OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4761/01**

MATHEMATICS (MEI)

Mechanics 1

MONDAY 21 MAY 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

Section A (36 marks)

1 Fig. 1 shows four forces in equilibrium.

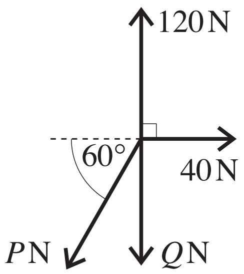

Fig. 1

(i) Find the value of P. [3]

(ii) Hence find the value of Q. [2]

2 A car passes a point A travelling at 10 m s^{-1}. Its motion over the next 45 seconds is modelled as follows.

- The car's speed increases uniformly from 10 m s^{-1} to 30 m s^{-1} over the first 10 s.
- Its speed then increases uniformly to 40 m s^{-1} over the next 15 s.
- The car then maintains this speed for a further 20 s at which time it reaches the point B.

(i) Sketch a speed-time graph to represent this motion. [3]

(ii) Calculate the distance from A to B. [3]

(iii) When it reaches the point B, the car is brought uniformly to rest in T seconds. The total distance from A is now 1700 m. Calculate the value of T. [2]

3 Fig. 3 shows a system in equilibrium. The rod is firmly attached to the floor and also to an object, P. The light string is attached to P and passes over a smooth pulley with an object Q hanging freely from its other end.

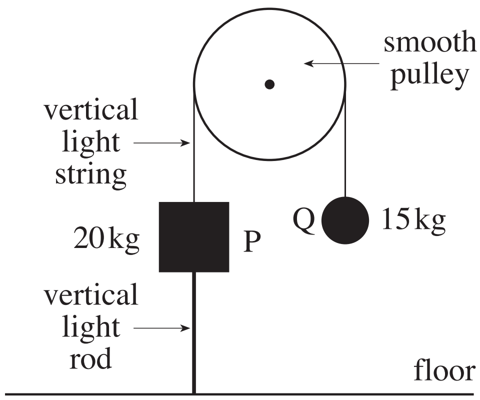

Fig. 3

(i) Why is the tension the same throughout the string? [1]

(ii) Calculate the force in the rod, stating whether it is a tension or a thrust. [3]

4 Two trucks, A and B, each of mass 10 000 kg, are pulled along a straight, horizontal track by a constant, horizontal force of P N. The coupling between the trucks is light and horizontal. This situation and the resistances to motion of the trucks are shown in Fig. 4.

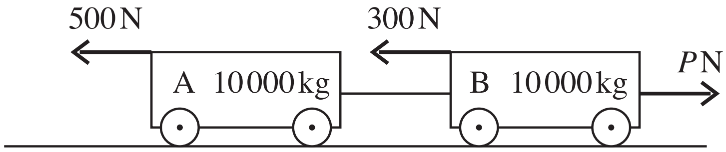

Fig. 4

The acceleration of the system is 0.2 m s^{-2} in the direction of the pulling force of magnitude P.

(i) Calculate the value of P. [3]

Truck A is now subjected to an extra resistive force of 2000 N while P does not change.

(ii) Calculate the new acceleration of the trucks. [2]

(iii) Calculate the force in the coupling between the trucks. [2]

5 A block of weight 100 N is on a rough plane that is inclined at 35° to the horizontal. The block is in equilibrium with a horizontal force of 40 N acting on it, as shown in Fig. 5.

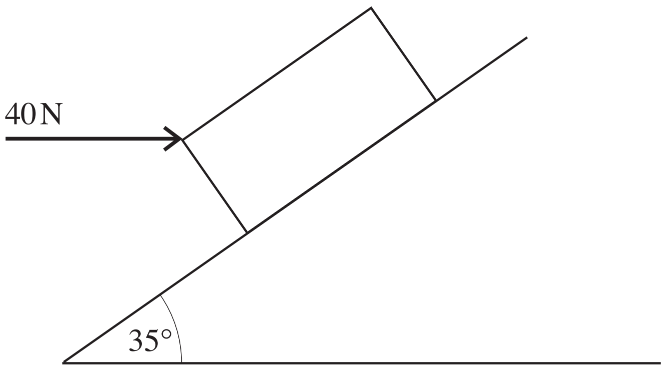

Fig. 5

Calculate the frictional force acting on the block. [4]

6 A rock of mass 8 kg is acted on by just the two forces $-80\mathbf{k}$ N and $(-\mathbf{i} + 16\mathbf{j} + 72\mathbf{k})$ N, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane and $\mathbf{k}$ is a unit vector vertically upward.

