RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

MONDAY 11 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

  [Turn over

Section A (36 marks)

1 You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$.

(i) Find the inverse of $\mathbf{M}$. [2]

(ii) A triangle of area 2 square units undergoes the transformation represented by the matrix $\mathbf{M}$. Find the area of the image of the triangle following this transformation. [1]

2 Write down the equation of the locus represented by the circle in the Argand diagram shown in Fig. 2. [3]

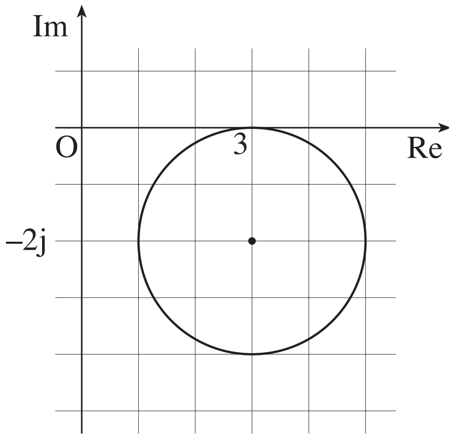


Fig. 2

3 Find the values of the constants A, B, C and D in the identity

$$x^3 - 4 \equiv (x-1)(Ax^2 + Bx + C) + D.$$ [5]

4 Two complex numbers, α and β, are given by $\alpha = 1 - 2\text{j}$ and $\beta = -2 - \text{j}$.

(i) Represent β and its complex conjugate β^* on an Argand diagram. [2]

(ii) Express $\alpha\beta$ in the form $a + b\text{j}$. [2]

(iii) Express $\dfrac{\alpha + \beta}{\beta}$ in the form $a + b\text{j}$. [3]

5 The roots of the cubic equation $x^3 + 3x^2 - 7x + 1 = 0$ are α, β and γ. Find the cubic equation whose roots are 3α, 3β and 3γ, expressing your answer in a form with integer coefficients. [6]

6 **(i)** Show that $\dfrac{1}{r+2} - \dfrac{1}{r+3} = \dfrac{1}{(r+2)(r+3)}$. [2]

(ii) Hence use the method of differences to find $\dfrac{1}{3 \times 4} + \dfrac{1}{4 \times 5} + \dfrac{1}{5 \times 6} + \ldots + \dfrac{1}{52 \times 53}$. [4]

7 Prove by induction that $\displaystyle\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$. [6]

Section B (36 marks)

8 A curve has equation $y = \dfrac{x^2 - 4}{(x-3)(x+1)(x-1)}$.

(i) Write down the coordinates of the points where the curve crosses the axes. [3]

(ii) Write down the equations of the three vertical asymptotes and the one horizontal asymptote. [4]

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. [3]

(iv) Sketch the curve. [4]

9 The cubic equation $x^3 + Ax^2 + Bx + 15 = 0$, where A and B are real numbers, has a root $x = 1 + 2\text{j}$.

(i) Write down the other complex root. [1]

(ii) Explain why the equation must have a real root. [1]

(iii) Find the value of the real root and the values of A and B. [9]

[Question 10 is printed overleaf.]

10 You are given that $\mathbf{A} = \begin{pmatrix} 1 & -2 & k \\ 2 & 1 & 2 \\ 3 & 2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -5 & -2+2k & -4-k \\ 8 & -1-3k & -2+2k \\ 1 & -8 & 5 \end{pmatrix}$ and that $\mathbf{AB}$ is of the form $\mathbf{AB} = \begin{pmatrix} k-n & 0 & 0 \\ 0 & k-n & 0 \\ 0 & 0 & k-n \end{pmatrix}$.

