RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

MATHEMATICS (MEI)

Decision Mathematics 2

WEDNESDAY 20 JUNE 2007

4772/01

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

1 **(a)** A joke has it that army recruits used to be instructed: "If it moves, salute it. If it doesn't move, paint it."

Assume that this instruction has been carried out completely in the local universe, so that everything that doesn't move has been painted.

(i) A recruit encounters something which is not painted. What should he do, and why? [3]

(ii) A recruit encounters something which is painted. Do we know what he or she should do? Justify your answer. [3]

(b) Use a truth table to prove $(((m \Rightarrow s) \wedge (\sim m \Rightarrow p)) \wedge \sim p) \Rightarrow s$. [6]

(c) You are given the following two rules.

1 $(a \Rightarrow b) \Leftrightarrow (\sim b \Rightarrow \sim a)$

2 $(x \wedge (x \Rightarrow y)) \Rightarrow y$

Use Boolean algebra to prove that $(((m \Rightarrow s) \wedge (\sim m \Rightarrow p)) \wedge \sim p) \Rightarrow s$. [4]

2 Bill is at a horse race meeting. He has £2 left with two races to go. He only ever bets £1 at a time. For each race he chooses a horse and then decides whether or not to bet on it. In both races Bill's horse is offered at "evens". This means that, if Bill bets £1 and the horse wins, then Bill will receive back his £1 plus £1 winnings. If Bill's horse does not win then Bill will lose his £1.

(i) Draw a decision tree to model this situation. Show Bill's payoffs on your tree, i.e. how much money Bill finishes with under each possible outcome. [8]

Assume that in each race the probability of Bill's horse winning is the same, and that it has value p.

(ii) Find Bill's EMV when

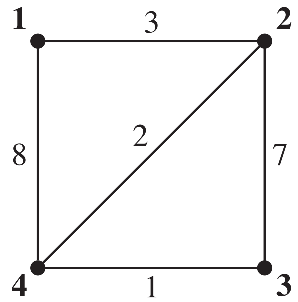
(A) $p = 0.6$,

(B) $p = 0.4$.

Give his best course of action in each case. [5]

(iii) Suppose that Bill uses the utility function $utility = (money)^x$, to decide whether or not to bet £1 on one race. Show that, with $p = 0.4$, Bill will not bet if $x = 0.5$, but will bet if $x = 1.5$. [3]

3 Floyd's algorithm is applied to the following network:



At the end of the third iteration of the algorithm the distance and route matrices are as follows:

	1	**2**	**3**	**4**
1	6	3	10	5
2	3	6	7	2
3	10	7	14	1
4	5	2	1	2

	1	**2**	**3**	**4**
1	2	2	2	2
2	1	1	3	4
3	2	2	2	4
4	2	2	3	3

(i) Perform the fourth (final) iteration of the algorithm. [7]

(ii) Explain how to use the final matrices to find the shortest distance and the shortest route from vertex **1** to vertex **3**, and give the distance and route. [4]

(iii) Draw the complete network of shortest distances. [1]

(iv) Apply the nearest neighbour algorithm, starting at vertex **1**, to your complete network of shortest distances. Give the Hamilton cycle it produces, its length, and the corresponding route through the original network. [3]

(v) By considering vertex **2** and its arcs, construct a lower bound for the length of the solution to the travelling salesperson problem in the original network. [3]

(vi) Explain what you can deduce from your answers to parts **(iv)** and **(v)**. [2]

[Question 4 is printed overleaf.]

4 Noel is designing a hotel patio. It will consist of decking and paving.

Decking costs £4 per m^2 and paving costs £2 per m^2. He has a budget of £2500.

Noel prefers paving to decking, and he wants the area given to paving to be at least twice that given to decking.

He wants to have as large a patio as possible.

Noel's problem is formulated as the following LP.

Let x be the number of m^2 of decking.

Let y be the number of m^2 of paving.

$$\begin{aligned} \text{Maximise} \quad & P = x + y \\ \text{subject to} \quad & 2x + y \leqslant 1250 \\ & 2x - y \leqslant 0 \\ & x \geqslant 0 \\ & y \geqslant 0 \end{aligned}$$

(i) Use the simplex algorithm to solve this LP. Pivot first on the positive element in the y column. [6]

Noel would like to have at least 200 m^2 of decking.

(ii) Add a line corresponding to this constraint to your solution tableau from part **(i)**, and modify the resulting table either for two-stage simplex or the big-M method. Hence solve the problem. [9]

Noel finally decides that he will minimise the annual cost of maintenance, which is given by £$(0.75x + 1.25y)$, subject to the additional constraint that there is at least 1000 m^2 of patio.

