OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4756/01**

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

THURSDAY 7 JUNE 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

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Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = a(1 - \cos\theta)$, where a is a positive constant.

(i) Sketch the curve. [2]

(ii) Find the area of the region enclosed by the section of the curve for which $0 \leqslant \theta \leqslant \frac{1}{2}\pi$ and the line $\theta = \frac{1}{2}\pi$. [6]

(b) Use a trigonometric substitution to show that $\displaystyle\int_0^1 \frac{1}{\left(4 - x^2\right)^{\frac{3}{2}}}\,dx = \frac{1}{4\sqrt{3}}$. [4]

(c) In this part of the question, $f(x) = \arccos(2x)$.

(i) Find $f'(x)$. [2]

(ii) Use a standard series to expand $f'(x)$, and hence find the series for $f(x)$ in ascending powers of x, up to the term in x^5. [4]

2 **(a)** Use de Moivre's theorem to show that $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$. [5]

(b) **(i)** Find the cube roots of $-2 + 2j$ in the form $re^{j\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. [6]

These cube roots are represented by points A, B and C in the Argand diagram, with A in the first quadrant and ABC going anticlockwise. The midpoint of AB is M, and M represents the complex number w.

(ii) Draw an Argand diagram, showing the points A, B, C and M. [2]

(iii) Find the modulus and argument of w. [2]

(iv) Find w^6 in the form $a + bj$. [3]

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3 Let $\mathbf{M} = \begin{pmatrix} 3 & 5 & 2 \\ 5 & 3 & -2 \\ 2 & -2 & -4 \end{pmatrix}$.

(i) Show that the characteristic equation for $\mathbf{M}$ is $\lambda^3 - 2\lambda^2 - 48\lambda = 0$. [4]

You are given that $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$ corresponding to the eigenvalue 0.

(ii) Find the other two eigenvalues of $\mathbf{M}$, and corresponding eigenvectors. [8]

(iii) Write down a matrix $\mathbf{P}$, and a diagonal matrix $\mathbf{D}$, such that $\mathbf{P}^{-1}\mathbf{M}^2\mathbf{P} = \mathbf{D}$. [3]

(iv) Use the Cayley-Hamilton theorem to find integers a and b such that $\mathbf{M}^4 = a\mathbf{M}^2 + b\mathbf{M}$. [3]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(a)** Find $\displaystyle\int_0^1 \frac{1}{\sqrt{9x^2+16}}\,\mathrm{d}x$, giving your answer in an exact logarithmic form. [5]

(b) **(i)** Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\sinh 2x = 2\sinh x \cosh x.$$ [2]

(ii) Show that one of the stationary points on the curve

$$y = 20\cosh x - 3\cosh 2x$$

has coordinates $(\ln 3, \frac{59}{3})$, and find the coordinates of the other two stationary points. [7]

(iii) Show that $\displaystyle\int_{-\ln 3}^{\ln 3} (20\cosh x - 3\cosh 2x)\,\mathrm{d}x = 40$. [4]

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 The curve with equation $y = \dfrac{x^2 - kx + 2k}{x + k}$ is to be investigated for different values of k.

(i) Use your graphical calculator to obtain rough sketches of the curve in the cases $k = -2$, $k = -0.5$ and $k = 1$. [6]

(ii) Show that the equation of the curve may be written as $y = x - 2k + \dfrac{2k(k+1)}{x+k}$.

Hence find the two values of k for which the curve is a straight line. [4]

(iii) When the curve is not a straight line, it is a conic.

