RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Mark Schemes for the Units

January 2007

3895-8/7895-8/MS/R/07J

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MARK SCHEME ON THE UNITS

Mark Scheme 4751
January 2007

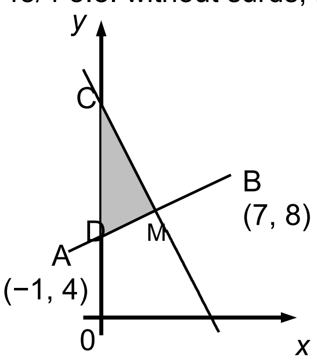
Section A

1	$y = 2x + 4$	3	M1 for $m = 2$ stated [M0 if go on to use $m = -½$] or M1 for $y = 2x + k$, $k \neq 7$ and M1indep for $y - 10 = m(x - 3)$ or (3, 10) subst in $y = mx + c$; allow 3 for $y = 2x + k$ and $k = 4$	3
2	neg quadratic curve intercept (0, 9) <u>through</u> (3, 0) and (−3, 0)	1 1 1	condone (0, 9) seen eg in table	3
3	$[a =]\dfrac{2c}{2-f}$ or $\dfrac{-2c}{f-2}$ as final answer	3	M1 for attempt to collect *a*s and *c*s on different sides and M1 ft for $a(2 - f)$ or dividing by $2 - f$; allow M2 for $\dfrac{7c-5c}{2-f}$ etc	3
4	f(2) = 3 seen or used $2^3 + 2k + 5 = 3$ o.e. $k = -5$	M1 M1 B1	allow M1 for divn by $(x - 2)$ with $x^2 + 2x + (k + 4)$ or $x^2 + 2x - 1$ obtained <u>alt:</u> M1 for $(x - 2)(x^2 + 2x - 1) + 3$ (may be seen in division) then M1dep (and B1) for $x^3 - 5x + 5$ <u>alt</u> divn of $x^3 + kx + 2$ by $x - 2$ with no rem.	3
5	375	3	allow $375x^4$; M1 for 5^2 or 25 used or seen with x^4 and M1 for 15 or $\dfrac{6\times5}{2}$ oe eg $\dfrac{6!}{4!2!}$ or 1 6 15 … seen [$^6C_4$ not sufft]	3
6	(i) 125 (ii) $\dfrac{9}{49}$ as final answer	2 2	M1 for $25^{\frac{1}{2}} = \sqrt{25}$ soi or for $\sqrt{25^3}$ M1 for $a^{-1} = \dfrac{1}{a}$ soi eg by 3/7 or 3/49	4
7	showing $a + b + c = 6$ o.e $bc = \dfrac{9^2 - 17}{16}$ =64/16 o.e. correctly obtained completion showing $abc = 6$ o.e.	1 M1 A1 A1	simple equiv fraction eg 192/32 or 24/4 correct expansion of numerator; may be unsimplified 4 term expansion; M0 if get no further than $\left(\sqrt{17}\right)^2$; M0 if no evidence before 64/16 o.e. may be implicit in use of factors in completion	4

8	$b^2 - 4ac$ soi use of $b^2 - 4ac$ **<** 0 $k^2 < 16$ [may be implied by $k < 4$] $-4 < k < 4$ or $k > -4$ and $k < 4$ isw	M1 M1 A1 A1	may be implied by $k^2 < 16$ deduct one mark in qn for ≤ instead of <; allow equalities earlier if final inequalities correct; condone b instead of k; if M2 not earned, give SC2 for qn [or M1 SC1] for k [=] 4 and − 4 as answer]	4
9	(i) $12a^5b^3$ as final answer (ii) $\frac{(x+2)(x-2)}{(x-2)(x-3)}$ $\frac{x+2}{x-3}$ as final answer	2 M2 A1	1 for 2 'terms' correct in final answer M1 for each of numerator or denom. correct or M1, M1 for correct factors seen separately	5
10	correct expansion of both brackets seen (may be unsimplified), or difference of squares used $4m^2$ correctly obtained [p =] [±]$2m$ cao	M2 A1 A1	M1 for one bracket expanded correctly; for M2, condone done together and lack of brackets round second expression if correct when we insert the pair of brackets	4

Section B

11	**iA**	0.2 to 0.3 and 3.7 to 3.8	1+1	[tol. 1mm or 0.05 throughout qn]; if 0, allow M1 for drawing down lines at both values	2
	iB	$x + \frac{1}{x} = 4 - x$ their $y = 4 - x$ drawn 0.2 to 0.35 and 1.65 to 1.8	M1 M1 B2	condone one error allow M2 for plotting positive branch of $y = 2x + 1/x$ [plots at (1,3) and (2,4.5) and above other graph] or for plot of $y = 2x^2 - 4x + 1$ 1 each	4
	ii	(0, ±√3)	2	condone y = ±√3 isw; 1 each or M1 for $1 + y^2 = 4$ or $y^2 = 3$ o.e.	2
	iii	centre (1, 0) radius 2 touches at (1, 2) [which is distance 2 from centre] all points on other branch > 2 from centre	1+1 1 1	allow seen in (ii) allow ft for both these marks for centre at (−1, 0), rad 2; allow 2 for good sketch or compass-drawn circle of rad 2 centre (±1, 0)	4

12	**i**	(3, 6)	2	1 each coord	
		grad AB = (8 − 4)/(7 − −1) or 4/8 grad normal = −2 or ft	M1 M1	indep obtained for use of $m_1m_2 = -1$; condone stated/used as −2 with no working only if 4/8 seen	
		perp bisector is $y - 6 = -2(x - 3)$ or ft their grad. of normal (not AB) and/or midpoint correct step towards completion	M1 A1	or M1 for showing grad given line = −2 and M1 for showing (3, 6) fits given line	6
	ii	Bisector crosses y axis at C (0, 12) seen or used	M1	may be implicit in their area calcn	
		AB crosses y axis at D (0, 4.5) seen or used	B2	M1 for 4 + their grad AB or for eqn AB is $y - 8$ = their ½ $(x - 7)$ oe with coords of A or their M used	
		½ × (12 − their 4.5) × 3 (may be two triangles M1 each)	M2	or M1 for $[MC]^2 = 3^2 + 6^2$ or 45 or $[MD]^2 = 3^2 + 1.5^2$ or 11.25 oe and M1 for ½ × their MC × MD; all ft their M	
		45/4 o.e. without surds, isw 	A1	<u>MR</u>: AMC used not DMC: lose B2 for D but then allow ft M1 for MC^2 or MA^2 $[=4^2 + 2^2]$ and M1 for ½ × MA × MC and A1 for 15 <u>MR</u>: intn used as D(0, 4) can score a max of M1, B0, M2 (eg M1 for their DM = √13), A0	
		<u>alt</u> allow integration used: $\int_0^3 (-2x+12)\mathrm{d}x\ [=27]$	M1	condone poor notation	
		obtaining AB is $y - 8$ = their ½ $(x - 7)$ oe $[y = \frac{1}{2}x + 4.5]$	M1	allow if seen, with correct line and limits seen/used as above	
		$\int_0^3 (\frac{1}{2}x+4.5)\mathrm{d}x$	M1	ft from their AB	
		= 63/4 o.e. cao	A1		
		their area under CB − their area under AB	M1	allow only if at least some valid integration/area calculations for these trapezia seen	
		= 45/4 o.e. cao	A1	if combined integration, so 63/4 not found separately, mark equivalently for Ms and allow A2 for final answer	6
13	**i**	$x - 2$ is factor soi attempt at divn by $x - 2$ as far as $x^3 - 2x^2$ seen in working	M1 M1	eg may be implied by divn or other factor $(x^2 \ldots -1)$ or $(x^2 + 2x\ldots)$	
		$x^2 + 2x - 1$ obtained	A1	or B3 www	
		attempt at quad formula or comp square	M1	ft their quadratic	
		$-1 \pm \sqrt{2}$ as final answer	A2	A1 for $\frac{-2 \pm \sqrt{8}}{2}$ seen; or B3 www	6

	ii	$f(x-3) = (x-3)^3 - 5(x-3) + 2$ $(x-3)(x^2-6x+9)$ or other constructive attempt at expanding $(x-3)^3$ eg 1 3 3 1 soi $x^3 - 9x^2 + 27x - 27$ $-5x + 15$ [+2]	B1 M1 A1 B1	or $(x-5)(x-2+\sqrt{2})(x-2-\sqrt{2})$ soi or ft from their (i) for attempt at multiplying out 2 brackets or valid attempt at multiplying all 3 alt: A2 for correct full unsimplified expansion or A1 for correct 2 bracket expansion eg $(x-5)(x^2-4x+2)$	4
	iii	5 $2 \pm \sqrt{2}$ or ft	B1 B1	condone factors here, not roots if B0 in this part, allow SC1 for their roots in (i) − 3	2