(iii) Starting from your solution to part **(ii)**, use simplex to solve this problem. [5]

Mark Scheme 4772
June 2007

1.

(a)(i) He should salute it. — B1

Since all objects which don't move are painted any unpainted object must move, and anything that moves must be saluted. — M1 A1

(ii) We do not know.

We do not know about painted objects. Some will have been painted because they do not move, but there may be some objects which move which are painted. We do not know whether this object moves or not. — B1, M1 A1

(b)

((m	⇒	s)	∧	(~	m	⇒	p))	∧	~	p	⇒	s
1	1	1	**1**	0	1	1	1	0	**0**	1	**1**	1
1	1	1	**1**	0	1	1	0	1	**1**	0	**1**	1
1	0	0	**0**	0	1	1	1	0	**0**	1	**1**	0
1	0	0	**0**	0	1	1	0	0	**1**	0	**1**	0
0	1	1	**1**	1	0	1	1	0	**0**	1	**1**	1
0	1	1	**0**	1	0	0	0	0	**1**	0	**1**	1
0	1	0	**1**	1	0	1	1	0	**0**	1	**1**	0
0	1	0	**0**	1	0	0	0	0	**1**	0	**1**	0

M1 8 rows
A1 m⇒s
A1 ~m⇒p
A1 first ∧
A1 second ∧
A1 result

(c) $\left(\left(m \Rightarrow s\right) \wedge \left(\sim m \Rightarrow p\right)\right) \wedge \sim p$

$\Leftrightarrow \left(\sim p \wedge \left(\sim m \Rightarrow p\right)\right) \wedge \left(m \Rightarrow s\right)$

$\Leftrightarrow \left(\sim p \wedge \left(\sim p \Rightarrow m\right)\right) \wedge \left(m \Rightarrow s\right)$ (contrapositive)

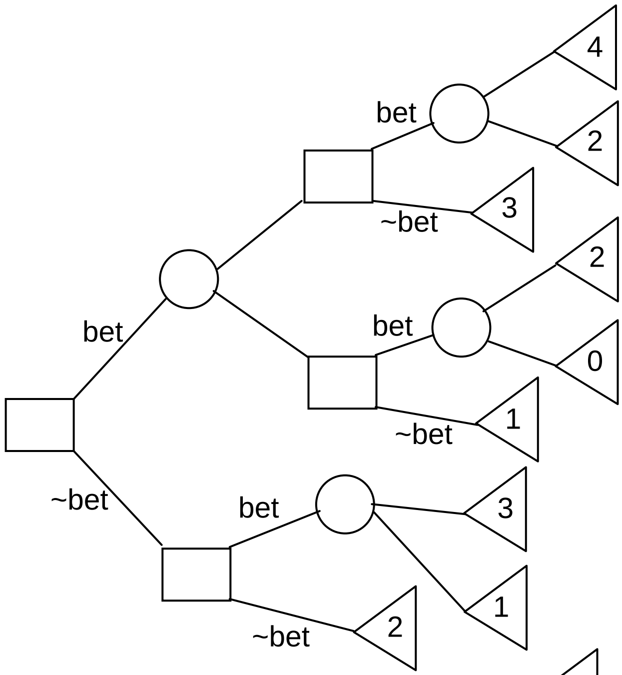
$\Rightarrow m \wedge \left(m \Rightarrow s\right)$ (modus ponens)

$\Rightarrow s$ (modus ponens)

M1
A1 reordering
A1 contrapositive
A1 modus ponens

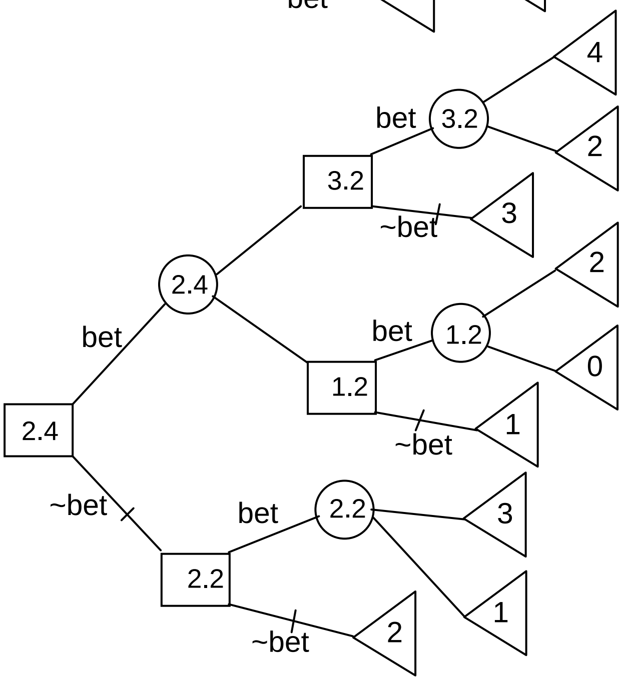
2.