(A) Name the type of conic. [1]

(B) Write down the equations of the asymptotes. [2]

(iv) Draw a sketch to show the shape of the curve when $1 < k < 8$. This sketch should show where the curve crosses the axes and how it approaches its asymptotes. Indicate the points A and B on the curve where $x = 1$ and $x = k$ respectively. [5]

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Mark Scheme 4756
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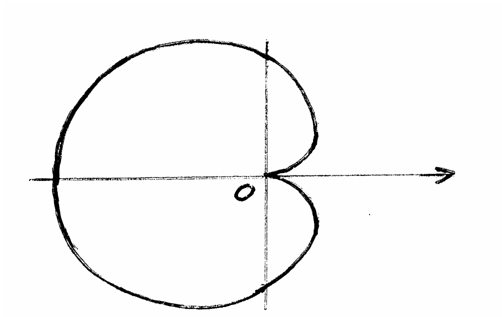
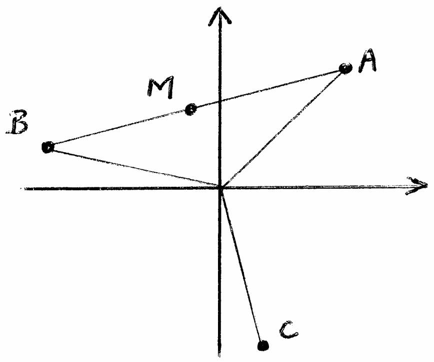
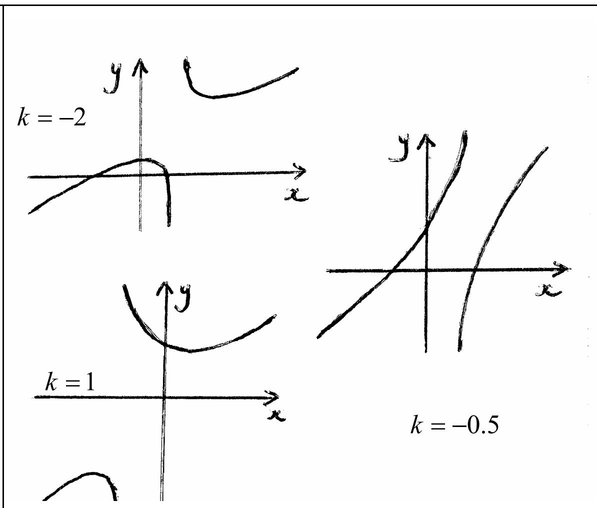
1(a)(i)		B2 **2**	Must include a sharp point at O and have infinite gradient at $\theta = \pi$ Give B1 for r increasing from zero for $0 < \theta < \pi$, or decreasing to zero for $-\pi < \theta < 0$
(ii)	Area is $\int \frac{1}{2} r^2 \, d\theta = \int_0^{\frac{1}{2}\pi} \frac{1}{2} a^2 (1-\cos\theta)^2 \, d\theta$ $= \frac{1}{2} a^2 \int_0^{\frac{1}{2}\pi} \left(1 - 2\cos\theta + \frac{1}{2}(1+\cos 2\theta) \right) d\theta$ $= \frac{1}{2} a^2 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta \right]_0^{\frac{1}{2}\pi}$ $= \frac{1}{2} a^2 (\frac{3}{4}\pi - 2)$	M1 A1 B1 B1B1 ft B1 **6**	For integral of $(1-\cos\theta)^2$ For a correct integral expression including limits *(may be implied by later work)* Using $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ Integrating $a + b\cos\theta$ and $k\cos 2\theta$ Accept $0.178a^2$
(b)	Put $x = 2\sin\theta$ Integral is $\int_0^{\frac{1}{6}\pi} \frac{1}{(4-4\sin^2\theta)^{\frac{3}{2}}} (2\cos\theta) \, d\theta$ $= \int_0^{\frac{1}{6}\pi} \frac{2\cos\theta}{8\cos^3\theta} \, d\theta = \int_0^{\frac{1}{6}\pi} \frac{1}{4}\sec^2\theta \, d\theta$ $= \left[\frac{1}{4}\tan\theta \right]_0^{\frac{1}{6}\pi}$ $= \frac{1}{4} \times \frac{1}{\sqrt{3}} = \frac{1}{4\sqrt{3}}$	M1 A1 M1 A1 ag **4**	or $x = 2\cos\theta$ Limits not required For $\int \sec^2\theta \, d\theta = \tan\theta$ *SR* If $x = 2\tanh u$ is used M1 for $\frac{1}{4}\sinh(\frac{1}{2}\ln 3)$ A1 for $\frac{1}{8}(\sqrt{3} - \frac{1}{\sqrt{3}}) = \frac{1}{4\sqrt{3}}$ (max 2 / 4)
(c)(i)	$f'(x) = \frac{-2}{\sqrt{1-4x^2}}$	B2 **2**	Give B1 for any non-zero real multiple of this (or for $\frac{-2}{\sin y}$ etc)
(ii)	$f'(x) = -2(1-4x^2)^{-\frac{1}{2}}$ $= -2(1 + 2x^2 + 6x^4 + ...)$ $f(x) = C - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + ...$ $f(0) = \frac{1}{2}\pi \Rightarrow C = \frac{1}{2}\pi$ $f(x) = \frac{1}{2}\pi - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + ...$	M1 A1 M1 A1 **4**	Binomial expansion (3 terms, $n = -\frac{1}{2}$) Expansion of $(1-4x^2)^{-\frac{1}{2}}$ correct (accept unsimplified form) Integrating series for $f'(x)$ Must obtain a non-zero x^5 term C not required