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Section A

1	$\frac{5}{2} \times 6x^{\frac{3}{2}}$	1+1	- 1 if extra term	2
2	-0.2	3	M1 for $5 = \frac{6}{1-r}$ and M1 dep for correct constructive step	3
3	√8 or 2√2 not ±√8	3	M1 for use of $\sin^2 \theta + (1/3)^2 = 1$ and M1for sinθ = √8/3 (ignore ±) Diag.: hypot = 3, one side =1 M1 3rd side √8 M1	3
4	(i) C (ii) B (iii) 2^{n-1}	1 1 1		3
5	(i) −0.93, -0.930, -0.9297… (ii) answer strictly between 1.91 and 2 or 2 and 2.1 (iii) $y' = -8/x^3$, gradient = −1	2 B1 M1A1	M1 for grad = (1 – their y_B)/(2 − 2.1) if M0, SC1 for 0.93 don't allow 1.9 recurring	5
6	At least one cycle from (0, 0) amplitude 1 and period 360[°] indicated 222.8 to 223 and 317 to 317.2 [°]	G1 G1dep 2	1 each, ignore extras	4
7	$x < 0$ and $x > 6$	3	B2 for one of these or for 0 and 6 identified or M1 for $x^2 - 6x > 0$ seen (M1 if y found correctly and sketch drawn)	3
8	$a + 6d = 6$ correct $30 = \frac{10}{2}(2a+9d)$ correct o.e. elimination using their equations $a = -6$ and $d = 2$ 5th term = 2	M1 M1 M1f.t. A1 A1	Two equations in a and d	5
9	$(y =)\ 2x^3 + 4x^2 - 1$ accept $2x^3 + 4x^2 + c$ <u>and</u> c = - 1	4	M2 for $(y =)\ 2x^3 + 4x^2 + c$ (M1 if one error) and M1 for subst of (1, 5) dep on their y =, +c, integration attempt.	4
10	(i) 3 $\log_a x$ ii) $b = \frac{1000}{c}$	2 2	M1 for 4 $\log_a x$ or − $\log_a x$; or $\log x^3$ M1 for 1000 or 10^3 seen	4

Section B

11	**i**	Correct attempt at cos rule correct full method for C C = 141.1… bearing = [0]38.8 cao	M1 M1 A1 A1	any vertex, any letter or B4	 4
	ii	½ × 118 × 82 × sin their C or supp. 3030 to 3050 [m^2]	M1 A1	or correct use of angle A or angle B	2
	iiiA	sin (θ/2) = (½ × 189)/130 1.6276 → 1.63	M1 A1	or cosθ = ($130^2+130^2-189^2$)/(2x130x130) In all methods, the more accurate number to be seen.	2
	iiiB	$0.5 \times 130^2 \times \sin 1.63$ $0.5 \times 130^2 \times 1.63$ their sector – their triangle AOB 5315 to 5340	M1 M1 M1 A1	condone their θ (8435) condone their θ in radians (13770) dep on sector > triangle	 4
12	**i**	(2*x* − 3)(*x* − 4) *x* = 4 or 1.5	M1 A1A1	or (11 ±√(121 – 96))/4 if M0, then B1 for showing *y* = 0 when *x* = 4 and B2 for x = 1.5	3
	ii	*y*′ = 4*x* − 11 = 5 when *x* = 4 c.a.o. grad of normal = −1/their *y*′ *y*[− 0]=<u>their</u> −0.2 (*x* − 4) y-intercept for <u>their</u> normal area = ½ × 4 × 0.8 c.a.o.	M1 A1 M1f.t. M1 B1f.t. A1	condone one error or 0 = their (-0.2)x4 + c dep on normal attempt s.o.i. normal must be linear or integrating <u>their</u> f(x) from 0 to 4 M1	6
	iii	$\frac{2}{3}x^3 - \frac{11}{2}x^2 + 12x$ attempt difference between value at 4 and value at 1.5 [−]$5\frac{5}{24}$ o.e. or [−]5.2(083..)	M1 M1 A1	condone one error, ignore + c ft their (i), dep on integration attempt. c.a.o.	3
13	**i**	$\log_{10} y = \log_{10} k + \log_{10} 10^{ax}$ $\log_{10} y = ax + \log_{10} k$ compared to y = mx+c	M1 M1		2
	ii	2.9(0), 3.08, 3.28, 3.48, 3.68 plots [tol 1 mm] ruled line of best fit drawn	T1 P1f.t L1f.t.	condone one error	3
	iii	intercept = 2.5 approx gradient = 0.2 approx y = their 300x $10^{x(\text{their } 0.2)}$ or y = $10^{(\text{their } 2.5 + \text{their } 0.2x)}$	M1 M1 M1f.t.	or y – 2.7 = m(x – 1)	3
	iv	subst 75000 in any x/y eqn subst in a correct form of the relationship 11,12 or 13	M1 M1 A1	 B3 with evidence of valid working	3
	v	"Profits change" or any reason for this.	R1	too big, too soon	1

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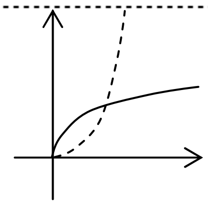
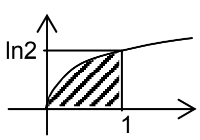
Section A

1 **(i)** P is (2, 1)	B1	
(ii) $\lvert x\rvert = 1\frac{1}{2}$ $\Rightarrow x = (-1\frac{1}{2})$ or $1\frac{1}{2}$ $\lvert x-2\rvert + 1 = 1\frac{1}{2} \Rightarrow \lvert x-2\rvert = \frac{1}{2}$ $\Rightarrow x = (2\frac{1}{2})$ or $1\frac{1}{2}$	M1 A1 M1 E1	allow $x = 1½$ unsupported or $\left\lvert 1\frac{1}{2} - 2\right\rvert + 1 = \frac{1}{2} + 1 = 1\frac{1}{2}$
or by solving equation directly: $\lvert x-2\rvert + 1 = \lvert x\rvert$ $\Rightarrow 2 - x + 1 = x$ $\Rightarrow x = 1½$ $\Rightarrow y = \lvert x\rvert = 1½$	M1 M1 A1 E1 [4]	equating from graph or listing possible cases
2 $\int_1^2 x^2 \ln x dx$ $u = \ln x$ $dv/dx = x^2 \Rightarrow v = \frac{1}{3}x^3$ $= \left[\frac{1}{3}x^3 \ln x\right]_1^2 - \int_1^2 \frac{1}{3}x^3 . \frac{1}{x} dx$ $= \frac{8}{3}\ln 2 - \int_1^2 \frac{1}{3}x^2 dx$ $= \frac{8}{3}\ln 2 - \left[\frac{1}{9}x^3\right]_1^2$ $= \frac{8}{3}\ln 2 - \frac{8}{9} + \frac{1}{9}$ $= \frac{8}{3}\ln 2 - \frac{7}{9}$	M1 A1 A1 M1 A1 cao [5]	Parts with $u = \ln x$ $dv/dx = x^2 \Rightarrow v = x^3/3$ $\left[\frac{1}{9}x^3\right]$ substituting limits o.e. – not ln 1
3 **(i)** When $t = 0$, $V = 10\,000$ $\Rightarrow 10\,000 = Ae^0 = A$ When $t = 3$, $V = 6000$ $\Rightarrow 6000 = 10\,000\, e^{-3k}$ $\Rightarrow -3k = \ln(0.6) = -0.5108…$ $\Rightarrow k = 0.17(02…)$	M1 A1 M1 M1 A1 [5]	$10\,000 = Ae^0$ $A = 10\,000$ taking lns (correctly) on their exponential equation - not logs unless to base 10 art 0.17 or $-(\ln 0.6)/3$ oe
(ii) $2000 = 10\,000e^{-kt}$ $\Rightarrow -kt = \ln 0.2$ $\Rightarrow t = -\ln 0.2 / k = 9.45$ (years)	M1 A1 [2]	taking lns on correct equation (consistent with their k) allow art 9.5, but not 9.

