OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4771/01**

MATHEMATICS (MEI)

Decision Mathematics 1

TUESDAY 23 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Printed Answer Book
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the printed answer book.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

[Turn over

Answer all the questions in the printed answer book provided.

Section A (24 marks)

1 Each of the following symbols consists of boundaries enclosing regions.

The symbol representing zero has three regions, the outside, that between the two boundaries and the inside.

To classify the symbols a graph is produced for each one. The graph has a vertex for each region, with arcs connecting regions which share a boundary. Thus the graph for

 is .

(i) Produce the graph for the symbol  . [1]

(ii) Give two symbols each having the graph . [2]

(iii) Produce the graph for the symbol  . [2]

(iv) Produce a single graph for the composite symbol  . [2]

(v) Give the name of a connected graph with n nodes and $n - 1$ arcs. [1]

2 The following algorithm is a version of bubble sort.

Step 1 Store the values to be sorted in locations L(1), L(2), … , L(n) and set i to be the number, n, of values to be sorted.

Step 2 Set j = 1.

Step 3 Compare the values in locations L(j) and L(j+1) and swap them if that in L(j) is larger than that in L(j+1).

Step 4 Add 1 to j.

Step 5 If j is less than i then go to step 3.

Step 5 Write out the current list, L(1), L(2), … , L(n).

Step 6 Subtract 1 from i.

Step 7 If i is larger than 1 then go to step 2.

Step 8 Stop.

(i) Apply this algorithm to sort the following list.

109 32 3 523 58.

Count the number of comparisons and the number of swaps which you make in applying the algorithm. [4]

(ii) Put the five values into the order which maximises the number of swaps made in applying the algorithm, and give that number. [2]

(iii) Bubble sort has quadratic complexity. Using bubble sort it takes a computer 1.5 seconds to sort a list of 1000 values. Approximately how long would it take to sort a list of 100 000 values? (Give your answer in hours and minutes.) [2]

3 A bag contains five pieces of paper labelled A, B, C, D and E. One piece is drawn at random from the bag. If the piece is labelled with a vowel (A or E) then the process stops. Otherwise the piece of paper is replaced, the bag is shaken, and the process is repeated. You are to simulate this process to estimate the mean number of draws needed to get a vowel.

(i) Show how to use single digit random numbers to simulate the process efficiently. You need to describe exactly how your simulation will work. [3]

(ii) Use the random numbers in your answer book to run your simulation 5 times, recording your results. [2]

(iii) From your results compute an estimate of the mean number of draws needed to get a vowel. [2]

(iv) State how you could produce a more accurate estimate. [1]

[Turn over

Section B (48 marks)

4 Cassi is managing the building of a house. The table shows the major activities that are involved, their durations and their precedences.

Activity		Duration (days)	Immediate predecessors
A	Build concrete frame	10	–
B	Lay bricks	7	A
C	Lay roof tiles	10	A
D	First fit electrics	5	B
E	First fit plumbing	4	B
F	Plastering	6	C, D, E
G	Second fit electrics	3	F
H	Second fit plumbing	2	F
I	Tiling	10	G, H
J	Fit sanitary ware	2	H
K	Fit windows and doors	5	I

(i) Draw an activity-on-arc network to represent this information. [5]

(ii) Find the early time and the late time for each event. Give the project duration and list the critical activities. [6]

(iii) Calculate total and independent floats for each non-critical activity. [2]

Cassi's clients wish to take delivery in 42 days. Some durations can be reduced, at extra cost, to achieve this.

- The tiler will finish activity I in 9 days for an extra £250, or in 8 days for an extra £500.
- The bricklayer will cut his total of 7 days on activity B by up to 3 days at an extra cost of £350 per day.
- The electrician could be paid £300 more to cut a day off activity D, or £600 more to cut two days.

(iv) What is the cheapest way in which Cassi can get the house built in 42 days? [3]

5 Leone is designing her new garden. She wants to have at least 1000 m^2, split between lawn and flower beds.

Initial costs are £0.80 per m^2 for lawn and £0.40 per m^2 for flowerbeds. Leone's budget is £500.

Leone prefers flower beds to lawn, and she wants the area for flower beds to be at least twice the area for lawn. However, she wants to have at least 200 m^2 of lawn.

Maintenance costs each year are £0.15 per m^2 for lawn and £0.25 per m^2 for flower beds. Leone wants to minimize the maintenance costs of her garden.