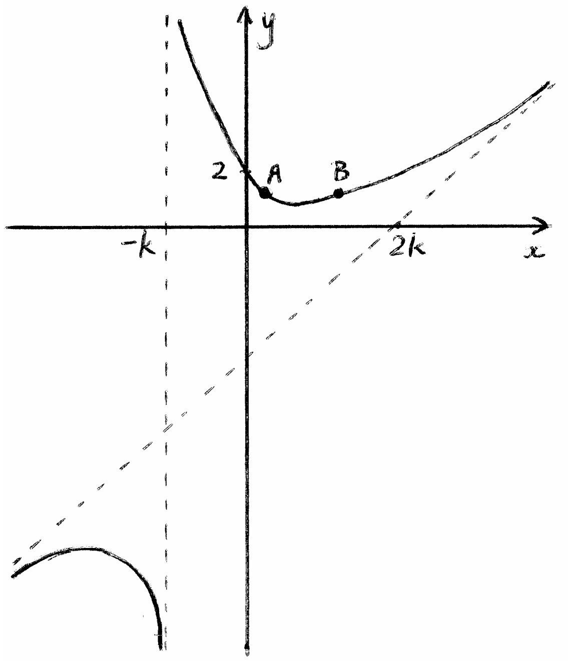
	OR by repeated differentiation Finding $\mathrm{f}^{(5)}(x)$ M1 Evaluating $\mathrm{f}^{(5)}(0)$ $(=-288)$ M1 $\mathrm{f}'(x) = -2 - 4x^2 - 12x^4 + \ldots$ A1 ft $\mathrm{f}(x) = \frac{1}{2}\pi - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + \ldots$ A1		Must obtain a non-zero value ft from (c)(i) when B1 given