(i) Show that the acceleration of the rock is $\left(-\frac{1}{8}\mathbf{i} + 2\mathbf{j} - \mathbf{k}\right) \text{m s}^{-2}$. [2]

The rock passes through the origin of position vectors, O, with velocity $(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$ m s^{-1} and 4 seconds later passes through the point A.

(ii) Find the position vector of A. [3]

(iii) Find the distance OA. [1]

(iv) Find the angle that OA makes with the horizontal. [2]

Section B (36 marks)

7 Fig. 7 is a sketch of part of the velocity-time graph for the motion of an insect walking in a straight line. Its velocity, $v \text{ m s}^{-1}$, at time t seconds for the time interval $-3 \leqslant t \leqslant 5$ is given by

$$v = t^2 - 2t - 8.$$

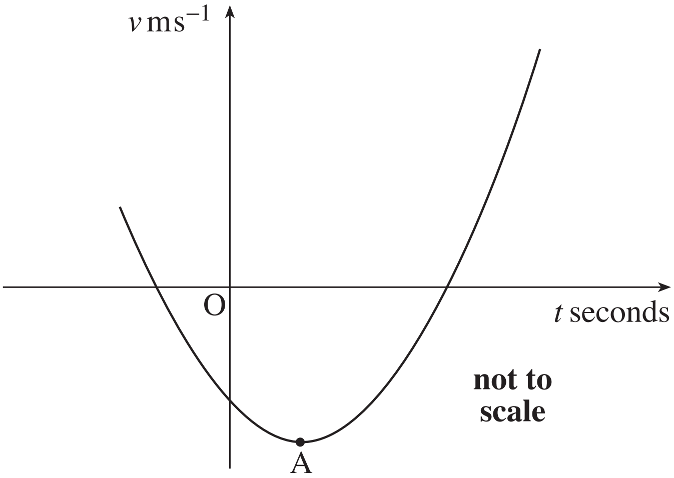

Fig. 7

(i) Write down the velocity of the insect when $t = 0$. [1]

(ii) Show that the insect is instantaneously at rest when $t = -2$ and when $t = 4$. [2]

(iii) Determine the velocity of the insect when its acceleration is zero.

Write down the coordinates of the point A shown in Fig. 7. [5]

(iv) Calculate the distance travelled by the insect from $t = 1$ to $t = 4$. [5]

(v) Write down the distance travelled by the insect in the time interval $-2 \leqslant t \leqslant 4$. [1]

(vi) How far does the insect walk in the time interval $1 \leqslant t \leqslant 5$? [3]

8 A ball is kicked from ground level over horizontal ground. It leaves the ground at a speed of $25\,\text{m s}^{-1}$ and at an angle θ to the horizontal such that $\cos\theta = 0.96$ and $\sin\theta = 0.28$.

(i) Show that the height, y m, of the ball above the ground t seconds after projection is given by $y = 7t - 4.9t^2$. Show also that the horizontal distance, x m, travelled by this time is given by $x = 24t$. [3]

(ii) Calculate the maximum height reached by the ball. [2]

(iii) Calculate the times at which the ball is at half its maximum height.

Find the horizontal distance travelled by the ball between these times. [4]

(iv) Determine the following when $t = 1.25$.

(A) The vertical component of the velocity of the ball.

(B) Whether the ball is rising or falling. (You should give a reason for your answer.)

(C) The speed of the ball. [5]

(v) Show that the equation of the trajectory of the ball is

$$y = \frac{0.7x}{576}(240 - 7x).$$

Hence, or otherwise, find the range of the ball. [5]

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Mark Scheme 4761
June 2007

Q1				
(i)	$\rightarrow\ 40 - P\cos 60 = 0$ $P = 80$	M1 A1 A1	For any resolution in an equation involving *P*. Allow for *P* = 40 cos 60 or *P* = 40 cos 30 or *P* = 40 sin 60 or *P* = 40 sin 30 Correct equation cao	3
(ii)	$\downarrow\ Q + P\cos 30 = 120$ $Q = 40\left(3-\sqrt{3}\right)$ = 50.7179… so 50.7 (3 s. f.)	M1 A1	Resolve vert. All forces present. Allow sin ↔ cos No extra forces. Allow wrong signs. cao	2
				5