(i) Formulate Leone's problem as a linear programming problem. [7]

(ii) Produce a graph to illustrate the inequalities. [6]

(iii) Solve Leone's problem. [2]

(iv) If Leone had more than £500 available initially, how much extra could she spend to minimize maintenance costs? [1]

6 In a factory a network of pipes connects 6 vats, A, B, C, D, E and F. Two separate connectors need to be chosen from the network The table shows the lengths of pipes (metres) connecting the 6 vats.

	A	B	C	D	E	F
A	–	7	–	–	12	–
B	7	–	5	3	6	6
C	–	5	–	8	4	7
D	–	3	8	–	1	5
E	12	6	4	1	–	7
F	–	6	7	5	7	–

(i) Use Kruskal's algorithm to find a minimum connector. Show the order in which you select pipes, draw your connector and give its total length. [5]

(ii) Produce a new table excluding the pipes which you selected in part **(i)**. Use the tabular form of Prim's algorithm to find a second minimum connector from this reduced set of pipes. Show your working, draw your connector and give its total length. [7]

(iii) The factory manager prefers the following pair of connectors:

{AB, BC, BD, BE, BF} and {AE, BF, CE, DE, DF}.

Give two possible reasons for this preference. [4]

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MATHEMATICS (MEI)

Decision Mathematics 1

ANSWER BOOK

TUESDAY 23 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Candidate Name	

Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided.
- Write your answers in the spaces provided on the answer book. If extra space is required use the blank pages making sure that you label your work clearly.

This answer book consists of **8** printed pages and **4** blank pages.

1 (i)

(ii)

(iii)

(iv)

(v)

2 (i)

Original list	109	32	3	523	58	Comparisons	Swaps

(ii)

Maximum number of swaps = …….

(iii)

3 (i)

(ii) Run 1 random numbers 2 2 1 7 6 4 3 4 0 9 6 1 5 8 8

Run 1 outcome ……

Run 2 random numbers 6 2 3 8 7 4 6 1 0 9 0 6 4 2 5

Run 2 outcome ……

Run 3 random numbers 5 6 4 9 0 1 8 3 5 4 7 2 1 0 0

Run 3 outcome ……

Run 4 random numbers 5 3 6 1 0 8 4 7 9 8 2 2 2 7 4

Run 4 outcome ……

Run 5 random numbers 1 2 1 6 5 0 9 7 7 6 9 8 4 3 3

Run 5 outcome ……

(iii)

(iv)

4 (i) & (ii)

Project duration: …….....

Critical activities: ……

(iii)

(iv)