2 (a)	$(\cos\theta + \mathrm{j}\sin\theta)^5$ $= c^5 + 5\mathrm{j}c^4s - 10c^3s^2 - 10\mathrm{j}c^2s^3 + 5cs^4 + \mathrm{j}s^5$ Equating imaginary parts $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$ $= 5(1-s^2)^2s - 10(1-s^2)s^3 + s^5$ $= 5s - 10s^3 + 5s^5 - 10s^3 + 10s^5 + s^5$ $= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$	M1 M1 A1 M1 A1 ag **5**	
(b)(i)	$\lvert -2+2\mathrm{j}\rvert = \sqrt{8}$, $\arg(-2+2\mathrm{j}) = \frac{3}{4}\pi$ $r = \sqrt{2}$ $\theta = \frac{1}{4}\pi$ $\theta = \frac{11}{12}\pi,\ -\frac{5}{12}\pi$	B1B1 B1 ft B1 ft M1 A1 **6**	Accept 2.8; 2.4, 135° (Implies B1 for $\sqrt{8}$) One correct (Implies B1 for $\frac{3}{4}\pi$) Adding or subtracting $\frac{2}{3}\pi$ Accept $\theta = \frac{1}{4}\pi + \frac{2}{3}k\pi,\ k = 0, 1, -1$
(ii)	 	B2 **2**	Give B1 for two of B, C, M in the correct quadrants Give B1 ft for all four points in the correct quadrants
(iii)	$\lvert w\rvert = \frac{1}{2}\sqrt{2}$ $\arg w = \frac{1}{2}(\frac{1}{4}\pi + \frac{11}{12}\pi) = \frac{7}{12}\pi$	B1 ft B1 **2**	Accept 0.71 Accept 1.8
(iv)	$\lvert w^6\rvert = (\frac{1}{2}\sqrt{2})^6 = \frac{1}{8}$ $\arg(w^6) = 6\times\frac{7}{12}\pi = \frac{7}{2}\pi$ $w^6 = \frac{1}{8}(\cos\frac{7}{2}\pi + \mathrm{j}\sin\frac{7}{2}\pi)$ $= -\frac{1}{8}\mathrm{j}$	M1 A1 ft A1 **3**	Obtaining either modulus or argument Both correct (ft) Allow from $\arg w = \frac{1}{4}\pi$ etc
			SR If B, C interchanged on diagram (ii) B1 (iii) B1 B1 for $-\frac{1}{12}\pi$ (iv) M1A1A1

3 (i)	$\det(\mathbf{M}-\lambda\mathbf{I}) = (3-\lambda)[(3-\lambda)(-4-\lambda)-4]$ $-5[5(-4-\lambda)+4]+2[-10-2(3-\lambda)]$ $=(3-\lambda)(-16+\lambda+\lambda^2)-5(-16-5\lambda)+2(-16+2\lambda)$ $=-48+19\lambda+2\lambda^2-\lambda^3+80+25\lambda-32+4\lambda$ $=48\lambda+2\lambda^2-\lambda^3$ Characteristic equation is $\lambda^3-2\lambda^2-48\lambda=0$	M1 A1 M1 A1 ag **4**	Obtaining $\det(\mathbf{M}-\lambda\mathbf{I})$ Any correct form Simplification
(ii)	$\lambda(\lambda-8)(\lambda+6)=0$ Other eigenvalues are $8,\ -6$ When $\lambda=8$, $3x+5y+2z=8x$ $(\ 5x+3y-2z=8y\ )$ $2x-2y-4z=8z$ $y=x$ and $z=0$; eigenvector is $\begin{pmatrix}1\\1\\0\end{pmatrix}$ When $\lambda=-6$, $3x+5y+2z=-6x$ $5x+3y-2z=-6y$ $y=-x$, $z=-2x$; eigenvector is $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$	M1 A1 M1 M1 A1 M1 M1 A1 **8**	Solving to obtain a non-zero value Two independent equations Obtaining a non-zero eigenvector *($-5x+5y+2z=8x$ etc can earn M0M1)* Two independent equations Obtaining a non-zero eigenvector
(iii)	$\mathbf{P}=\begin{pmatrix}1&1&1\\-1&1&-1\\1&0&-2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}0&0&0\\0&8&0\\0&0&-6\end{pmatrix}^2$ $=\begin{pmatrix}0&0&0\\0&64&0\\0&0&36\end{pmatrix}$	B1 ft M1 A1 **3**	B0 if **P** is clearly singular Order must be consistent with **P** when B1 has been earned
(iv)	$\mathbf{M}^3-2\mathbf{M}^2-48\mathbf{M}=\mathbf{0}$ $\mathbf{M}^3=2\mathbf{M}^2+48\mathbf{M}$ $\mathbf{M}^4=2\mathbf{M}^3+48\mathbf{M}^2$ $=2(2\mathbf{M}^2+48\mathbf{M})+48\mathbf{M}^2$ $=52\mathbf{M}^2+96\mathbf{M}$	M1 M1 A1 **3**	