4 Perfect squares are 0, 1, 4, 9, 16, 25, 36, 49, 64, 81 none of which end in a 2, 3, 7 or 8. Generalisation: no perfect squares end in a 2, 3, 7 or 8.	M1 E1 B1 [3]	Listing all 1- and 2- digit squares. Condone absence of 0^2, and listing squares of 2 digit nos (i.e. $0^2 – 19^2$) For extending result to include further square numbers.
5 (i) $y=\frac{x^2}{2x+1}$ $\Rightarrow \frac{dy}{dx}=\frac{(2x+1)2x-x^2.2}{(2x+1)^2}$ $=\frac{2x^2+2x}{(2x+1)^2}=\frac{2x(x+1)}{(2x+1)^2}$ *	M1 A1 A1 E1 [4]	Use of quotient rule (or product rule) Correct numerator – condone missing bracket provided it is treated as present Correct denominator www –do not condone missing brackets
(ii) $\frac{dy}{dx}=0$ when $2x(x + 1) = 0$ $\Rightarrow$ $x = 0$ or -1 $y = 0$ or -1	B1 B1 B1 B1 [4]	Must be from correct working: SC −1 if denominator = 0
6(i) QA = $3 – y$, PA = $6 – (3 – y) = 3 + y$ By Pythagoras, $PA^2 = OP^2 + OA^2$ $\Rightarrow (3 + y)^2 = x^2 + 3^2 = x^2 + 9$. *	B1 B1 E1 [3]	must show some working to indicate Pythagoras (e.g. $x^2 + 3^2$)
(ii) Differentiating implicitly: $2(y+3)\frac{dy}{dx}=2x$ $\Rightarrow \frac{dy}{dx}=\frac{x}{y+3}$ *	M1 E1	Allow errors in RHS derivative (but not LHS) - notation should be correct brackets must be used
or $9 + 6y + y^2 = x^2 + 9$ $\Rightarrow 6y + y^2 = x^2$ $\Rightarrow (6+2y)\frac{dy}{dx}=2x$ $\Rightarrow \frac{dy}{dx}=\frac{x}{y+3}$	M1 E1	Allow errors in RHS derivative (but not LHS) - notation should be correct brackets must be used
or $y = \sqrt{(x^2+ 9)} – 3 \Rightarrow dy/dx = ½(x^2+9)^{-1/2}.2x$ $=\frac{x}{\sqrt{x^2+9}}=\frac{x}{y+3}$	M1 E1	(cao)
(iii) $\frac{dy}{dt}=\frac{dy}{dx}.\frac{dx}{dt}$ $=\frac{4}{2+3}\times 2$ $=\frac{8}{5}$	M1 A1 A1 [3]	chain rule (soi)

Section B

7(i) When $x = -1$, $y = -1\sqrt{0} = 0$ Domain $x \geq -1$	E1 B1 [2]	Not $y \geq -1$
(ii) $\frac{dy}{dx} = x.\frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}$ $= \frac{1}{2}(1+x)^{-1/2}[x + 2(1+x)]$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	B1 B1 M1 E1	$x.\frac{1}{2}(1+x)^{-1/2}$ … + $(1+x)^{1/2}$ taking out common factor or common denominator www
or $u = x + 1 \Rightarrow du/dx = 1$ $\Rightarrow y = (u-1)u^{1/2} = u^{3/2} - u^{1/2}$ $\Rightarrow \frac{dy}{du} = \frac{3}{2}u^{\frac{1}{2}} - \frac{1}{2}u^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{3}{2}(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}$ $= \frac{1}{2}(x+1)^{-\frac{1}{2}}(3x+3-1)$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	M1 A1 M1 E1 [4]	 taking out common factor or common denominator
(iii) $dy/dx = 0$ when $3x + 2 = 0$ $\Rightarrow x = -2/3$ $\Rightarrow y = -\frac{2}{3}\sqrt{\frac{1}{3}}$ Range is $y \geq -\frac{2}{3}\sqrt{\frac{1}{3}}$	M1 A1cao A1 B1 ft [4]	 o.e. not $x \geq -\frac{2}{3}\sqrt{\frac{1}{3}}$ (ft their y value, even if approximate)
(iv) $\int_{-1}^{0} x\sqrt{1+x}dx$ let $u = 1 + x$, $du/dx = 1 \Rightarrow du = dx$ when $x = -1$, $u = 0$, when $x = 0$, $u = 1$ $= \int_0^1 (u-1)\sqrt{u}du$ $= \int_0^1 (u^{3/2} - u^{1/2})du$ *	 M1 B1 M1 E1	 $du = dx$ or $du/dx = 1$ or $dx/du = 1$ changing limits – allow with no working shown provided limits are present and consistent with dx and du. $(u-1)\sqrt{u}$ www – condone only final brackets missing, otherwise notation must be correct
$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$ $= \pm\frac{4}{15}$	B1 B1 M1 A1cao [8]	$\frac{2}{5}u^{5/2}, -\frac{2}{3}u^{3/2}$ (oe) substituting correct limits (can imply the zero limit) $\pm\frac{4}{15}$ or ± 0.27 or better, not 0.26

8 (i) $f'(x) = 2(e^x - 1)e^x$ When $x = 0$, $f'(0) = 0$ When $x = \ln 2$, $f'(\ln 2) = 2(2 - 1)2$ $= 4$	M1 A1 B1dep M1 A1cao [5]	or $f(x) = e^{2x} - 2e^x + 1$ M1 (or $(e^x)^2 - 2e^x + 1$ plus correct deriv of $(e^x)^2$) $\Rightarrow f'(x) = 2e^{2x} - 2e^x$ A1 derivative must be correct, www $e^{\ln 2} = 2$ soi
(ii) $y = (e^x - 1)^2$ $x \leftrightarrow y$ $x = (e^y - 1)^2$ $\Rightarrow \sqrt{x} = e^y - 1$ $\Rightarrow 1 + \sqrt{x} = e^y$ $\Rightarrow y = \ln(1 + \sqrt{x})$	M1 M1 E1	reasonable attempt to invert formula taking lns similar scheme of inverting $y = \ln(1 + \sqrt{x})$
or $gf(x) = g((e^x - 1)^2)$ $= \ln(1 + e^x - 1)$ $= x$	M1 M1 E1	constructing gf or fg $\ln(e^x) = x$ or $e^{\ln(1+\sqrt{x})} = 1 + \sqrt{x}$
 Gradient at $(1, \ln 2) = ¼$	B1 B1ft [5]	reflection in $y = x$ (must have infinite gradient at origin)
(iii) $\int (e^x - 1)^2 dx = \int (e^{2x} - 2e^x + 1)dx$ $= \frac{1}{2}e^{2x} - 2e^x + x + c$ * $\int_0^{\ln 2} (e^x - 1)^2 dx = \left[\frac{1}{2}e^{2x} - 2e^x + x\right]_0^{\ln 2}$ $= ½ e^{2\ln 2} - 2e^{\ln 2} + \ln 2 - (½ - 2)$ $= 2 - 4 + \ln 2 - ½ + 2$ $= \ln 2 - ½$	M1 E1 M1 M1 A1 [5]	expanding brackets (condone e^{x^2}) substituting limits $e^{\ln 2} = 2$ used must be exact
(iv)  Area $= 1 \times \ln 2 - (\ln 2 - ½)$ $= ½$	M1 B1 A1cao [3]	subtracting area in (iii) from rectangle rectangle area $= 1 \times \ln 2$ must be supported

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Paper A – Section A

1 $\frac{1}{x}+\frac{x}{x+2}=1$ $\Rightarrow x+2+x^2=x(x+2)$ $=x^2+2x$ $\Rightarrow x=2$	M1 A1 DM1 A1 [4]	Clearing fractions solving cao
2(i) <table><tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr><tr><td>y</td><td>1</td><td>1.0607</td><td>1.4142</td><td>2.0917</td><td>3</td></tr></table> $A \approx 0.5[(1+3)/2 + 1.0607 + 1.4142 + 2.0917]$ $= 3.28$ (3 s.f.)	B1 M1 A1 [3]	At least one value calculated correctly or 13.13…or 6.566… seen
(ii) 3.25 (or Chris) area should decrease with the number of strips used.	B1 B1 [2]	ft (i) or area should decrease as concave upwards
3(i) sin 60 = √3/2, cos 60 = 1/2 , sin 45 = 1/√2, cos 45 = 1/√2 sin(105°) = sin(60°+45°) = sin60°cos45° + cos60°sin45° $= \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}+\frac{1}{2}.\frac{1}{\sqrt{2}}$ $= \frac{\sqrt{3}+1}{2\sqrt{2}}$ *	M1 M1 A1 E1 [4]	 splitting into 60° and 45°, and using the compound angle formulae
(ii) Angle B = 105° By the sine rule: $\frac{AC}{\sin B}=\frac{1}{\sin 30}$ $\Rightarrow$ AC = $\frac{\sin 105}{\sin 30}=\frac{\sqrt{3}+1}{2\sqrt{2}}.2$ $= \frac{\sqrt{3}+1}{\sqrt{2}}$ *	 M1 A1 E1 [3]	 Sine rule with exact values www
4 $\frac{1+\tan^2\theta}{1-\tan^2\theta}=\frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}$ $=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}$ $=\frac{1}{\cos 2\theta}$ $=\sec 2\theta$ $\sec 2\theta = 2 \Rightarrow \cos 2\theta = ½$ $\Rightarrow 2\theta = 60°, 300°$ $\Rightarrow \theta = 30°, 150°$	M1 M1 M1 E1 M1 B1 B1 [7]	$\tan\theta=\frac{\sin\theta}{\cos\theta}$ or $1+\tan^2\theta=\sec^2\theta$ used simplifying to a simple fraction in terms of sinθ and/or cos θ only $\cos^2\theta - \sin^2\theta = \cos 2\theta$ oe used or $1+\tan^2\theta = 2(1-\tan^2\theta) \Rightarrow \tan\theta = \pm 1/\sqrt{3}$oe 30° 150° and no others in range