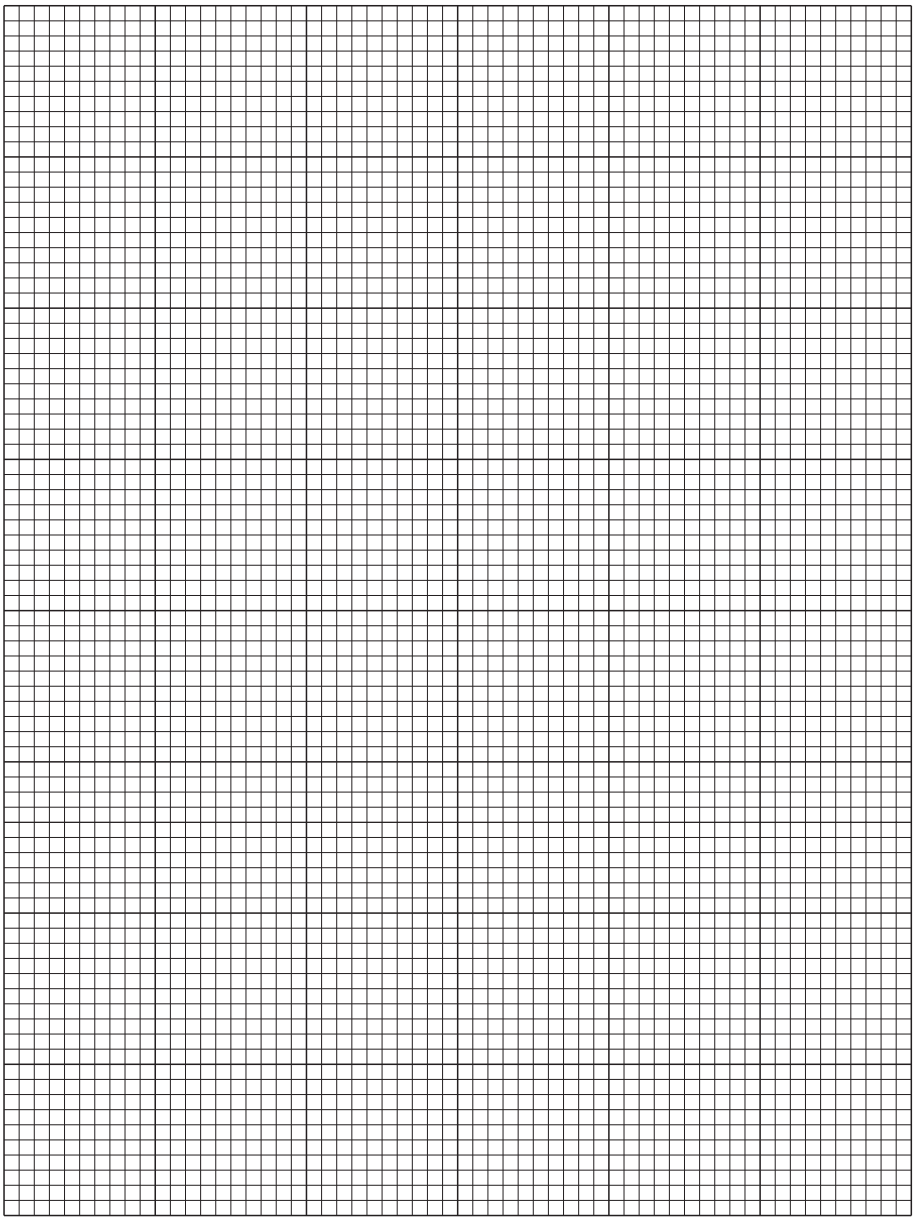
5 **(i)**

(ii) (See opposite for graph paper.)

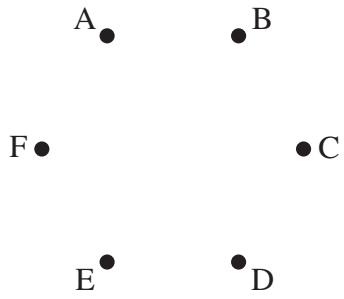
(iii)

(iv)

5 (ii)



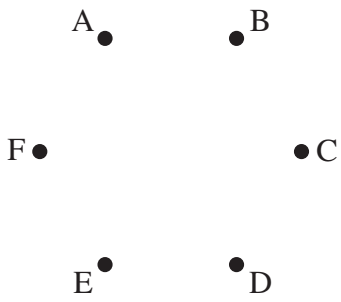
6 **(i)** Order of arc selection:



Total length of connector = ……..

(ii)

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						



Total length of connector = ……..

(iii)

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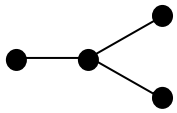
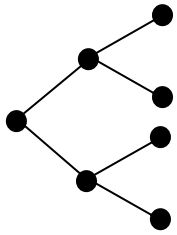
1.

(i) 	B1
(ii) Any two of 1 or 2 or 3 or 5 or 7	B1 B1
(iii) 	M1 branching tree A1
(iv) 	M1 branching tree A1
(v) A tree	B1

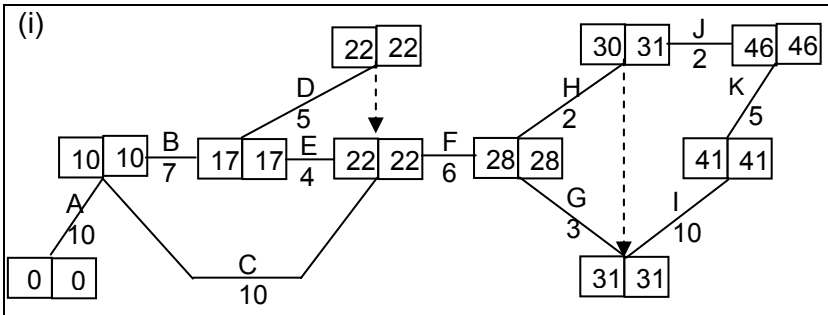
2.