(i) Find the value of n. [2]

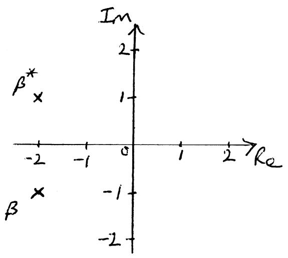
(ii) Write down the inverse matrix $\mathbf{A}^{-1}$ and state the condition on k for this inverse to exist. [4]

(iii) Using the result from part **(ii)**, or otherwise, solve the following simultaneous equations.

$$\begin{aligned} x - 2y + z &= 1 \\ 2x + y + 2z &= 12 \\ 3x + 2y - z &= 3 \end{aligned}$$

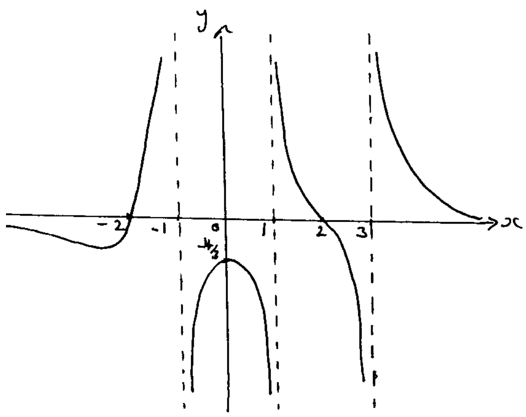
[5]

Mark Scheme 4755
June 2007

Section A			
1(i)	$\mathbf{M}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$	M1 A1 **[2]**	Attempt to find determinant
1(ii)	20 square units	B1 **[1]**	$2\times$ their determinant
2	$\lvert z-(3-2\mathrm{j})\rvert = 2$	B1 B1 B1 **[3]**	$z \pm (3-2\mathrm{j})$ seen radius = 2 seen Correct use of modulus
3	$x^3-4=(x-1)(Ax^2+Bx+C)+D$ $\Rightarrow x^3-4 = Ax^3+(B-A)x^2+(C-B)x-C+D$ $\Rightarrow A=1, B=1, C=1, D=-3$	M1 B1 B1 B1 B1 **[5]**	Attempt at equating coefficients or long division (may be implied) For A = 1 B1 for each of B, C and D
4(i)		B1 B1 **[2]**	One for each correctly shown. s.c. B1 if not labelled correctly but position correct
4(ii)	$\alpha\beta=(1-2\mathrm{j})(-2-\mathrm{j})=-4+3\mathrm{j}$	M1 A1 **[2]**	Attempt to multiply
4(iii)	$\frac{\alpha+\beta}{\beta}=\frac{(\alpha+\beta)\beta^*}{\beta\beta^*}=\frac{\alpha\beta^*+\beta\beta^*}{\beta\beta^*}=\frac{5\mathrm{j}+5}{5}=\mathrm{j}+1$	M1 A1 A1 **[3]**	Appropriate attempt to use conjugate, or other valid method 5 in denominator or correct working consistent with their method All correct

5	**Scheme A** $w = 3x \Rightarrow x = \frac{w}{3}$ $\Rightarrow \left(\frac{w}{3}\right)^3 + 3\left(\frac{w}{3}\right)^2 - 7\left(\frac{w}{3}\right) + 1 = 0$ $\Rightarrow w^3 + 9w^2 - 63w + 27 = 0$ **OR**	B1 M1 A3 A1 **[6]**	Substitution. For substitution $x = 3w$ give B0 but then follow through for a maximum of 3 marks Substitute into cubic Correct coefficients consistent with x^3 coefficient, minus 1 each error Correct cubic equation c.a.o.
	Scheme B $\alpha + \beta + \gamma = -3$ $\alpha\beta + \alpha\gamma + \beta\gamma = -7$ $\alpha\beta\gamma = -1$ Let new roots be *k*, *l*, *m* then $k + l + m = 3(\alpha + \beta + \gamma) = -9 = \frac{-B}{A}$ $kl + km + lm = 9(\alpha\beta + \alpha\gamma + \beta\gamma) = -63 = \frac{C}{A}$ $klm = 27\alpha\beta\gamma = -27 = \frac{-D}{A}$ $\Rightarrow \omega^3 + 9\omega^2 - 63\omega + 27 = 0$	M1 M1 A3 A1 **[6]**	Attempt to find sums and products of roots (at least two of three) Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation Correct coefficients consistent with x^3 coefficient, minus 1 each error Correct cubic equation c.a.o.
6(i) **6(ii)**	$\frac{1}{r+2} - \frac{1}{r+3} = \frac{r+3-(r+2)}{(r+2)(r+3)} = \frac{1}{(r+2)(r+3)}$ $\sum_{r=1}^{50} \frac{1}{(r+2)(r+3)} = \sum_{r=1}^{50}\left[\frac{1}{r+2} - \frac{1}{r+3}\right]$ $= \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + ...$ $+ \left(\frac{1}{51} - \frac{1}{52}\right) + \left(\frac{1}{52} - \frac{1}{53}\right)$ $= \frac{1}{3} - \frac{1}{53} = \frac{50}{159}$	M1 A1 **[2]** M1 M1, M1 A1 **[4]**	Attempt at common denominator Correct use of part (i) (may be implied) First two terms in full Last two terms in full (allow in terms of *n*) Give B4 for correct without working Allow 0.314 (3s.f.)