5 $(1+3x)^{\frac{1}{3}} =$ $= 1+\frac{1}{3}(3x)+\frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(3x)^2+\frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(3x)^3+...$ $=1+x-x^2+\frac{5}{3}x^3+...$ Valid for $-1 < 3x < 1 \Rightarrow -1/3 < x < 1/3$	M1 B1 A2,1,0 B1 [5]	binomial expansion (at least 3 terms) correct binomial coefficients (all) x, $-x^2$, $5x^3/3$
6(i) $\frac{1}{(2x+1)(x+1)} = \frac{A}{2x+1}+\frac{B}{x+1}$ $\Rightarrow$ $1 = A(x + 1) + B(2x + 1)$ $x = -1$: $1 = -B \Rightarrow B = -1$ $x = -½$: $1 = ½ A \Rightarrow A = 2$	M1 A1 A1 [3]	or cover up rule for either value
(ii) $\frac{dy}{dx}=\frac{y}{(2x+1)(x+1)}$ $\Rightarrow$ $\int\frac{1}{y}dy=\int\frac{1}{(2x+1)(x+1)}dx$ $=\int(\frac{2}{2x+1}-\frac{1}{x+1})dx$ $\Rightarrow$ $\ln y = \ln(2x + 1) - \ln(x + 1) + c$ When $x = 0$, $y = 2$ $\Rightarrow$ $\ln 2 = \ln 1 - \ln 1 + c \Rightarrow c = \ln 2$ $\Rightarrow$ $\ln y = \ln(2x + 1) - \ln(x + 1) + \ln 2$ $=\ln\frac{2(2x+1)}{x+1}$ $\Rightarrow$ $y=\frac{4x+2}{x+1}$ *	M1 A1 B1ft M1 E1 [5]	separating variables correctly condone omission of c. ft A,B from (i) calculating c , no incorrect log rules combining lns www

Section B

7(i) At A, cos $\theta = 1 \Rightarrow \theta = 0$ At B, cos $\theta = -1 \Rightarrow \theta = \pi$ At C $x = 0, \Rightarrow$ cos $\theta = 0 \Rightarrow \theta = \pi/2$ $\Rightarrow y = \sin\frac{\pi}{2} - \frac{1}{8}\sin\pi = 1$	B1 B1 M1 A1 [4]	or subst in both x and y allow 180°
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$ $= \frac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$ dy/dx = 0 when $\cos 2\theta - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 1 - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 4\cos\theta - 1 = 0$ *	M1 A1 A1 M1 E1 [5]	finding $dy/d\theta$ and $dx/d\theta$ correct numerator correct denominator =0 or their num=0
(iii) $\cos\theta = \frac{4 \pm \sqrt{16+8}}{4} = 1 \pm \frac{1}{2}\sqrt{6}$ (1 + ½ √6 > 1 so no solution) $\Rightarrow \theta = 1.7975$ $y = \sin\theta - \frac{1}{8}\sin 2\theta = 1.0292$	M1 A1ft A1 cao M1 A1 cao [5]	$1 \pm \frac{1}{2}\sqrt{6}$ or (2.2247,-.2247) both or –ve their quadratic equation 1.80 or 103° their angle 1.03 or better
(iv) $V = \int_{-1}^{1} \pi y^2 dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2)(1 - x^2)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2 - 16x^2 + 8x^3 - x^4)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x - 15x^2 + 8x^3 - x^4)dx$ * $= \frac{1}{16}\pi\left[16x - 4x^2 - 5x^3 + 2x^4 - \frac{1}{5}x^5\right]_{-1}^{1}$ $= \frac{1}{16}\pi(32 - 10 - \frac{2}{5})$ = 1.35π = 4.24	M1 M1 E1 B1 M1 A1cao [6]	correct integral and limits expanding brackets correctly integrated substituting limits

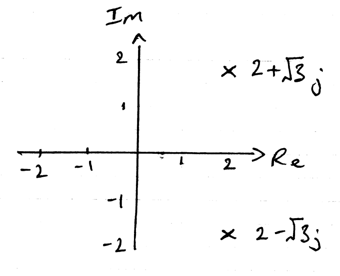
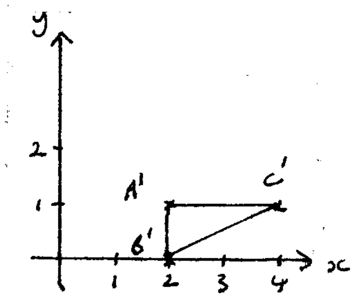
8 (i) $\sqrt{(40-0)^2+(0+40)^2+(-20-0)^2}$ = 60 m	M1 A1 [2]	
(ii) $\overrightarrow{BA} = \begin{pmatrix}-40\\-40\\20\end{pmatrix} = 20\begin{pmatrix}-2\\-2\\1\end{pmatrix}$ $\cos\theta = \dfrac{\begin{pmatrix}-2\\-2\\1\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix}}{\sqrt{9}\sqrt{26}} = -\dfrac{13}{3\sqrt{26}}$ $\Rightarrow \quad \theta = 148°$	M1 A1 A1 A1 [4]	or $\overrightarrow{AB}$ −13 oe eg -260 √9√26 oe eg 60√26 cao (or radians)
(iii) $\mathbf{r} = \begin{pmatrix}40\\0\\-20\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ At C, $z = 0 \Rightarrow \lambda = 20$ $\Rightarrow \quad a = 40 + 3 \times 20 = 100$ $b = 0 + 4 \times 20 = 80$	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix}40\\0\\-20\end{pmatrix} + ...$ $... + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ or….$+\lambda\begin{pmatrix}a-40\\b\\20\end{pmatrix}$ 100 80
(iv) $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}-2\\-2\\1\end{pmatrix} = -12+10+2=0$ $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix} = 18-20+2=0$ $\Rightarrow \begin{pmatrix}6\\-5\\2\end{pmatrix}$ is perpendicular to plane. Equation of plane is $6x - 5y + 2z = c$ At B (say) $6 \times 40 - 5 \times 0 + 2 \times -20 = c$ $\Rightarrow c = 200$ so $6x - 5y + 2z = 200$	B1 B1 M1 M1 A1 [5]	(alt. method finding vector equation of plane M1 eliminating both parameters DM1 correct equation A1 stating Normal hence perpendicular B2)

Paper B Comprehension

<table>
<tr><td>1(i)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>6</td><td>4</td><td>2</td><td>2</td><td>2</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
</table>
</td><td>B1 Table</td></tr>
<tr><td>(ii)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>7</td><td>3</td><td>2</td><td>3</td><td>1</td><td>2</td><td>1</td><td>1</td><td>0</td></tr>
</table>
</td><td>M1 A1 Table</td></tr>
<tr><td>(iii)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>6.0</td><td>3.5</td><td>2.5</td><td>1.9</td><td>1.6</td><td>1.3</td><td>1.2</td><td>1.0</td><td>0.9</td></tr>
</table>
</td><td>B1any 4 correct
B1 other 4 correct</td></tr>
<tr><td>(iv)</td><td>Any sensible comment such as:
• The general pattern of the frequencies/results is the same for all three tables.
• Due to the small number of data items we cannot expect the pattern to follow Benford's Law very closely.</td><td>E1</td></tr>
<tr><td>2</td><td>Evidence of 4+3+4+2+2 from Table 4 frequencies is the same as 15 in Table 6</td><td>B1</td></tr>
<tr><td>3</td><td>$p_1 = p_3 + p_4 + p_5$: on multiplication by 3, numbers with a leading digit of 1 will be mapped to numbers with a leading digit of 3, 4 or 5 and no other numbers have this property.</td><td>B1 Multiplication

B1… by 3</td></tr>
<tr><td>4</td><td>$\log_{10}(n+1) - \log_{10} n = \log_{10}\left(\frac{n+1}{n}\right) = \log_{10}\left(\frac{n}{n} + \frac{1}{n}\right) = \log_{10}\left(1 + \frac{1}{n}\right)$</td><td>M1
E1</td></tr>
<tr><td>5</td><td>Substitute $\mathrm{L}(4) = 2\times\mathrm{L}(2)$ and $\mathrm{L}(6) = \mathrm{L}(3) + \mathrm{L}(2)$ in
$\mathrm{L}(8) - \mathrm{L}(6) = \mathrm{L}(4) - \mathrm{L}(3)$:
this gives $\mathrm{L}(8) = \mathrm{L}(6) - \mathrm{L}(3) + \mathrm{L}(4) = \mathrm{L}(2) + 2\times\mathrm{L}(2) = 3\times\mathrm{L}(2)$</td><td>M1
M1 subst
E1
(or alt
M1 for 2 or more Ls used
M1 use of at least 2 given results oe
E1)</td></tr>
<tr><td>6</td><td>$a = 28$. All entries with leading digit 2 or 3 will, on multiplying by 5, have leading digit 1. None of the other original daily wages would have this property.</td><td>B1 B1</td></tr>
<tr><td></td><td>$b = 9$. Similarly, all entries with leading digit 8 or 9 will, on multiplying by 5, have leading digit 4. None of the other original daily wages would have this property.</td><td>B1 B1</td></tr>
<tr><td></td><td></td><td>Total 18</td></tr>
</table>