Q2				
(i)	Straight lines connecting (0, 10), (10, 30), (25, 40) and (45, 40)	B1 B1 B1	Axes with labels (words or letter). Scales indicated. Accept no arrows. Use of straight line segments and horiz section All correct with salient points clearly indicated	3
(ii)	$0.5(10+30)\times 10 + 0.5(30+40)\times 15 + 40\times 20$ = 200 + 525 + 800 = 1525	M1 M1 A1	Attempt at area(s) or use of appropriate *uvast* Evidence of attempt to find whole area cao	3
(iii)	$0.5\times 40\times T = 1700 - 1525$ so 20*T* = 175 and *T* = 8.75	M1 F1	Equating triangle area to 1700 – **their** (ii) (1700 – **their** (ii))/20. Do not award for – ve answer.	2
				8

Q3				
(i)	String light and pulley smooth	E1	Accept pulley smooth alone	1
(ii)	5*g* (49) N thrust	M1 B1 A1	Three forces in equilibrium. Allow sign errors. for 15*g* (147) N used as a tension 5*g* (49) N thrust. Accept ±5*g* (49). Ignore diagram. [Award SC2 for ±5*g* (49) N without 'thrust' and SC3 if it is]	3
				4

Q4				
(i)	$P - 800 = 20000 \times 0.2$ $P = 4800$	M1 A1 A1	N2L. Allow $F = mga$. Allow wrong or zero resistance. No extra forces. Allow sign errors. If done as 1 equn need m = 20 000. If A and B analysed separately, must have 2 equns with 'T'. N2L correct.	3
(ii)	New accn $4800 - 2800 = 20000a$ $a = 0.1$	M1 A1	$F = ma$. Finding new accn. No extra forces. Allow 500 N but not 300 N omitted. Allow sign errors. FT **their** P	2
(iii)	$T - 2500 = 10000 \times 0.1$ $T = 3500$ so 3500 N	M1 A1	N2L with new a. Mass 10000. All forces present for A or B except allow 500 N omitted on A. No extra forces cao	2
				7

Q5				
	Take F +ve up the plane $F + 40\cos 35 = 100\sin 35$ F = 24.5915… so 24.6 N (3 s. f.) up the plane	 M1 B1 A1 A1	Resolve // plane (or horiz or vert). All forces present. At least one resolved. Allow sin ↔ cos and sign errors. Allow $100g$ used. Either $\pm 40\cos 35$ or $\pm 100\sin 35$ or equivalent seen Accept ± 24.5915… or ±90.1237... even if inconsistent or wrong signs used. 24.6 N up the plane (specified or from diagram) or equiv all obtained from consistent and correct working.	4
				4

Q6				
(i)	$\left(-\mathbf{i}+16\mathbf{j}+72\mathbf{k}\right)+\left(-80\mathbf{k}\right)=8\mathbf{a}$	M1	Use of N2L. All forces present.	
	$\mathbf{a}=\left(-\frac{1}{8}\mathbf{i}+2\mathbf{j}-\mathbf{k}\right)\text{m s}^{-2}$	E1	Need at least the **k** term clearly derived	2
(ii)	$\mathbf{r}=4\left(\mathbf{i}-4\mathbf{j}+3\mathbf{k}\right)+0.5\times 16\left(-\frac{1}{8}\mathbf{i}+2\mathbf{j}-\mathbf{k}\right)$	M1	Use of appropriate **uvas***t* or integration (twice)	
		A1	Correct substitution (or limits if integrated)	
	$=3\mathbf{i}+4\mathbf{k}$	A1		3
(iii)	$\sqrt{3^2+4^2}=5$ so 5 m	B1	FT **their** (ii) even if it not a displacement. Allow surd form	1
(iv)	$\arctan\frac{4}{3}$	M1	Accept $\arctan\frac{3}{4}$. FT **their** (ii) even if not a displacement. Condone sign errors. (May use arcsin4/5 or equivalent. FT **their** (ii) and (iii) even if not displacement. Condone sign errors)	
	= 53.130… so 53.1° (3 s. f.)	A1	cao	2
				8