7	$\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$ $n = 1$, LHS = RHS = 1 Assume true for $n = k$ Next term is 3^k Add to both sides $\text{RHS} = \frac{3^k - 1}{2} + 3^k$ $= \frac{3^k - 1 + 2 \times 3^k}{2}$ $= \frac{3 \times 3^k - 1}{2}$ $= \frac{3^{k+1} - 1}{2}$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$. Since it is true for $k = 1$, it is true for $k = 1, 2, 3$ and so true for all positive integers.	B1 E1 M1 A1 E1 E1 **[6]**	 Assuming true for k Attempt to add 3^k to RHS c.a.o. with correct simplification Dependent on previous E1 and immediately previous A1 Dependent on B1 and both previous E marks
			Section A Total: 36

Section B			
8(i)	$(2,\ 0)$, $(-2,\ 0)$, $\left(0,\ \frac{-4}{3}\right)$	B1 B1 B1 **[3]**	1 mark for each s.c. B2 for $2,\ -2,\ \frac{-4}{3}$
8(ii)	$x=3,\ x=-1,\ x=1,\ y=0$	B4 **[4]**	Minus 1 for each error
8(iii)	Large positive x, $y \to 0^+$, approach from above (e.g. consider $x=100$) Large negative x, $y \to 0^-$, approach from below (e.g. consider $x=-100$)	B1 B1 M1 **[3]**	Direction of approach must be clear for each B mark Evidence of method required
8(iv)	Curve 4 branches correct Asymptotes correct and labelled Intercepts labelled 	B2 B1 B1 **[4]**	Minus 1 each error, min 0

9(i)	$x = 1 - 2\text{j}$	B1 **[1]**	
9(ii)	Complex roots occur in conjugate pairs. A cubic has three roots, so one must be real. Or, valid argument involving graph of a cubic or behaviour for large positive and large negative x.	E1 **[1]**	
9(iii)	**Scheme A**		
	$(x-1-2\text{j})(x-1+2\text{j}) = x^2 - 2x + 5$	M1 A1	Attempt to use factor theorem Correct factors
	$(x-\alpha)(x^2-2x+5) = x^3 + Ax^2 + Bx + 15$	A1(ft)	Correct quadratic(using their factors)
	comparing constant term:	M1	Use of factor involving real root
	$-5\alpha = 15 \Rightarrow \alpha = -3$	M1	Comparing constant term
	So real root is $x = -3$	A1(ft)	From their quadratic
	$(x+3)(x^2-2x+5) = x^3 + Ax^2 + Bx + 15$	M1	Expand LHS
	$\Rightarrow x^3 + x^2 - x + 15 = x^3 + Ax^2 + Bx + 15$	M1	Compare coefficients
	$\Rightarrow A = 1,\ B = -1$	A1 **[9]**	1 mark for both values
	OR		
	Scheme B		
	Product of roots $= -15$	M1 A1	Attempt to use product of roots Product is –15
	$(1+2\text{j})(1-2\text{j}) = 5$	M1 A1	Multiplying complex roots
	$\Rightarrow 5\alpha = -15$	A1	
	$\Rightarrow \alpha = -3$	A1	c.a.o.
	Sum of roots $= -A$ $\Rightarrow -A = 1 + 2j + 1 - 2j - 3 = -1 \Rightarrow A = 1$	M1	Attempt to use sum of roots
	Substitute root $x = -3$ into cubic $(-3)^3 + (-3)^2 - 3B + 15 = 0 \Rightarrow B = -1$	M1	Attempt to substitute, or to use sum
	$A = 1$ and $B = -1$	A1 **[9]**	c.a.o.
	OR		
	Scheme C		
	$\alpha = -3$	6	As scheme A, or other valid method
	$(1+2\text{j})^3 + A(1+2\text{j})^2 + B(1+2\text{j}) + 15 = 0$	M1	Attempt to substitute root
	$\Rightarrow A(-3+4\text{j}) + B(1+2\text{j}) + 4 - 2\text{j} = 0$ $\Rightarrow -3A + B + 4 = 0$ and $4A + 2B - 2 = 0$	M1	Attempt to equate real and imaginary parts, or equivalent.
	$\Rightarrow A = 1$ and $B = -1$	A1 **[9]**	c.a.o.