4 (a)	$\int_0^1 \frac{1}{\sqrt{9x^2+16}}\,dx = \left[\frac{1}{3}\operatorname{arsinh}\frac{3x}{4}\right]_0^1$ $= \frac{1}{3}\operatorname{arsinh}\frac{3}{4}$ $= \frac{1}{3}\ln(\frac{3}{4}+\sqrt{\frac{9}{16}+1})$ $= \frac{1}{3}\ln 2$	M1 A1 A1 M1 A1 **5**	For arsinh or for any sinh substitution For $\frac{3}{4}x$ or for $3x = 4\sinh u$ For $\frac{1}{3}$ or for $\int \frac{1}{3}\,du$
	OR M2 $\left[\frac{1}{3}\ln(3x+\sqrt{9x^2+16})\right]_0^1$ A1A1 $= \frac{1}{3}\ln 8 - \frac{1}{3}\ln 4$ $= \frac{1}{3}\ln 2$ A1		For $\ln(kx+\sqrt{k^2x^2+\dots})$ [Give M1 for $\ln(ax+\sqrt{bx^2+\dots})$] or $\frac{1}{3}\ln(x+\sqrt{x^2+\frac{16}{9}})$
(b)(i)	$2\sinh x\cosh x = 2\times\frac{1}{2}(e^x-e^{-x})\frac{1}{2}(e^x+e^{-x})$ $= \frac{1}{2}(e^{2x}-e^{-2x})$ $= \sinh 2x$	M1 A1 **2**	$(e^x-e^{-x})(e^x+e^{-x}) = (e^{2x}-e^{-2x})$ For completion
(ii)	$\frac{dy}{dx} = 20\sinh x - 6\sinh 2x$ For stationary points, $20\sinh x - 12\sinh x\cosh x = 0$ $4\sinh x(5-3\cosh x) = 0$ $\sinh x = 0$ or $\cosh x = \frac{5}{3}$ $x=0,\quad y=17$ $x = (\pm)\ln(\frac{5}{3}+\sqrt{\frac{25}{9}-1}) = \ln 3$ $y = 10\left(3+\frac{1}{3}\right)-\frac{3}{2}\left(9+\frac{1}{9}\right) = \frac{59}{3}$ $x = -\ln 3,\quad y = \frac{59}{3}$	B1B1 M1 A1 A1 ag A1 ag B1 **7**	When exponential form used, give B1 for any 2 terms correctly differentiated Solving $\frac{dy}{dx}=0$ to obtain a value of $\sinh x$, $\cosh x$ or e^x (or $x=0$ stated) Correctly obtained Correctly obtained *The last A1A1 ag can be replaced by B1B1 ag for a full verification*
(iii)	$\left[20\sinh x - \frac{3}{2}\sinh 2x\right]_{-\ln 3}^{\ln 3}$ $= \left\{10\left(3-\frac{1}{3}\right)-\frac{3}{4}\left(9-\frac{1}{9}\right)\right\}\times 2$ $= \left(\frac{80}{3}-\frac{20}{3}\right)\times 2 = 40$	B1B1 M1 A1 ag **4**	When exponential form used, give B1 for any 2 terms correctly integrated Exact evaluation of $\sinh(\ln 3)$ and $\sinh(2\ln 3)$