Q7				
(i)	8 m s^{-1} (in the negative direction)	B1	Allow $\pm$ and no direction indicated	1
(ii)	$(t+2)(t-4)=0$ so $t = -2$ or 4	M1 A1	Equating v to zero and solving or subst If subst used then both must be clearly shown	2
(iii)	$a = 2t - 2$ $a = 0$ when $t = 1$ $v(1) = 1 - 2 - 8 = -9$ so 9 m s^{-1} in the negative direction $(1, -9)$	M1 A1 F1 A1 B1	Differentiating Correct Accept -9 but not 9 without comment FT	5
(iv)	$\int_1^4 \left(t^2 - 2t - 8\right)\,dx$ $= \left[\frac{t^3}{3} - t^2 - 8t\right]_1^4$ $= \left(\frac{64}{3} - 16 - 32\right) - \left(\frac{1}{3} - 1 - 8\right)$ $= -18$ distance is 18 m	M1 A1 M1 A1 A1	Attempt at integration. Ignore limits. Correct integration. Ignore limits. Attempt to sub correct limits and subtract Limits correctly evaluated. Award if -18 seen but no need to evaluate Award even if -18 not seen. Do not award for -18. cao	5
(v)	$2 \times 18 = 36$ m	F1	Award for $2\times$ **their** (iv).	1
(vi)	$\int_4^5 \left(t^2 - 2t - 8\right)\,dx = \left[\frac{t^3}{3} - t^2 - 8t\right]_4^5$ $= \left(\frac{125}{3} - 25 - 40\right) - \left(-\frac{80}{3}\right) = 3\frac{1}{3}$ so $3\frac{1}{3} + 18 = 21\frac{1}{3}$ m	M1 A1 A1	$\int_4^5$ attempted or, otherwise, complete method seen. Correct substitution Award for $3\frac{1}{3}$ + **their** (positive) (iv)	3
				17

Q8				
(i)	$y = 25\sin\theta t + 0.5\times(-9.8)t^2$	M1	Use of $s = ut + \frac{1}{2}at^2$ .Accept sin, cos, 0.96, 0.28, ± 9.8, ± 10, $u = 25$ and derivation of -4.9 not clear.	
	$= 7t - 4.9t^2$	E1	Shown including deriv of – 4.9. Accept $25\sin\theta t = 7t$ WW	
	$x = 25\cos\theta t = 25\times 0.96t = 24t$	B1	Accept $25\times 0.96t$ or $25\cos\theta t$ seen WW	3
(ii)	$0 = 7^2 - 19.6s$	M1	Accept sequence of *uvast*. Accept u=24 but not 25. Allow $u \leftrightarrow v$ and ± 9.8 and ± 10	
	s = 2.5 so 2.5 m	A1	+ve answer obtained by correct manipulation.	2
(iii)	Need $7t - 4.9t^2 = 1.25$ so $4.9t^2 - 7t + 1.25 = 0$	M1	Equate y to **their** (ii)/2 or equivalent.	
		M1	Correct sub into quad formula of their 3 term quadratic being solved (i.e. allow manipulation errors before using the formula).	
	t = 0.209209…. and 1.219361…	A1	Both. cao. [Award M1 A1 for two correct roots WW]	
	need $24\times$ (1.219… - 0.209209…) $= 24\times 1.01...$ so 24.2 m (3 s.f.)	B1	FT **their** roots (only if both positive)	4
(iv) (A)	$\dot{y} = 7 - 9.8t$	M1	Attempt at $\dot{y}$. Accept sign errors and u = 24 but not 25	
	$\dot{y}(1.25) = 7 - 9.8\times 1.25 = -5.25$ m s^{-1}	A1		
(B)	Falling as velocity is negative	E1	Reason must be clear. FT **their** $\dot{y}$ even if not a velocity Could use an argument involving time.	
(C)	Speed is $\sqrt{24^2 + (-5.25)^2}$	M1	Use of Pythag and 24 or 7 with **their** $\dot{y}$	
	= 24.5675… so 24.6 m s^{-1} (3 s. f.)	A1	cao	5

(v)	$y = 7t - 4.9t^2,\ x = 24t$	M1	Elimination of t	
	so $y = \frac{7x}{24} - 4.9\left(\frac{x}{24}\right)^2$	A1	Elimination correct. Condone wrong notation with interpretation correct for the problem.	
	$y = \frac{7x}{24} - 4.9 \times \frac{x^2}{576} = \frac{0.7x}{576}(240 - 7x)$	E1	If not wrong accept as long as $24^2 = 576$ seen. Condone wrong notation with interpretation correct for the problem.	
	either Need $y = 0$	M1		
	so $x = 0$ or $\frac{240}{7}$ so $\frac{240}{7}$ m	A1	Accept $x = 0$ not mentioned. Condone $0 \le X \le \frac{240}{7}$.	
	or	B1	Time of flight $^{10}/_{7}$ s	
		B1	Range $^{240}/_{7}$ m. Condone $0 \le X \le \frac{240}{7}$.	5
				19