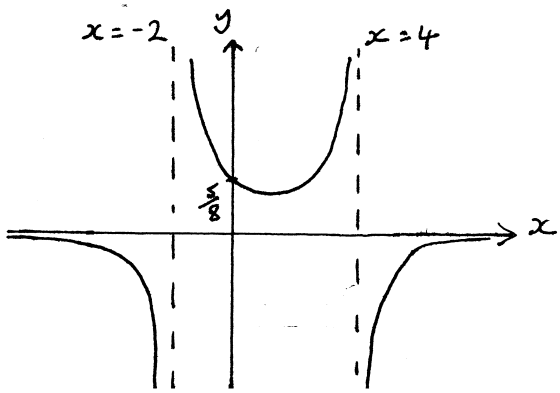
Mark Scheme 4755
January 2007

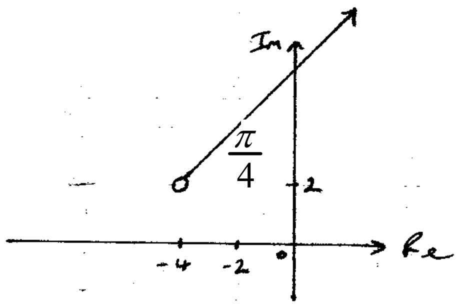
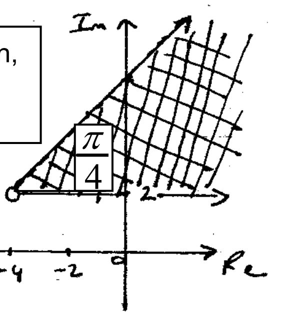
Qu	Answer	Mark	Comment
Section A			
1	The statement is false. The 'if' part is true, but the 'only if' is false since $x = -2$ also satisfies the equation.	M1 A1 **[2]**	'False', with attempted justification (may be implied) Correct justification
2(i)	$\frac{4 \pm \sqrt{16-28}}{2}$ $= \frac{4 \pm \sqrt{12}}{2}\text{j} = 2 \pm \sqrt{3}\text{j}$	M1 A1 A1 A1 **[4]**	Attempt to use quadratic formula or other valid method Correct Unsimplified form. Fully simplified form.
2(ii)		B1(ft) B1(ft) **[2]**	One correct, with correct labelling Other in correct relative position s.c. give B1 if both points consistent with (i) but no/incorrect labelling
3(i)	 $\begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \end{pmatrix}$	B3 B1 ***ELSE*** M1 A1 **[4]**	Points correctly plotted Points correctly labelled Applying matrix to points Minus 1 each error
3(ii)	Stretch, factor 2 in *x*-direction, stretch factor half in *y*-direction.	B1 B1 B1 **[3]**	1 mark for stretch (withhold if rotation, reflection or translation mentioned incorrectly) 1 mark for each factor and direction

4	$\sum_{r=1}^{n} r(r^2+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r$	M1	Separate into two sums (may be implied by later working)
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)$	M1 A1	Use of standard results Correct
	$= \frac{1}{4}n(n+1)[n(n+1)+2]$	M1 A1	Attempt to factorise (dependent on previous M marks) Factor of $n(n + 1)$
	$= \frac{1}{4}n(n+1)(n^2+n+2)$	A1	c.a.o.
		[6]	
5	$\omega = 2x+1 \Rightarrow x = \frac{\omega-1}{2}$	M1 A1	Attempt to give substitution Correct
	$2\left(\frac{\omega-1}{2}\right)^3 - 3\left(\frac{\omega-1}{2}\right)^2 + \left(\frac{\omega-1}{2}\right) - 4 = 0$	M1	Substitute into cubic
	$\Rightarrow \frac{1}{4}(\omega^3 - 3\omega^2 + 3\omega - 1) - \frac{3}{4}(\omega^2 - 2\omega + 1)$ $+\frac{1}{2}(\omega - 1) - 4 = 0$	A1(ft) A1(ft)	Cubic term Quadratic term
	$\Rightarrow \omega^3 - 6\omega^2 + 11\omega - 22 = 0$	A2	Minus 1 each error (missing '= 0' is an error)
		[7]	
5	OR $\alpha + \beta + \gamma = \frac{3}{2}$ $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{1}{2}$ $\alpha\beta\gamma = 2$	M1 A1	Attempt to find sums and products of roots All correct
	Let new roots be *k*, *l*, *m* then $k + l + m = 2(\alpha + \beta + \gamma) + 3 = 6 = \frac{-B}{A}$	M1	Use of sum of roots
	$kl + km + lm = 4(\alpha\beta + \alpha\gamma + \beta\gamma) +$ $4(\alpha + \beta + \gamma) + 3 = 11 = \frac{C}{A}$	M1	Use of sum of product of roots in pairs
	$klm = 8\alpha\beta\gamma + 4(\alpha\beta + \beta\gamma + \beta\gamma)$ $+2(\alpha + \beta + \gamma) + 1 = 22 = \frac{-D}{A}$	M1	Use of product of roots
	$\Rightarrow \omega^3 - 6\omega^2 + 11\omega - 22 = 0$	A2 **[7]**	Minus 1 each error (missing '= 0' is an error)

6	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ n = 1, LHS = RHS = 1 Assume true for $n = k$ Next term is $(k+1)^2$ Add to both sides $\text{RHS} = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ $= \frac{1}{6}(k+1)[k(2k+1)+6(k+1)]$ $= \frac{1}{6}(k+1)[2k^2+7k+6]$ $= \frac{1}{6}(k+1)(k+2)(2k+3)$ $= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$ But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for k = 1, 2, 3 and so true for all positive integers.	B1 M1 B1 M1 M1 A1 E1 E1 **[8]**	 Assuming true for k. (k + 1)th term. Add to both sides Attempt to factorise Correct brackets required – also allow correct unfactorised form Showing this is the expression with $n = k + 1$ Only if both previous E marks awarded
			Section A Total: 36

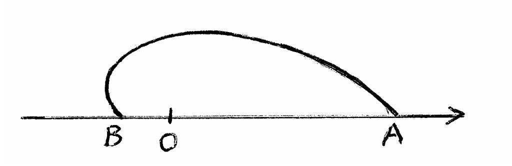
Section B

Question	Answer	Marks	Notes
7(i)	$y = \frac{5}{8}$	B1 **[1]**	
7(ii)	$x = -2,\ x = 4,\ y = 0$	B1, B1 B1 **[3]**	
7(iii)	3 correct branches Correct, labelled asymptotes *y*-intercept labelled 	B1 B1 B1 **[3]**	 Ft from (ii) Ft from (i)
7(iv)	$\frac{5}{(x+2)(4-x)} = 1$ $\Rightarrow 5 = (x+2)(4-x)$ $\Rightarrow 5 = -x^2 + 2x + 8$ $\Rightarrow x^2 - 2x - 3 = 0$ $\Rightarrow (x-3)(x+1) = 0$ $\Rightarrow x = 3$ or $x = -1$ From graph: $x < -2$ or $-1 < x < 3$ or $x > 4$	M1 A1 B1 B1 B1 **[5]**	Or evidence of other valid method Both values Ft from previous A1 Penalise inclusive inequalities only once

8(i)	$\frac{1}{m}=\frac{1}{-4+2\mathrm{j}}=\frac{-4-2\mathrm{j}}{(-4+2\mathrm{j})(-4-2\mathrm{j})}$	M1	Attempt to multiply top and bottom by conjugate
	$=\frac{-1}{5}-\frac{1}{10}\mathrm{j}$	A1 **[2]**	Or equivalent
8(ii)	$\lvert m\rvert=\sqrt{(-4)^2+2^2}=\sqrt{20}$	B1	
	$\arg m=\pi-\arctan\left(\frac{1}{2}\right)=2.68$	M1 A1	Attempt to calculate angle Accept any correct expression for angle, including 153.4 degrees, –206 degrees and –3.61 (must be at least 3s.f.)
	So $m=\sqrt{20}\left(\cos 2.68+\mathrm{j}\sin 2.68\right)$	A1(ft) **[4]**	Also accept $(r,\ \theta)$ form
8(iii) (*A*)		B1 B1 **[2]**	Correct initial point Half-line at correct angle
8(iii) (*B*)	Shaded region, excluding boundaries 	B1(ft) B1(ft) B1(ft) **[3]**	Correct horizontal half-line from starting point Correct region indicated Boundaries excluded (accept dotted lines)