5 (i)		B1 B1 B1 B1 B1 B1 **6**	Maximum on LH branch and minimum on RH branch Crossing axes correctly Two branches with positive gradient Crossing axes correctly Maximum on LH branch and minimum on RH branch Crossing positive y-axis and minimum in first quadrant
(ii)	$y=\dfrac{(x+k)(x-2k)+2k^2+2k}{x+k}$ $=x-2k+\dfrac{2k(k+1)}{x+k}$ Straight line when $2k(k+1)=0$ $k=0,\quad k=-1$	M1 A1 (ag) B1B1 **4**	Working in either direction For completion
(iii)(A)	Hyperbola	B1 **1**	
(B)	$x=-k$ $y=x-2k$	B1 B1 **2**	

(iv)		B1 B1 B1 B1 B1 **5**	Asymptotes correctly drawn Curve approaching asymptotes correctly (both branches) Intercept 2 on y-axis, and not crossing the x-axis Points A and B marked, with minimum point between them Points A and B at the same height ($y = 1$)

4756: Further Methods for Advanced Mathematics (FP2)

General Comments

On the whole, the candidates seemed to find this paper quite hard; and the full range of marks, from 0 to 72, was obtained. There were many excellent scripts, with about 15% of candidates scoring 60 marks or more, as well as some who were clearly not ready for an examination at this level; about 20% of candidates scored less than 30 marks. Quite a few candidates appeared to be short of time, especially when they had used inefficient methods such as repeated differentiation in Q1(c)(ii) or multiplying out brackets (instead of using the binomial theorem) in Q2(a). In Section A, the question on matrices (Q3) was much the best answered; and in Section B the overwhelming majority of candidates chose the hyperbolic functions option.

Comments on Individual Questions

1 This question, on polar coordinates and calculus, had an average mark of about 10 (out of 18).

(a)(i) The sketch was usually drawn well. Most showed the cusp at $\theta = 0$ clearly, but some also had a sharp point at $\theta = \pi$, instead of an infinite gradient.

(a)(ii) Most candidates knew how to find the area, and the integration was very often carried out accurately. Sometimes the factors ½ and a^2 were missing, and some candidates were unable to integrate $\cos^2\theta$.

(b) About half the candidates used the correct substitution $x = 2\sin\theta$ (or $x = 2\cos\theta$) to obtain $\int \frac{1}{4\cos^2\theta}\mathrm{d}\theta$ (or $\int \frac{-1}{4\sin^2\theta}\mathrm{d}\theta$), but many could not proceed beyond this point. Other candidates tried inappropriate substitutions such as $x = \sin\theta$, $x = 2\tan\theta$, $u = 4 - x^2$, and achieved nothing useful.

(c)(i) The differentiation of $\arccos 2x$ was often done correctly, although $\frac{-1}{\sqrt{1-4x^2}}$ was a very common error. The denominator was sometimes given as $\sqrt{1-x^2}$.

(c)(ii) This part was poorly answered. Few candidates realised that all they needed to do was to find the first three terms of the binomial expansion of $-2(1-4x^2)^{-\frac{1}{2}}$ and then integrate; and those that did, often forgot about the constant term after integrating. There were very many attempts to differentiate $\arccos 2x$ five times, and hardly any of these were successful.

2 This question, on complex numbers, had an average mark of about 10.

(a) For many candidates this seemed to be a familiar piece of work which was carried out confidently and accurately. Quite a few candidates confused this with the process of expressing powers in terms of multiple angles and considered $(z - 1/z)^5$, without any success.

(b)(i) The process of finding the cube roots of a complex number was quite well understood, and there were very many correct answers. The argument of one of the roots was sometimes given as $\frac{19}{12}\pi$, outside the required range, instead of $-\frac{5}{12}\pi$; but the most common error was to start by giving the argument of $-2+2\mathrm{j}$ as $-\frac{1}{4}\pi$ instead of $\frac{3}{4}\pi$.

(b)(ii) The Argand diagram was usually drawn correctly with ABC forming an equilateral triangle, although the midpoint M was sometimes placed on the imaginary axis instead of in the second quadrant.