(i)



M1	
A1	first D box
A1	D box on ~bet branch
A1	P box on bet branch
A1	D boxes following P box
A1	remaining P boxes
M1	outcomes
A1	

(ii)(A)

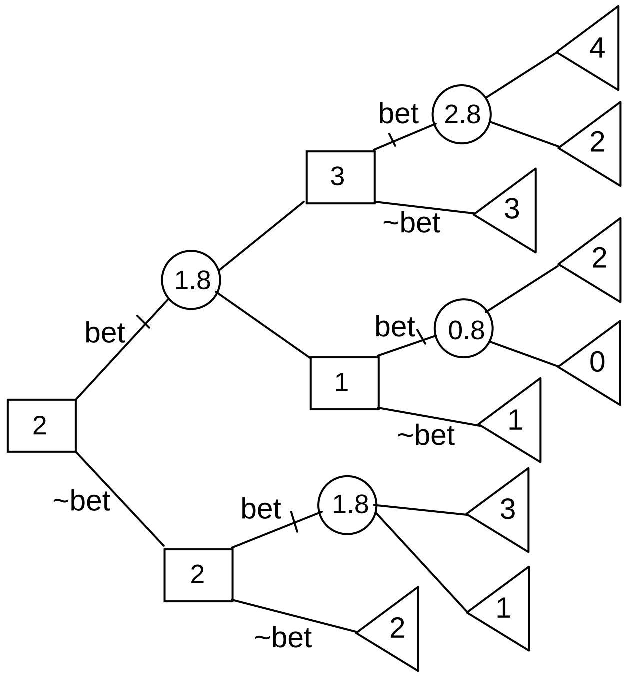


M1	
A1	

EMV = 2.4 by betting and betting again

B1	course of action

2(cont).

(ii)(B) 	A1
EMV = 2 by not betting	B1 course of action
(iii) $2^{0.5}\times 0.4 = 0.566 < 1$, but $2^{1.5}\times 0.4 = 1.131 > 1$	M1 A1 A1

3.

(i)

	1	**2**	**3**	**4**
1	6	3	6	5
2	3	4	3	2
3	6	3	2	1
4	5	2	1	2

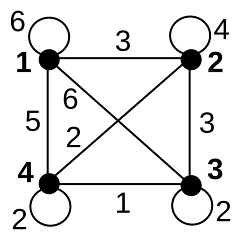
	1	**2**	**3**	**4**
1	2	2	2	2
2	1	4	4	4
3	4	4	4	4
4	2	2	3	3

M1 distances
A2 6 changes
(–1 each error)
M1 a correct update
A1 1 to 3 route (2)
A2 rest
(–1 each error)

(ii) Distance from row 1 col 3 of distance matrix (6)
Route from row 1 col 3 of route matrix (2), then from row 2 col 3 (4), then from row 4 col 3 (3). So 1 2 4 3.

B1 B1
B1
B1

(iii)

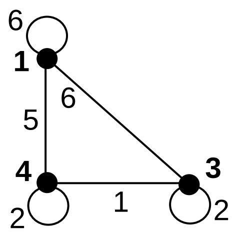


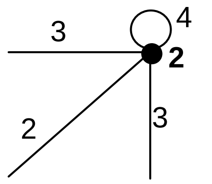
B1 whether or not loops included

(iv) **1 2 4 3 1**
length = 12
1 2 4 3 4 2 1

B1
B1
B1

(v)





MST has length 6, so lower bound = 6 + 2 + 3 = 11

M1
A1 MST
A1 add back

(vi) TSP length is either 11 or 12

B1 11 to 12
B1 either 11 or 12

4.