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}^{-1} = \frac{1}{3}\begin{pmatrix}1 & -2\\0 & 3\end{pmatrix}$ $\mathbf{N}^{-1} = \frac{1}{7}\begin{pmatrix}4 & 3\\-1 & 1\end{pmatrix}$	M1 A1 A1 [3]	Dividing by determinant One for each inverse c.a.o.
9(ii)	$\mathbf{MN} = \begin{pmatrix}3 & 2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -3\\1 & 4\end{pmatrix} = \begin{pmatrix}5 & -1\\1 & 4\end{pmatrix}$	M1 A1	Must multiply in correct order
	$(\mathbf{MN})^{-1} = \frac{1}{21}\begin{pmatrix}4 & 1\\-1 & 5\end{pmatrix}$	A1	Ft from **MN**
	$\mathbf{N}^{-1}\mathbf{M}^{-1} = \frac{1}{7}\begin{pmatrix}4 & 3\\-1 & 1\end{pmatrix}\times\frac{1}{3}\begin{pmatrix}1 & -2\\0 & 3\end{pmatrix}$ $= \frac{1}{21}\begin{pmatrix}4 & 1\\-1 & 5\end{pmatrix}$ $= (\mathbf{MN})^{-1}$	M1 A1 A1 [6]	Multiplication in correct order Ft from (i) Statement of equivalence to $(\mathbf{MN})^{-1}$
9(iii)	$\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1} = \mathbf{IQ}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PI} = \mathbf{Q}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{P} = \mathbf{Q}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PP}^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{I} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$	M1 M1 M1 A1 [4]	$\mathbf{QQ}^{-1} = \mathbf{I}$ Correctly eliminate **I** from LHS Post-multiply both sides by $\mathbf{P}^{-1}$ at an appropriate point Correct and complete argument
			Section B Total: 36
			Total: 72

Mark Scheme 4756
January 2007

1(a)(i)		B1 B1 **2**	Correct shape for $0 \le \theta \le \frac{1}{2}\pi$ Correct shape for $\frac{1}{2}\pi \le \theta \le \pi$ *Requires decreasing r on at least one axis* *Ignore other values of θ*
(ii)	Area is $\int \frac{1}{2} r^2 \, d\theta = \int_0^{\pi} \frac{1}{2} a^2 (e^{-k\theta})^2 \, d\theta$ $= \left[-\frac{a^2}{4k} e^{-2k\theta} \right]_0^{\pi}$ $= \frac{a^2}{4k}(1 - e^{-2k\pi})$	M1 A1 M1 A1 **4**	For $\int (e^{-k\theta})^2 \, d\theta$ For a correct integral expression including limits *(may be implied by later work) (Condone reversed limits)* Obtaining a multiple of $e^{-2k\theta}$ as the integral
(b)	$\int_0^{\frac{1}{2}} \frac{1}{3+4x^2} \, dx = \left[\frac{1}{2\sqrt{3}} \arctan\left(\frac{2x}{\sqrt{3}}\right) \right]_0^{\frac{1}{2}}$ $= \frac{1}{2\sqrt{3}} \arctan\left(\frac{1}{\sqrt{3}}\right)$ $= \frac{\pi}{12\sqrt{3}}$	M1 A1A1 M1 A1 **5**	For arctan For $\frac{1}{2\sqrt{3}}$ and $\frac{2x}{\sqrt{3}}$ *Dependent on first M1*
	OR Putting $2x = \sqrt{3} \tan \theta$ Integral is $\int_0^{\frac{1}{6}\pi} \frac{1}{2\sqrt{3}} \, d\theta$ $= \frac{\pi}{12\sqrt{3}}$	M1 A1 A1 M1 A1	For any tan substitution For $\int \frac{1}{2\sqrt{3}} \, d\theta$ For changing to limits of θ *Dependent on first M1*
(c)(i)	$f(x) = \tan x, \quad f(0) = 0$ $f'(x) = \sec^2 x, \quad f'(0) = 1$ $f''(x) = 2\sec^2 x \tan x, \quad f''(0) = 0$ $f'''(x) = 2\sec^4 x + 4\sec^2 x \tan^2 x, \quad f'''(0) = 2$ $\tan x = x + \frac{x^3}{3!}(2) + \ldots \quad (= x + \frac{1}{3}x^3 + \ldots)$	B1 M1 A1 B1 ft **4**	Obtaining $f'''(x)$ For $f''(0)$ and $f'''(0)$ correct ft requires x^3 term and at least one other to be non-zero
(ii)	$\int_h^{4h} \frac{\tan x}{x} \, dx \approx \int_h^{4h} (1 + \frac{1}{3}x^2) \, dx$ $= \left[x + \frac{1}{9}x^3 \right]_h^{4h}$ $= (4h + \frac{64}{9}h^3) - (h + \frac{1}{9}h^3)$ $= 3h + 7h^3$	M1 A1 ft A1 ag **3**	Obtaining a polynomial to integrate For $x + \frac{1}{9}x^3$ ft requires at least two non-zero terms

2(a)(i)	$\lvert w \rvert = 3, \quad \arg w = -\frac{1}{12}\pi$ $\lvert z \rvert = 2, \quad \arg z = -\frac{1}{3}\pi$ $\left\lvert \frac{w}{z} \right\rvert = \frac{3}{2}, \quad \arg\frac{w}{z} = (-\frac{1}{12}\pi) - (-\frac{1}{3}\pi) = \frac{1}{4}\pi$	B1 B1B1 B1B1 ft **5**	*Deduct 1 mark if answers given in form $r(\cos\theta + \mathrm{j}\sin\theta)$ but modulus and argument not stated.* Accept degrees and decimal approxs
(ii)	$\frac{w}{z} = \frac{3}{2}(\cos\frac{1}{4}\pi + \mathrm{j}\sin\frac{1}{4}\pi)$ $= \frac{3}{2\sqrt{2}} + \frac{3}{2\sqrt{2}}\mathrm{j}$	M1 A1 **2**	Accept $\sqrt{1.125} + \sqrt{1.125}\,\mathrm{j}$
(b)(i)	$\mathrm{e}^{-\frac{1}{2}\mathrm{j}\theta} + \mathrm{e}^{\frac{1}{2}\mathrm{j}\theta}$ $= (\cos\frac{1}{2}\theta - \mathrm{j}\sin\frac{1}{2}\theta) + (\cos\frac{1}{2}\theta + \mathrm{j}\sin\frac{1}{2}\theta)$ $= 2\cos\frac{1}{2}\theta$	M1 A1	For either bracketed expression
	$1 + \mathrm{e}^{\mathrm{j}\theta} = \mathrm{e}^{\frac{1}{2}\mathrm{j}\theta}(\mathrm{e}^{-\frac{1}{2}\mathrm{j}\theta} + \mathrm{e}^{\frac{1}{2}\mathrm{j}\theta})$ $= \mathrm{e}^{\frac{1}{2}\mathrm{j}\theta}(2\cos\frac{1}{2}\theta)$	M1 A1 ag **4**	
	OR $1 + \mathrm{e}^{\mathrm{j}\theta} = 1 + \cos\theta + \mathrm{j}\sin\theta$ $= 2\cos^2\frac{1}{2}\theta + 2\mathrm{j}\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$ M1 $= 2\cos\frac{1}{2}\theta(\cos\frac{1}{2}\theta + \mathrm{j}\sin\frac{1}{2}\theta)$ $= 2\mathrm{e}^{\frac{1}{2}\mathrm{j}\theta}\cos\frac{1}{2}\theta$ A1		
(ii)	$C + \mathrm{j}S = 1 + \binom{n}{1}\mathrm{e}^{\mathrm{j}\theta} + \binom{n}{2}\mathrm{e}^{2\mathrm{j}\theta} + \ldots + \binom{n}{n}\mathrm{e}^{n\mathrm{j}\theta}$ $= (1 + \mathrm{e}^{\mathrm{j}\theta})^n$ $= 2^n \mathrm{e}^{\frac{1}{2}n\theta\mathrm{j}}\cos^n\frac{1}{2}\theta$ $C = 2^n\cos(\frac{1}{2}n\theta)\cos^n\frac{1}{2}\theta$ $S = 2^n\sin(\frac{1}{2}n\theta)\cos^n\frac{1}{2}\theta$ $\frac{S}{C} = \frac{2^n\sin(\frac{1}{2}n\theta)\cos^n\frac{1}{2}\theta}{2^n\cos(\frac{1}{2}n\theta)\cos^n\frac{1}{2}\theta} = \frac{\sin(\frac{1}{2}n\theta)}{\cos(\frac{1}{2}n\theta)} = \tan(\frac{1}{2}n\theta)$	M1 M1A1 M1 A1 A1 B1 ag **7**	Using (i) to obtain a form from which the real and imaginary parts can be written down Allow ft from $C + \mathrm{j}S = \mathrm{e}^{\frac{1}{2}n\theta\mathrm{j}} \times$ any real function of *n* and θ

