{(x-3)(x+1)(x-1)}$.

(i) Write down the coordinates of the points where the curve crosses the axes. [3]

(ii) Write down the equations of the three vertical asymptotes and the one horizontal asymptote. [4]

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. [3]

(iv) Sketch the curve. [4]

9 The cubic equation $x^3 + Ax^2 + Bx + 15 = 0$, where A and B are real numbers, has a root $x = 1 + 2\mathrm{j}$.

(i) Write down the other complex root. [1]

(ii) Explain why the equation must have a real root. [1]

(iii) Find the value of the real root and the values of A and B. [9]

[Question 10 is printed overleaf.]

10 You are given that $\mathbf{A} = \begin{pmatrix} 1 & -2 & k \\ 2 & 1 & 2 \\ 3 & 2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -5 & -2+2k & -4-k \\ 8 & -1-3k & -2+2k \\ 1 & -8 & 5 \end{pmatrix}$ and that $\mathbf{AB}$ is of the form $\mathbf{AB} = \begin{pmatrix} k-n & 0 & 0 \\ 0 & k-n & 0 \\ 0 & 0 & k-n \end{pmatrix}$.

(i) Find the value of n. [2]

(ii) Write down the inverse matrix $\mathbf{A}^{-1}$ and state the condition on k for this inverse to exist. [4]

(iii) Using the result from part **(ii)**, or otherwise, solve the following simultaneous equations.

$$\begin{aligned} x - 2y + z &= 1 \\ 2x + y + 2z &= 12 \\ 3x + 2y - z &= 3 \end{aligned}$$

[5]