(i)

P	x	y	s_1	s_2	RHS
1	–1	–1	0	0	0
0	2	1	1	0	1250
0	2	–1	0	1	0
1	1	0	1	0	1250
0	2	1	1	0	1250
0	4	0	1	1	1250

M1 initial tableau
A1

M1 pivot
A2 (–1 each error)

1250 m^2 of paving and no decking — B1 interpretation

(ii) 2-phase

A	P	x	y	s_1	s_2	s_3	a	RHS
1	0	1	0	0	0	–1	0	200
0	1	1	0	1	0	0	0	1250
0	0	2	1	1	0	0	0	1250
0	0	4	0	1	1	0	0	1250
0	0	1	0	0	0	–1	1	200
1	0	0	0	0	0	0	–1	0
0	1	0	0	1	0	1	–1	1050
0	0	0	1	1	0	2	–2	850
0	0	0	0	1	1	4	–4	450
0	0	1	0	0	0	–1	1	200

M1 A1 new objective
B1 surplus
B1 artificial
B1 new constraint

M1
A2

Big-M alternative

P	*x*	*y*	s_1	s_2	s_3	*a*	*RHS*
1	*1–M*	*0*	*1*	*0*	*M*	*0*	*1250–2M*
0	*2*	*1*	*1*	*0*	*0*	*0*	*1250*
0	*4*	*0*	*1*	*1*	*0*	*0*	*1250*
0	*1*	*0*	*0*	*0*	*–1*	*1*	*200*
1	*0*	*0*	*1*	*0*	*1*	*M–1*	*1050*
0	*0*	*1*	*1*	*0*	*2*	*–2*	*850*
0	*0*	*0*	*1*	*1*	*4*	*–4*	*450*
0	*1*	*0*	*0*	*0*	*–1*	*1*	*200*

M1 A1 new objective
B1 surplus
B1 artificial
B1 new constraint

M1
A2

850 m^2 of paving and 200 m^2 of decking. — A1 interpretation

(iii)

C	x	y	s_1	s_2	s_3	s_4	RHS	Mark	Comment
1	0	0	1.25	0	1.75	0	1212.5	B1	new objective
0	0	1	1	0	2	0	850		
0	0	0	1	1	4	0	450		
0	1	0	0	0	–1	0	200		
0	0	0	1	0	1	1	50	B1	new constraint
1	0	0	–0.5	0	0	–1.75	1125		
0	0	1	–1	0	0	–2	750		
0	0	0	–3	1	0	–4	250	M1	
0	1	0	1	0	0	1	250	A1	
0	0	0	1	0	1	1	50		

750 m^2 of paving and 250 m^2 of decking at an annual cost of £1125 — A1 interpretation

4772: Decision Mathematics 2

General Comments

Again, candidates were mostly able and well-prepared, and gave good performances.

Comments on Individual Questions

1 **Logic**

(a) Most candidates were able to answer part (i) correctly, but many were floored by part (ii). Not all of those who gave the correct answer could justify it, and there were even some who produced correct arguments having given an incorrect answer.

(b) The truth table work was well done. Most attempting it had 8 rows to their tables. Many had their entries completely correct and some just made the odd slip.

(c) Few candidates were able to see through what was required here. This was the case even in instances where a thorough and correct line of reasoning had been supplied in part (a)(i).

2 **Decision Analysis**

Most candidates were able to score heavily on this question, but a substantial minority failed to produce a correct tree at the beginning of part (i). Many of those had an initial bifurcation for "first race/second race", with attendant confusion over the node type. Such candidates were struggling for marks after a misunderstanding of that magnitude.

The utility analysis in part (ii) required that candidates both applied the utility function to a payoff and multiplied by a probability so as to give an expected utility.

3 **Networks**

(i) Almost all candidates scored these marks.

(ii) Most were able to give both answers and explanations. Some gave the route as **1-2-3**, presumably thinking that the matrix represents the first vertex en route.

(iii) An easy mark, scored by most.

(iv) Also relatively easy, and high scoring. A few fell at the interpretation.

(v) Many candidates came adrift here by incorrectly applying the technique to the original network, instead of to the complete network of shortest distances. The resulting "lower bound" is 14, which is bigger than the upper bound of 12 found in part (iv). Candidates who found themselves in this situation seemed to be unconcerned or oblivious to the problem, and went on to make strange comments in part (vi).

(vi) Surprisingly few candidates were able to make the correct deduction that the answer is 11 or 12.

4 **LP**

Candidates were very competent in the basic techniques of simplex, as tested in part (i).

In part (ii) they were instructed to initiate an extended simplex application from their solution to part (i). This requires exactly the same skills as the setting up of a two-stage or big-M tableau, and many were able to do it. The instruction was intended as a help, since with the correctly deduced tableau, one iteration leads to optimality. There were many candidates who were not able to follow the instructions, and who set up a tableau "ab initio". For these candidates full marks were still available, but more iterations were needed, and many succeeded thus.

The structure of the final part was similar, but it would have been much more difficult to solve ab initio, and no-one did so.