4761: Mechanics 1

General Comments

There were many high scores and few candidates who were unable to make a reasonable start to most of the questions. Very many candidates obtained high marks on questions 2, 3 and 7. Success with the other questions was not uniform and it seemed that some of the candidates were either not familiar with the ideas being tested or were not familiar with the form of the question. Q6 was the question that caused most problems. A good number of candidates tackled it with confidence for full or nearly full marks but more were unable to deal with the vector forms and couldn't make any progress beyond the first part.

As always, there were many very good, well presented scripts and some where the untidiness had clearly handicapped the candidate. Many candidates lost marks because they did not make their reasoning clear when asked to *show* something. Many candidates produced some good, clear diagrams but many did not and few produced as many as would have been helpful to them.

Comments on Individual Questions

1 **Resolution of forces and equilibrium**

Many candidates knew exactly what to do and did it efficiently but quite a few failed to write down clear equations for the equilibrium or had no plan and struggled.

(i) Most of the candidates realised that resolution was required but many of them either did not resolve in the right direction or failed to resolve accurately. Some simply ignored Q and thought that $P = \sqrt{40^2 + 120^2}$.

(ii) Fewer candidates managed this part than part (i), mostly because they failed to resolve or omitted a force.

2 **Sketching and using a speed – time graph**

This was the best answered question on the paper with most candidates obtaining a good score.

(i) Most candidates scored full marks on this part. The marks lost were usually because curves were used in place of straight line segments or there was a mistake with one of the values used.

(ii) Most candidates knew that they should find the area under the graph. Of these more split the region up into triangles and rectangles than two trapezia and one rectangle. Many candidates did not indicate *how* they had split up the region and quite a few omitted one part (often a 10 by 45 rectangle). Candidates who tried to apply the constant acceleration formulae were generally less successful.

(iii) This was the least well done part. Some worked with the 1700 m instead of 1700 less the answer to part (ii); others equated the extra distance to $40T$ – effectively arguing that the car is brought to rest at a constant speed!

3 **Equilibrium involving a pulley**

Despite the unusual situation described, there were many correct answers.

(i) Most candidates knew that the pulley had to be smooth for the tension to be the same throughout the string but some thought that the system being in equilibrium was enough.

(ii) The presentation of the argument was not always very good but this did not prevent many candidates from obtaining the correct value for the force and correctly naming it as a thrust. The most common errors came from candidates who first set up a pair of equations as if the situation were dynamic with the rod not present.

4 **The use of Newton's second law and the force in a tow – bar**

There were very many correct answers to parts (i) and (ii) and most candidates used the right principles in both parts. There were rather more errors seen in the attempts at part (iii) and many candidates would have done better if they had drawn a diagram and set out their work in a conventional way.

(i) A few used $F = mga$ or $F - mg = ma$ but the most common error was to omit one (or both) of the resistances. Quite a few candidates went from $P - 800 = 4000$ to *P* = 3200.

(ii) This part was generally done correctly if part (i) was correct. The chief error was to omit one or both of the original resistances.

(iii) A few candidates used the wrong mass or acceleration but the chief error was to omit one or more resistances or to apply the extra resistance of 2000 N to the wrong truck. There were a few candidates who based their method on wrong principles that did not include the use of Newton's second law.

5 **A block in equilibrium on a rough slope**

This question was done very well by some candidates but others made little progress with it. Very many candidates did not attempt to draw a diagram and these were, of course, more likely to omit forces or fail to resolve properly or at all.

For candidates who resolved up (or down) the plane, the chief errors were to omit a force, resolve only one of the forces or confuse sine with cosine. Candidates who tried to resolve in another direction usually omitted the normal reaction as well as making one or more of the errors listed above. Candidates who did not have a diagram showing the frictional force and did not specify the direction of the frictional force were not awarded the final mark.

It is worth repeating that many candidates, often whole centres, tackled this problem efficiently and accurately, obtaining full marks in just two or three lines.