Section B (continued)			
10(i)	$\mathbf{AB}=\begin{pmatrix}1 & -2 & k\\ 2 & 1 & 2\\ 3 & 2 & -1\end{pmatrix}\begin{pmatrix}-5 & -2+2k & -4-k\\ 8 & -1-3k & -2+2k\\ 1 & -8 & 5\end{pmatrix}$ $=\begin{pmatrix}k-21 & 0 & 0\\ 0 & k-21 & 0\\ 0 & 0 & k-21\end{pmatrix}$	M1	Attempt to multiply matrices (can be implied)
	$n=21$	A1 **[2]**	
10(ii)	$\mathbf{A}^{-1}=\frac{1}{k-21}\begin{pmatrix}-5 & -2+2k & -4-k\\ 8 & -1-3k & -2+2k\\ 1 & -8 & 5\end{pmatrix}$	M1 M1 A1	Use of **B** Attempt to use their answer to (i) Correct inverse
	$k\neq 21$	A1 **[4]**	Accept n in place of 21 for full marks
10 (iii)	**Scheme A** $\frac{1}{-20}\begin{pmatrix}-5 & 0 & -5\\ 8 & -4 & 0\\ 1 & -8 & 5\end{pmatrix}\begin{pmatrix}1\\ 12\\ 3\end{pmatrix}=\frac{1}{-20}\begin{pmatrix}-20\\ -40\\ -80\end{pmatrix}=\begin{pmatrix}1\\ 2\\ 4\end{pmatrix}$	M1 M1	Attempt to use inverse Their inverse with $k=1$
	$x=1,\ y=2,\ z=4$	A3 **[5]**	One for each correct (ft)
	OR		
	Scheme B Attempt to eliminate 2 variables Substitute in their value to attempt to find others $x=1,\ y=2,\ z=4$	M1 M1 A3 **[5]**	s.c. award 2 marks only for $x=1,\ y=2,\ z=4$ with no working.
			Section B Total: 36
			Total: 72

4755: Further Concepts for Advanced Mathematics (FP1)

General Comments

This paper was of an appropriate standard, with some questions that almost all candidates could make a good attempt at and others that provided a challenge for the most able. It may, however, have been a little long; some strong candidates seemed to have run out of time.

The paper appeared to be slightly easier than recent papers for weaker candidates but a little harder for strong ones.

By far the majority of candidates were clearly well prepared for the examination.

Comments on Individual Questions

1 **Properties of a matrix**

In this question candidates were asked to find the inverse of the given 2×2 matrix, and the effect of the matrix on the area of a figure which was transformed by the matrix. Almost all candidates answered the first part correctly but a significant minority were unable to do the second part.

2 **Locus on argand diagram**

While there were many correct answers to this question from strong candidates, there were also plenty of mistakes. The commonest of these were sign errors, missing or incorrectly used modulus brackets, using $\leq$ instead of =, and attempting to give the cartesian equation of the circle.

3 **Identity**

By far the majority of candidates got this question right. The errors that did occur mostly resulted from careless mistakes involving signs.