(i) 109; 32; 3; 523; 58 32; 3; 109; 58; 523 4 comparisons and 3 swaps 3; 32; 58; 109; 523 3 and 2 3; 32; 58; 109; 523 2 and 0 3; 32; 58; 109; 523 1 and 0 10 and 5 in total	M1 A1 only if all iterations completed B1 B1
(ii) 523; 109; 58; 32; 3 10 swaps	B1 B1
(iii) 1.5×100^2 = 15000 seconds = 4 hrs 10 mins	M1 A1 hours and minutes

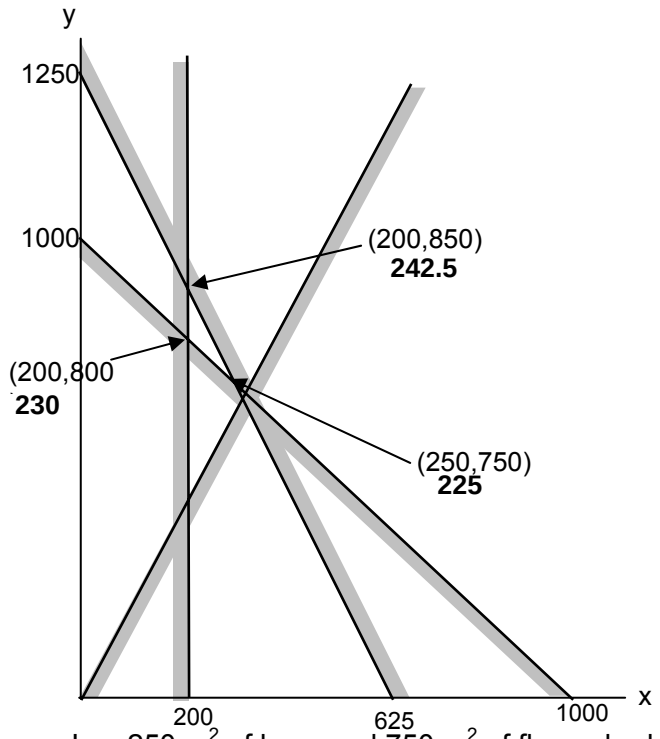
3.

(i) e.g. 0, 1 → A 2, 3 → B 4, 5 → C 6, 7 → D 8, 9 → E	M1 A1 proportions OK B1 efficient
(ii) e.g: 3, 4, 4, 4, 1	M1 A1
(iii) In the above simulation mean = 3.2 (Correct expectation is 2.5 – geometric rand variable)	M1 A1
(iv) More repetitions	B1

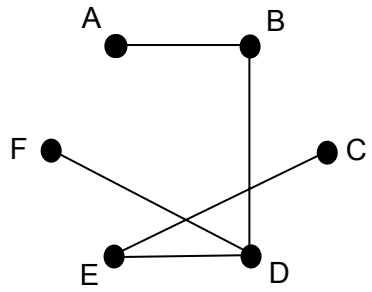
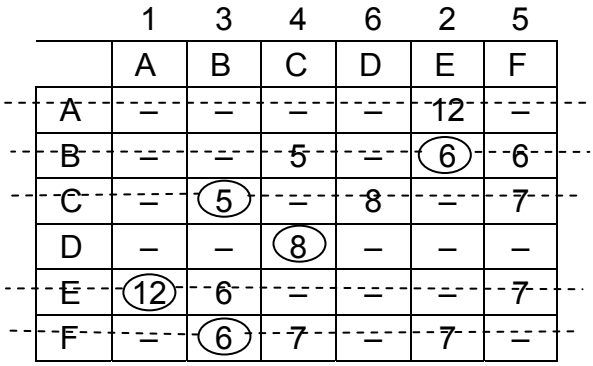
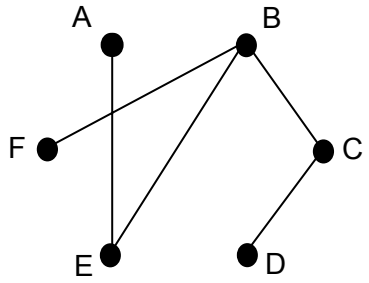
4.

(i)  	M1 activity-on-arc A1 single start and end A1 dummy 1 A1 dummy 2 A1 rest
(ii) See above Critical activities: A; B; D; F; G; I; K Duration = 46	M1 A1 forward pass M1 A1 backward pass B1 critical activities B1 duration
(iii) E: total float = 1; independent float = 1 H: 1 and 0 J: 14 and 13 C: 2 and 2	B1 total floats B1 independent floats
(iv) Tiler (I) – 2 days – £500 Electrician (D) – 1 day – £300 Bricklayer (B) – 1 day – £350	B1 tiler B1 electrician B1 bricklayer

5.