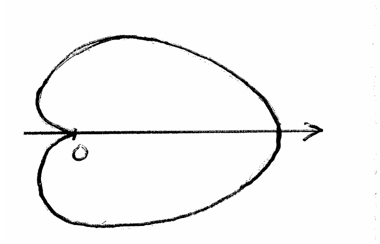
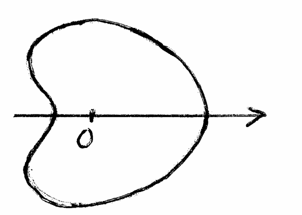
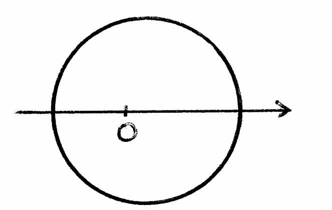
3 (i)	$\det \mathbf{P} = 1(6-k) - 1(4-2)$ $= 4-k$ $\mathbf{P}^{-1} = \frac{1}{4-k}\begin{pmatrix} -1 & 2 & 6-k \\ 4 & -4-k & k-12 \\ -1 & 2 & 2 \end{pmatrix}$ When $k=2$, $\mathbf{P}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 2 & 4 \\ 4 & -6 & -10 \\ -1 & 2 & 2 \end{pmatrix}$	M1 A1 M1 M1 A1 ft B1 ag **6**	Evaluating at least three cofactors Fully correct method for inverse Ft from wrong determinant Correctly obtained
(ii)	$\mathbf{M}\begin{pmatrix}4\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} = 0\begin{pmatrix}4\\1\\1\end{pmatrix}$ $\mathbf{M}\begin{pmatrix}2\\1\\0\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix} = 1\begin{pmatrix}2\\1\\0\end{pmatrix}$ $\mathbf{M}\begin{pmatrix}2\\3\\-1\end{pmatrix} = \begin{pmatrix}4\\6\\-2\end{pmatrix} = 2\begin{pmatrix}2\\3\\-1\end{pmatrix}$ Eigenvalues are 0, 1, 2	M1 A1A1A1 **4**	For one evaluation
	OR M1 Eigenvalues are 0, 1, 2 A2 A1		Obtaining an eigenvalue (e.g. by solving $-\lambda^3 + 3\lambda^2 - 2\lambda = 0$) Give A1 for one correct Verifying given eigenvectors, linking with eigenvalues correctly
(iii)	$\mathbf{M}^n = \begin{pmatrix} 4 & 2 & 2 \\ 1 & 1 & 3 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^n \end{pmatrix}\frac{1}{2}\begin{pmatrix} -1 & 2 & 4 \\ 4 & -6 & -10 \\ -1 & 2 & 2 \end{pmatrix}$ $= \frac{1}{2}\begin{pmatrix} 0 & 2 & 2^{n+1} \\ 0 & 1 & 3\times 2^n \\ 0 & 0 & -2^n \end{pmatrix}\begin{pmatrix} -1 & 2 & 4 \\ 4 & -6 & -10 \\ -1 & 2 & 2 \end{pmatrix}$ $= \begin{pmatrix} 4-2^n & -6+2^{n+1} & -10+2^{n+1} \\ 2-3\times 2^{n-1} & -3+3\times 2^n & -5+3\times 2^n \\ 2^{n-1} & -2^n & -2^n \end{pmatrix}$ $= \begin{pmatrix} 4 & -6 & -10 \\ 2 & -3 & -5 \\ 0 & 0 & 0 \end{pmatrix} + 2^{n-1}\begin{pmatrix} -2 & 4 & 4 \\ -3 & 6 & 6 \\ 1 & -2 & -2 \end{pmatrix}$	B1B1 M1A1 B1 ft M1 A1 A1 ag **8**	For $\begin{pmatrix} 4 & 2 & 2 \\ 1 & 1 & 3 \\ 1 & 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^n \end{pmatrix}$ seen (for B2, these must be consistent) For $\mathbf{S}\mathbf{D}^n\mathbf{S}^{-1}$ (M1A0 if order wrong) *or* $\frac{1}{2}\begin{pmatrix} 4 & 2 & 2 \\ 1 & 1 & 3 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 4 & -6 & -10 \\ -2^n & 2^{n+1} & 2^{n+1} \end{pmatrix}$ Evaluating product of 3 matrices Any correct form

	OR Prove $\mathbf{M}^n = \mathbf{A} + 2^{n-1}\mathbf{B}$ by induction			
	When $n=1$, $\mathbf{A}+\mathbf{B}=\mathbf{M}$	B1		
	Assuming $\mathbf{M}^k = \mathbf{A} + 2^{k-1}\mathbf{B}$,			
	$\mathbf{M}^{k+1} = \mathbf{A}\mathbf{M} + 2^{k-1}\mathbf{B}\mathbf{M}$	M1A2		or $\mathbf{M}^{k+1} = \mathbf{M}\mathbf{A} + 2^{k-1}\mathbf{M}\mathbf{B}$
	$= \mathbf{A} + 2^{k-1}(2\mathbf{B})$	A1A1		
	$= \mathbf{A} + 2^k\mathbf{B}$	A1		
	True for $n=k \Rightarrow$ True for $n=k+1$; hence true for all positive integers *n*	A1		*Dependent on previous 7 marks*

4 (i)	If $y = \text{arcosh}\, x$, $x = \cosh y = \frac{1}{2}(e^y + e^{-y})$ $e^{2y} - 2x e^y + 1 = 0$ $e^y = \dfrac{2x \pm \sqrt{4x^2 - 4}}{2}$ $= x \pm \sqrt{x^2 - 1}$ Since $y \ge 0$, $e^y \ge 1$, so $e^y = x + \sqrt{x^2 - 1}$ $\text{arcosh}\, x = y = \ln(x + \sqrt{x^2 - 1})$	M1 M1 M1 A1 A1 ag **5**	$\frac{1}{2}$ and + must be correct
(ii)	$\displaystyle\int_{2.5}^{3.9} \frac{1}{\sqrt{4x^2 - 9}}\, dx = \left[\frac{1}{2}\text{arcosh}\left(\frac{2x}{3}\right) \right]_{2.5}^{3.9}$ $= \frac{1}{2}(\text{arcosh}\, 2.6 - \text{arcosh}\, \frac{5}{3})$ $= \frac{1}{2}\left(\ln(2.6 + \sqrt{2.6^2 - 1}) - \ln(\frac{5}{3} + \sqrt{\frac{25}{9} - 1}) \right)$ $= \frac{1}{2}(\ln 5 - \ln 3)$ $= \frac{1}{2}\ln\frac{5}{3}$	M1 A1A1 M1 A1 **5**	For arcosh (or any cosh substitution) For $\frac{1}{2}$ and $\frac{2x}{3}$ (or $2x = 3\cosh u$ and $\int \frac{1}{2} du$) (or limits of *u* in logarithmic form)
	OR M2 $\left[\frac{1}{2}\ln(2x + \sqrt{4x^2 - 9}) \right]_{2.5}^{3.9}$ A1A1 $= \frac{1}{2}\ln 15 - \frac{1}{2}\ln 9$ $= \frac{1}{2}\ln\frac{5}{3}$ A1		For $\ln(kx + \sqrt{k^2x^2 - ...})$ Give M1 for $\ln(k_1x + \sqrt{k_2{}^2x^2 - ...})$ For $\frac{1}{2}$ and $\ln(2x + \sqrt{4x^2 - 9})$ (or $\ln(x + \sqrt{x^2 - \frac{9}{4}})$)
(iii)	$\dfrac{dy}{dx} = \dfrac{(2 + \sinh x)\sinh x - (\cosh x)(\cosh x)}{(2 + \sinh x)^2}$ $= \dfrac{2\sinh x - 1}{(2 + \sinh x)^2}$ $\dfrac{dy}{dx} = \dfrac{1}{9}$ when $18\sinh x - 9 = (2 + \sinh x)^2$ $\sinh^2 x - 14\sinh x + 13 = 0$ $\sinh x = 1,\ 13$ When $\sinh x = 1$, $\cosh x = \sqrt{2}$, $x = \ln(1 + \sqrt{2})$ Point is $\left(\ln(1 + \sqrt{2}),\ \dfrac{\sqrt{2}}{3} \right)$ When $\sinh x = 13$, $\cosh x = \sqrt{170}$, $x = \ln(13 + \sqrt{170})$ Point is $\left(\ln(13 + \sqrt{170}),\ \dfrac{\sqrt{170}}{15} \right)$	M1 A1 M1 M1 M1 A1 ag A1A1 **8**	Using quotient rule Any correct form Quadratic in $\sinh x$ (or product of two quadratics in e^x) Solving quadratic to obtain at least one value of $\sinh x$ (or e^x) Obtaining *x* in logarithmic form (must use a correct formula for arsinh) *SR* B1B1 for verifying $y = \frac{1}{3}\sqrt{2}$ and $\dfrac{dy}{dx} = \dfrac{1}{9}$ when $x = \ln(1+\sqrt{2})$