4 **Complex numbers**

Many candidates scored full marks on this question. There were, however, many poorly labelled Argand diagrams in part (i), careless mistakes in multiplying out $\alpha\beta$ in part (ii) and in dividing one complex number by another in part (iii). All but the weakest candidates knew what to do, but a significant proportion made simple slips.

5 **Roots of an equation**

Almost all candidates knew how to do this question and there were many correct answers. Most of the mistakes that occurred were careless errors in the manipulation, often involving signs. Both of the alternative methods were commonly used, but the substitution method was more efficient and resulted in fewer errors, though some failed to multiply the constant by 27.

6 **Method of differences**

This question was generally well answered. In part (i) a few candidates invalidated their establishment of the given result with sign errors when removing brackets. $\frac{r+3-r+2}{(r+2)(r+3)}=\frac{1}{(r+2)(r+3)}$ was seen quite often. The commonest mistake in part (ii) was to leave the answer as $\frac{1}{3}-\frac{1}{n+3}$, failing to substitute $n=50$.

7 **Proof by induction**

This was by far the least well answered question in Section A. Most candidates knew what they were trying to achieve but many failed to show that $3^k-1+2\times3^k=3^{k+1}-1$. There seemed to be a widespread belief that $2\times3^k=6^k$.

8 **Curve sketching**

This question was well answered. Many candidates scored full marks on it, or very nearly so.

(i) Candidates were asked for the points where the curve cuts the axes. Most scored full marks.

(ii) Candidates were asked for the vertical and horizontal asymptotes. The most common error was failing to recognise $y = 0$ as the horizontal asymptote.

(iii) Candidates were asked about approaches to the horizontal asymptote. A significant proportion failed to show any workings.

(iv) Candidates were asked to sketch the curve. Common errors were failing to label intercepts and asymptotes and incorrect approaches to the asymptotes.

9 **Cubic equation with two complex roots**

This question was often done well, but parts (ii) and (iii) differentiated well.

(i) Almost all candidates knew that if 1 + 2j was a root of the cubic, then 1 - 2j must also be a root.

(ii) Candidates were asked to explain why the third root must be real; they were expected to say that complex roots come in conjugate pairs and that because a cubic has three roots, the third must therefore be real. Many candidates omitted the word “conjugate”. Many produced entirely spurious arguments and a significant proportion failed to attempt an answer. A few gave alternative valid arguments, which were given full credit.

(iii) Part (iii) was an unstructured question for 9 marks to find the real root and the values of missing coefficients. Some candidates found efficient approaches to the work and took only a few lines to obtain the right answers. Others, by contrast, submitted several pages of work; in some cases there was nothing mathematically wrong but it was just not going anywhere useful. However, many candidates compounded a poor strategy with sign errors. The most efficient method was to consider the sums and products of the roots, but the majority used the factor theorem, which was more complicated and so prone to error. $2\text{j}\times 2\text{j} = 2\text{j}^2$ was seen too often.

10 **Inverse of a 3×3 matrix**

This question was rather low scoring. Time pressure may have been a factor for some.

Many did not see the structure of the question, failing to see the connection between parts (i) and (ii), and (ii) and (iii).

(i) Candidates were asked to find the value of a constant when multiplying two 3×3 matrices. Most candidates did this correctly but a common mistake was to write $n = -21$ instead of $n = 21$.

(ii) Candidates were asked to write down the inverse of a 3×3 matrix and state the condition on a constant for this inverse to exist. This all followed from (i). A mark of zero on this part of the question was quite common. Some candidates did, however, see what was happening and obtained the correct answer. The very best did seem to simply right it down, which was possible if they could see the connection with part (i). Many started again from scratch. A few earned some of the marks, but most were not successful.

(iii) Candidates were asked to solve a system of 3 simultaneous linear equations. Candidates were given the option of following the logic of the question, using the inverse 3×3 matrix to solve the equations, or of using another method. Many candidates chose Gaussian elimination and right answers obtained by this method were common. There were also some right answers using the matrix method, which followed easily from (ii). Incorrect inverse matrices were followed through from (ii), so most candidates who got this far earned at least some of the marks.