RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 11 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

Section A (36 marks)

1 **(i)** Differentiate $\sqrt{1+2x}$. [3]

(ii) Show that the derivative of $\ln(1 - e^{-x})$ is $\dfrac{1}{e^x - 1}$. [4]

2 Given that $f(x) = 1 - x$ and $g(x) = |x|$, write down the composite function $gf(x)$.

On separate diagrams, sketch the graphs of $y = f(x)$ and $y = gf(x)$. [3]

3 A curve has equation $2y^2 + y = 9x^2 + 1$.

(i) Find $\dfrac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at the point A $(1, 2)$. [4]

(ii) Find the coordinates of the points on the curve at which $\dfrac{dy}{dx} = 0$. [4]

4 A cup of water is cooling. Its initial temperature is 100°C. After 3 minutes, its temperature is 80°C.

(i) Given that $T = 25 + ae^{-kt}$, where T is the temperature in °C, t is the time in minutes and a and k are constants, find the values of a and k. [5]

(ii) What is the temperature of the water

(A) after 5 minutes,

(B) in the long term? [3]

5 Prove that the following statement is false.

For all integers n greater than or equal to 1, $n^2 + 3n + 1$ is a prime number. [2]

6 Fig. 6 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{1}{2}\arctan x$.

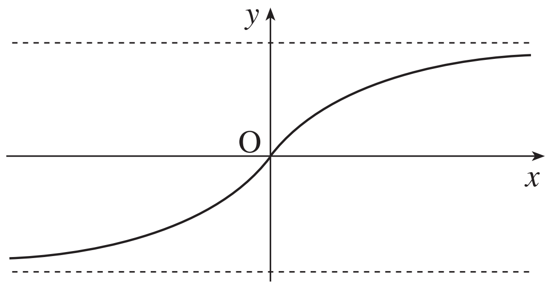

Fig. 6

(i) Find the range of the function $\mathrm{f}(x)$, giving your answer in terms of π. [2]

(ii) Find the inverse function $\mathrm{f}^{-1}(x)$. Find the gradient of the curve $y = \mathrm{f}^{-1}(x)$ at the origin. [5]

(iii) Hence write down the gradient of $y = \frac{1}{2}\arctan x$ at the origin. [1]

Section B (36 marks)

7 Fig. 7 shows the curve $y = \dfrac{x^2}{1+2x^3}$. It is undefined at $x = a$; the line $x = a$ is a vertical asymptote.

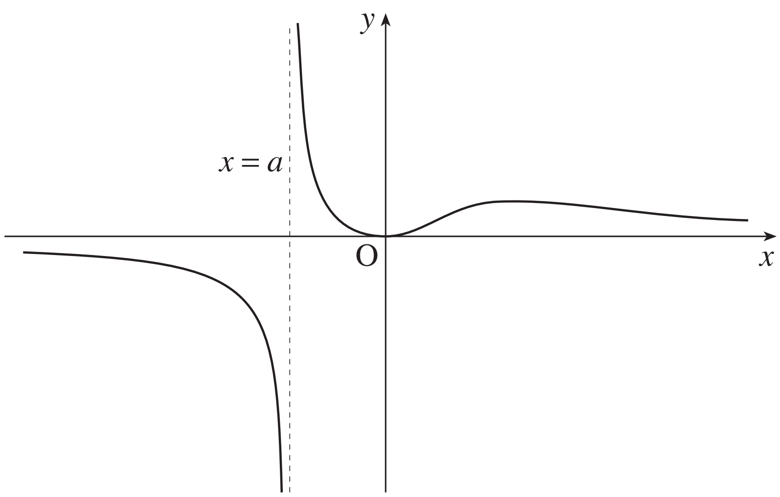

Fig. 7

(i) Calculate the value of a, giving your answer correct to 3 significant figures. [3]

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x - 2x^4}{(1+2x^3)^2}$. Hence determine the coordinates of the turning points of the curve. [8]

(iii) Show that the area of the region between the curve and the x-axis from $x = 0$ to $x = 1$ is $\frac{1}{6}\ln 3$. [5]

8 Fig. 8 shows part of the curve $y = x\cos 2x$, together with a point P at which the curve crosses the x-axis.

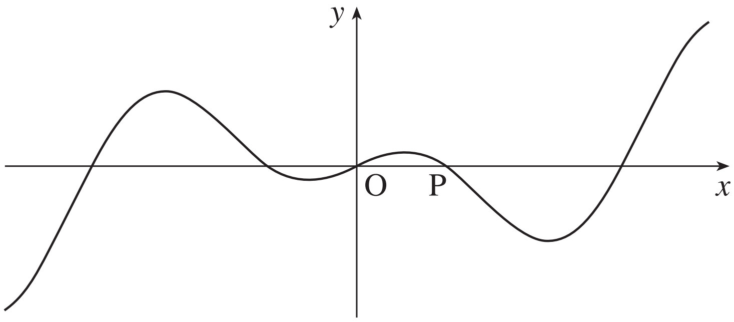

Fig. 8

(i) Find the exact coordinates of P. [3]

(ii) Show algebraically that $x\cos 2x$ is an odd function, and interpret this result graphically. [3]

(iii) Find $\dfrac{dy}{dx}$. [2]

(iv) Show that turning points occur on the curve for values of x which satisfy the equation $x\tan 2x = \frac{1}{2}$. [2]

(v) Find the gradient of the curve at the origin.

Show that the second derivative of $x\cos 2x$ is zero when $x = 0$. [4]

(vi) Evaluate $\displaystyle\int_0^{\frac{1}{4}\pi} x\cos 2x\,dx$, giving your answer in terms of π. Interpret this result graphically. [6]