(i) Let x be the number of m^2 of lawn. Let y be the number of m^2 of flower beds.	B1
$x + y \geq 1000$	B1
$0.80x + 0.40y \leq 500$, i.e. $2x + y \leq 1250$	B1
$y \geq 2x$	B1
$x \geq 200$	B1
Minimise $0.15x + 0.25y$	B1 B1
(ii) & (iii)  Lay 250 m^2 of lawn and 750 m^2 of flower beds. Annual maintenance = £225.	B1 axes labelled + scaled B4 lines B1 shading M1 A1
(iv) Intersection of $y \geq 2x$ & area constraint is at (333.33,666.67) so max useful capital is £533.33. So £33.33.	B1 (allow £533.33)

6.

(i) DtoE; BtoD; CtoE; DtoF; AtoB	M1 A1 no BC nor BE A1
	B1
Total length = 20	B1
(iii) e.g.	
	B1 reduced table M1 delete/select/delete A1 first 2 rows A1 rest of table A1 order
	B1
Total length = 37	B1
(iii) Lengths are 27 and 28. Shorter and more nearly equal.	B1 B1 B1 B1

4771 - Decision Mathematics 1

General Comments

This was the first session in which candidates were provided with a printed answer book. This seemed to work well. There were examples in which candidates did not have sufficient space, and had to use supplementary sheets, or where second attempts were needed. But such inconveniencies were greatly outweighed by the positive benefits for most students.

It is hoped that candidates will only need the printed answer book to write on, which is provided inside the question paper. For the summer session, 4-page answer booklets and graph paper will be available should the candidate request them.

Candidates were generally well prepared.

Comments on Individual Questions

1 **Graphs**

This was a very straightforward first question, and most candidates did very well on it.

2 **Algorithms**

(i) Examiners often found it difficult to see whether or not candidates had fully followed the algorithm, including the last two iterations on which no swaps were made.

(ii) Many candidates were able to give the correct ordering, and the majority of those were able to count 10 swaps.

(iii) Only about 25% of candidates were able to do the quadratic computation.

3 **Simulation**

Most candidates coped very well with this question. Quite a number insisted on giving their mean in part (iii) to the nearest integer, and were penalised when they did so. A minority erroneously thought that the accuracy could be improved by using 2-digit random numbers.

4 **CPA**

(i) & (ii) These parts were well done. Again, it was pleasing to see an aspect of modelling being tackled so well. (This copied from last summer's report.) Having said that, there was an unfortunate resurgence of "activity-on-node" from some centres. This gains no credit.
One recurrent minor error was having activities D and E share the same "i" and "j" events – a dummy was needed.

(iii) Fewer than 50% of candidates were able to demonstrate knowledge about both total and independent float. A few candidates who did have that knowledge proceeded to incorrect answers as a consequence of unnecessary dummy activities.

(iv) Able candidates found this very easy. Less able did not. It was a good discriminator.

5 **LP**

(i) The June 2006 report on this question started "How **do** we persuade candidates properly to define their variables?" Some improvement was seen this session, although far too many candidates stated "Let x = lawn …" etc. Nearly all candidates who failed in all or part of the subsequent formulation, and there were many, had failed properly to define their variables.

(ii) Graphs were often better than might have been expected from formulations. In particular the graph of y=2x (or equivalent) was seen more often than was the expression y=2x.

(iii) Surprisingly few candidates scored the marks here. They needed to be evaluating at vertices and comparing, or to be clearly applying an objective gradient.

(iv) Only a few succeeded with this, as had been expected.

6 **Networks**

(i) Most candidates were able to apply Kruskal successfully.

(ii) Candidates needed to convince the examiner that they were in fact applying the tabular form of Prim – not all did so.

(iii) This was the least satisfactory part of the paper. The point of the question was that a greedy approach – choose the minimum connector followed by the minimum connector of the remainder – does not produce the best answer. Allowing a suboptimal "first" connector allows, in this case, for the second connector more than to compensate.
The question, the last on the paper, was deliberately left very open-ended, and the outcomes were very poor. Students interpreted the invitation to give possible reasons as an excuse to let their imaginations run riot. Nearly all of the answers offered involved suppositions, with no basis to support them.
In such questions candidates should restrict themselves to that which is known, and that will almost always be what is given. (There are few cases in which knowledge of a real world situation can be assumed across the candidature.) In this case the knowns were pipe lengths, and it was there that candidates should have been focusing in constructing their answers.