(b)(iii) Relatively few candidates were able to use their Argand diagram to find the modulus and argument of *w* correctly. In particular, the modulus of *w* was often thought to be the same as that of the cube roots.

(b)(iv) Most candidates knew how to use the modulus and argument of *w* to find the modulus and argument of w^6 , and hence evaluate w^6 ; although errors in previous parts very often caused the final answer to be incorrect.

3 This question, on matrices, was the best answered question, with half the candidates scoring 14 marks or more. Candidates appeared to tackle it with confidence, and it was quite often the first question answered.

(i) Almost all candidates knew how to find the characteristic equation, although there were many algebraic, and especially sign, errors in the working.

(ii) The methods for finding eigenvalues and eigenvectors were very well known, and loss of marks was usually caused only by careless slips. However, some candidates did not appear to know that an eigenvector must be non-zero.

(iii) Almost all candidates correctly found P as the matrix with the eigenvectors as its columns (but this earned no marks if the resulting matrix was obviously singular, for example having a column of zeros or two identical columns). The diagonal matrix D very often contained the eigenvalues instead of their squares.

(iv) Most candidates knew the Cayley-Hamilton theorem, but many did not see how to obtain $\mathbf{M}^4$. Some attempts began by cancelling a factor M from the cubic equation, which is invalid since M is a singular matrix.

4 This question, on hyperbolic functions, was quite well answered, and the average mark was about 11. By far the most common reason for loss of marks was the failure to show sufficient working when the answer was given on the question paper.

(a) Most candidates realised that the integral involved arsinh, or went straight to the logarithmic form, although the factor $\frac{1}{3}$ was often omitted.

(b)(i) This was usually adequately, if not always elegantly, demonstrated.

(b)(ii) Most candidates differentiated correctly, although $\frac{\mathrm{d}}{\mathrm{d}x}(\cosh x)=-\sinh x$ and $\frac{\mathrm{d}}{\mathrm{d}x}(\cosh 2x)=\frac{1}{2}\sinh x$ were fairly common errors. However, it was rare to see full marks in this part. Having obtained $\cosh x=\frac{5}{3}$ it was not sufficient to write 'so $x=\ln 3$, $y=\frac{59}{3}$ ' since this point is given on the question paper. To earn the marks a candidate was expected to write (at least)

$$x=\ln(\tfrac{5}{3}+\sqrt{(\tfrac{5}{3})^2-1})=\ln 3,\ \ y=10(3+\tfrac{1}{3})-\tfrac{3}{2}(9+\tfrac{1}{9})=\tfrac{59}{3}$$

or an equivalent exact calculation using $\cosh x=\frac{5}{3}$ and $\cosh 2x=2(\frac{5}{3})^2-1$.

Candidates frequently failed to find one or other of the remaining stationary points $(0,\ 17)$ and $(-\ln 3,\ \frac{59}{3})$.

(b)(iii) The integration was usually done correctly (more often than the differentiation in part (b)(ii)), but very many candidates lost marks for not showing sufficient detail in the evaluation. Since the answer is given, it is not sufficient to state, for example $\sinh(2\ln 3) = \frac{40}{9}$; the calculation $\sinh(2\ln 3) = \frac{1}{2}(9 - \frac{1}{9})$ should be shown.

5 This question, on the investigation of curves, was more popular than in the past; and in a few centres all the candidates answered this question. Even so it was only attempted by about 4% of the candidates. There were a few very good complete solutions, but most attempts were fragmentary; the average mark was about 9.

(i) In the cases $k = -2$ and $k = 1$ it was quite common for one branch of the curve to be missing, presumably caused by poor choice of scales for the axes on the calculator.

(ii) Several candidates were unable to do this simple algebra.

(iii) Every type of conic was offered by at least one candidate, even though it is known to have asymptotes.

(iv) Few sketches were drawn carefully enough to show clearly all the details requested.