6 **Newton's second law and constant acceleration situation in vector form; the modulus and direction of a vector**

This question was found much harder than expected. Seemingly, candidates were not able to work out what was required of them because of the use of vector form and the term 'position vector'. It was for most candidates the question with the lowest percentage score. Many candidates scored only 1, 2 or 3 marks.

(i) Most candidates knew they should add the vectors and use Newton's second law. Some of them lost a mark because they did not show enough working to establish the *given* answer.

(ii) This was done well by few candidates. Many used **s** = *t* **u** and many others worked out **v**(4) instead of **s**(4); many of those who attempted **s**(4) wrongly used $\mathbf{s} = t\mathbf{v} - \frac{1}{2}t^2\mathbf{a}$. Those who used integration usually integrated **u** once instead of **a** twice and many integrated the i, j and k they used in place of **i**, **j** and **k**.

(iii) Any answer from part (ii) was followed but many candidates gave the distance OA as a vector, in many cases the correct value of **r** not found for part (ii).

(iv) Those candidates who had obtained 3**i** + 4**k** usually obtained the correct angle. Some of those with a **j** component knew what to do but many just used the **i** and **k** components.

7 **Kinematics using calculus**

This question was well understood by most of the candidates. Many used differentiation and integration appropriately and accurately and scored good marks. Very few candidates made little progress at all.

(i) Usually done correctly

(ii) Usually done correctly. More candidates used substitution than factorised.

(iii) Most candidates did this well but a few made slips and a fair number omitted the final coordinates.

(iv) Most candidates knew that they should integrate and obtained the correct indefinite integral with few falsely using the constant acceleration formulae. A major source of error was the incorrect evaluation of the definite integral either because of an error in the calculation of the value with one of the limits or because the minus signs led the candidates to add instead of subtract these values. Quite a few candidates gave the answer as a displacement of – 18 m instead of a distance of 18 m.

(v) Some candidates realised that they could use symmetry and double their answer to part (iv) but many worked out the value by starting again with new limits.

(vi) Some knew how to deal with regions above and below the axis but many just integrated from $t = 1$ to $t = 5$ and so found the displacement.

8 **Projectile motion**

Many candidates managed parts of this question but only a few obtained full marks. The answers to parts (iii) and (iv) (A) and (C) showed that many candidates have some misconceptions about projectile motion.

(i) Most candidates knew broadly what to do but many failed to give enough working to *show* a given answer. Quite a few candidates wasted time working out the value of the angle of projection so that they could then take the sine and cosine to obtain (sometimes only approximately) the values from which they had deduced the angle in the first place.

(ii) Most candidates obtained the correct answer, some by applying $v^2 = u^2 + 2as$ to the vertical motion and some by going via the time taken to reach the highest point. Most of the errors were with signs.

(iii) Many candidates set up the correct quadratic equation but quite a few made slips when evaluating the roots, either by misremembering the formula or by calculation errors. A surprisingly large number of candidates found the horizontal distances at their two times but did not then subtract them to find the *horizontal distance travelled between these times*.
The most common error was falsely to assume that the vertical height was a linear function of time and so half the greatest height was reached in half the time to reach the greatest height.

(iv) (A) Although many candidates got this right a large number did not and for many different reasons. Some correctly tried to find $v_y(1.25)$ but wrote $v_y = 7 - 9.8 \times 1.25 = 19.25$, others used the right formula but took the initial speed as 25 or even 24. A common mistake was to find the vertical *height* when *t* = 1.25 and quote this as a velocity.

(B) This part was done well by many candidates. Some interpreted the sign of their $v_y(1.25)$ and others correctly argued that 1.25 s is after the time taken to reach the highest point.

(C) This was poorly done by many candidates. Some thought they were required to find the vertical component of the speed of the ball, some just gave the modulus of their answer to (A) and many who had found a height in (A) now correctly calculated $v_y(1.25)$ but incorrectly gave it as the answer to this part. Quite a lot of candidates who were using Pythagoras' Theorem to find the speed took the horizontal component of speed to be 24×1.25 instead of 24.

(v) This part was done well by many candidates, including many who had not scored very well on the rest of the question or, indeed, on the rest of the paper.

Most knew how to eliminate *t* and did so properly (albeit with some poor notation on the way). Quite a few established the final result.

Many candidates knew how to find the horizontal range. Most calculations were correct with some candidates setting *y* = 0 in the trajectory equation and others finding the total time of flight and using this to find the range.

4762: Mechanics 2

General Comments