OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4776/01**

MATHEMATICS (MEI)

Numerical Methods

THURSDAY 25 JANUARY 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

Section A (36 marks)

1 A calculator gives the answer to a calculation as $1.711\,224\,5 \times 10^{98}$, correct to 8 significant figures. Find the largest possible absolute error and the largest possible relative error in this value.

Though the calculator displays numbers such as 1.711 224 5 to 8 digit accuracy, it stores them internally to 11 digit accuracy. Explain briefly why this is done. [5]

2 The approximation

$$\tan x \approx x + \tfrac{1}{3}x^3$$

is valid for small values of x in radians.

(i) Find the absolute and relative errors in the approximation for $x = 0.2$. [4]

A much more accurate approximation is given by

$$\tan x \approx x + \tfrac{1}{3}x^3 + kx^5,$$

where k is a constant.

(ii) Use the first result in part **(i)** to estimate k, giving your answer to 2 significant figures. [3]

3 An equation is being solved numerically using a fixed-point iteration of the form

$$x_{r+1} = \mathrm{g}(x_r).$$

The iteration has been used to obtain the values shown in the following table.

r	0	1	2	3
x_r	0.35	0.354 767	0.356 462	0.357 067
Differences				
Ratio of differences				

Copy and complete the table to show the differences in successive values of x_r and the ratios of those differences. Use extrapolation to estimate the root to which this iteration is converging, giving your answer to the accuracy that appears justified. [8]

4 Show, graphically or otherwise, that the equation $x^2 = \cos x$ where x is in radians has exactly one root for $x > 0$. Show further that the root lies in the interval $(0.7, 0.9)$.

Use the secant method to find the root correct to 3 decimal places. [8]

5 The function $f(x)$ has the values shown in the table.

x	0	0.25	0.5
$f(x)$	1.1105	1.2446	1.4065

(i) Use the forward difference formula with $h = 0.5$ and $h = 0.25$ to obtain two estimates of $f'(0)$. Comment on the likely accuracy of these results and on the number of decimal places that it would be safe to quote. [4]

(ii) Obtain the best estimate you can of the value of $f'(0.25)$. Comment on the likely accuracy of this result in relation to those in part (**i**). To how many decimal places would you quote the answer? [4]

Section B (36 marks)

6 The following values of x and y were obtained in an experiment. The values of x are exact; the values of y are correct to 2 decimal places. It is required to estimate α, the value of x for which $y = 0$.

x	0.9	1.1	1.2	1.4	1.5
y	−0.43	−0.09	0.15	0.78	1.15

(i) Use Lagrange's method to find the equation of the straight line joining the data points for $x = 1.1$ and $x = 1.2$. Hence estimate α.

By considering the maximum possible errors in the values of y obtain a range of possible values of α. Hence give the value of α to the accuracy that is justified. [10]

(ii) Obtain a further estimate of α by fitting a quadratic to the data points for $x = 1.1, 1.2$ and 1.4. [8]

7 This question concerns the function $f(x) = x^{-x}$. (This can also be written as $f(x) = \frac{1}{x^x}$.) The table below shows some values of the function.

x	1	1.5	2
$f(x)$	1	0.544331	0.25

(i) Use the values in the table to find the Simpson's rule estimate of $\int_1^2 f(x)\,dx$ with $h = 0.5$.

Find the Simpson's rule estimate with $h = 0.25$. [7]

You are now given that the Simpson's rule estimate with $h = 0.125$ is 0.572 344 to 6 dp. Let the three Simpson's rule estimates with $h = 0.5, 0.25, 0.125$ be denoted by a, b and c respectively.

(ii) Find the value of the ratio of differences $\frac{c - b}{b - a}$. State the theoretical value of this ratio and comment. [5]

(iii) Extrapolate from b and c to obtain a further estimate of the integral.

Give the value of the integral to the accuracy that appears to be justified, explaining your reasoning. [6]