Alternatives for Q4 (i)

	$\cosh\ln(x+\sqrt{x^2-1})=\frac{1}{2}(e^{\ln(x+\sqrt{x^2-1})}+e^{-\ln(x+\sqrt{x^2-1})})$ $=\frac{1}{2}(x+\sqrt{x^2-1}+\frac{1}{x+\sqrt{x^2-1}})$ $=\frac{1}{2}(x+\sqrt{x^2-1}+x-\sqrt{x^2-1})$ $=x$ Since $\ln(x+\sqrt{x^2-1})>0$, $\text{arcosh}\,x=\ln(x+\sqrt{x^2-1})$	M1 M1 M1 A1 A1 **5**	
	If $y=\text{arcosh}\,x$ then $\ln(x+\sqrt{x^2-1})=\ln(\cosh y+\sqrt{\cosh^2 y-1})$ $=\ln(\cosh y+\sinh y)$ since $\sinh y>0$ $=\ln(e^y)$ $=y$	M1 M1 A1 M1 A1 **5**	

5 (i)	$k=1$  $k=1.5$  $k=4$ 	B1 B1 B1 B1 B1 **5**	General shape correct Cusp at O clearly shown General shape correct 'Dimple' correctly shown
(ii)	Cusp	B1 **1**	
(iii)	When $k=1$, there are 3 points When $k=1.5$, there are 4 points When $k=4$, there are 2 points	B2 **2**	Give B1 for two cases correct
(iv)	$x=k\cos\theta+\cos^2\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=-k\sin\theta-2\cos\theta\sin\theta$ $=-\sin\theta(k+2\cos\theta)$ $=0$ when $\theta=0,\ \pi,$ or $\cos\theta=-\frac{1}{2}k$ For just two points, $k\geq 2$	B1 B1 M1 A1 **4**	Allow $k>2$
(v)	$d^2=r^2+1^2-2r\cos\theta$ $=(k+\cos\theta)^2+1-2(k+\cos\theta)\cos\theta$ $=k^2+1-\cos^2\theta \quad (=k^2+\sin^2\theta)$ Since $0\leq\cos^2\theta\leq 1$, $k^2\leq d^2\leq k^2+1$	M1 A1 M1 A1 ag **4**	or $0\leq\sin^2\theta\leq 1$
(vi)	When *k* is large, $\sqrt{k^2+1}\approx k$, so $d\approx k$ Curve is very nearly a circle, with centre $(1,\ 0)$ and radius *k*	M1 A1 **2**	

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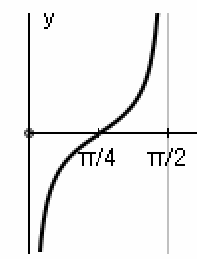
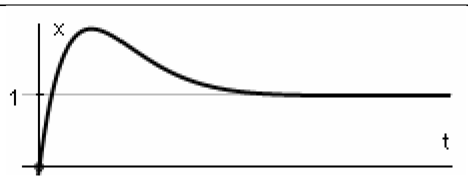
1(i)	$\lambda^2-\lambda-2=0$ $\lambda=-1$ or 2 CF $y=A\mathrm{e}^{-t}+B\mathrm{e}^{2t}$ PI $y=a\mathrm{e}^{-2t}$ $\dot{y}=-2a\mathrm{e}^{-2t}, \ddot{y}=4a\mathrm{e}^{-2t}$ $4a\mathrm{e}^{-2t}-\left(-2a\mathrm{e}^{-2t}\right)-2a\mathrm{e}^{-2t}=\mathrm{e}^{-2t}$ $4a=1$ $a=\frac{1}{4}$ $y=A\mathrm{e}^{-t}+B\mathrm{e}^{2t}+\frac{1}{4}\mathrm{e}^{-2t}$	M1 A1 F1 B1 M1 M1 M1 A1 F1	Auxiliary equation CF for their roots Differentiate twice Substitute Compare and solve Their CF with 2 constants + their PI	9
(ii)	$0=A+B+\frac{1}{4}$ $t\to\infty\Rightarrow \mathrm{e}^{-t}\to 0, \mathrm{e}^{-2t}\to 0, \mathrm{e}^{2t}\to\infty$ so $y\to 0\Rightarrow B=0$ $y=\frac{1}{4}\left(\mathrm{e}^{-2t}-\mathrm{e}^{-t}\right)$ $y=0\Leftrightarrow \mathrm{e}^{-t}=\mathrm{e}^{-2t}\Leftrightarrow \mathrm{e}^{t}=1\Leftrightarrow t=0$  	M1 M1 A1 M1 E1 B1 B1	Use initial condition Use asymptotic condition cao Valid method to establish 0 is *only* root Complete argument Curve satisfies both conditions $y\neq 0$ for $t>0$ and consistent with their solution	7
(iii)	CF $y=C\mathrm{e}^{-t}+D\mathrm{e}^{2t}$ PI $y=bt\mathrm{e}^{-t}$ $\dot{y}=b\mathrm{e}^{-t}-bt\mathrm{e}^{-t}, \ddot{y}=-2b\mathrm{e}^{-t}+bt\mathrm{e}^{-t}$ $-2b\mathrm{e}^{-t}+bt\mathrm{e}^{-t}-\left(b\mathrm{e}^{-t}-bt\mathrm{e}^{-t}\right)-2b\mathrm{e}^{-t}=\mathrm{e}^{-t}$ $\Rightarrow -2b-b=1\Rightarrow b=-\frac{1}{3}$ GS $y=C\mathrm{e}^{-t}+D\mathrm{e}^{2t}-\frac{1}{3}t\mathrm{e}^{-t}$ $y=0, t=0\Rightarrow C+D=0$ $y\to 0\Rightarrow D=0$ $y=-\frac{1}{3}t\mathrm{e}^{-t}$	F1 B1 M1 A1 F1 M1 M1 A1	Correct or same as in (i) Differentiate (product) and substitute cao Their CF + their non-zero PI Use condition Use condition cao	8

2(i)	$\frac{d}{dx}(\ln \sin x) = \frac{1}{\sin x} \cos x = \cot x$	E1	Differentiate (chain rule)	1
(ii)	$\frac{1}{y}\frac{dy}{dx} = -2\cot 2x$ $\int \frac{1}{y} dy = \int -2\cot 2x \, dx$ $\ln\|y\| = -\ln\|\sin 2x\| + c$ $y = A\,\text{cosec}\, 2x$	M1 M1 A1 A1 M1 A1	Rearrange Integrate One side correct (ignore constant) All correct, including constant Rearrange, dealing properly with constant	6
(iii)	$\frac{dy}{dx} + 2y\cot 2x = k$ $I = \exp\left(\int 2\cot 2x \, dx\right)$ $= \exp(\ln \sin 2x)$ $= \sin 2x$ $\frac{dy}{dx}\sin 2x + 2y\cos 2x = k\sin 2x$ $y\sin 2x = \int k\sin 2x \, dx$ $= -\frac{1}{2}k\cos 2x + A$ $y = A\,\text{cosec}\, 2x - \frac{1}{2}k\cot 2x$	M1 M1 A1 M1 M1 A1 E1	Attempt integrating factor Integrate Simplified form of IF Multiply by their IF Integrate both sides cao	7
(iv)	$x = \frac{1}{4}\pi, y = 0 \Rightarrow 0 = A$ $y = -\frac{1}{2}k\cot 2x$ 	M1 A1 B1 B1	Use condition Increasing and through $\left(\frac{1}{4}\pi, 0\right)$ Asymptote $x = 0$	4
(v)	$y = \frac{A - \frac{1}{2}k\cos 2x}{\sin 2x} = \frac{A - \frac{1}{2}k\left(1 - 2\sin^2 x\right)}{2\sin x\cos x}$ $A = \frac{1}{2}k \Rightarrow y = \frac{\frac{1}{2}k\sin x}{\cos x}$ which tends to zero as $x \to 0$	B1 M1 A1 M1 E1 B1	Both double angle formulae correct (or small angle approximations or series expansion) Use expressions in general solution Identify value of *A* Correct solution, fully justified Must be from correct solution	6

3(i)	$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - R$	B1	N2L equation (accept *ma*, allow sign errors)	
	$\frac{\mathrm{d}v}{\mathrm{d}t} = g - k_1 v$	E1	Must follow from correct N2L	
	$\int \frac{1}{g - k_1 v}\,\mathrm{d}v = \int \mathrm{d}t$	M1	Separate and integrate	
	$-\frac{1}{k_1}\ln\lvert g - k_1 v\rvert = t + c_1$	A1	LHS	
	$g - k_1 v = A\mathrm{e}^{-k_1 t}$	M1	Rearrange (dealing properly with constant)	
	Alternatively	*M1*	Attempt integrating factor	
		A1	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{k_1 t} v\right) = g\,\mathrm{e}^{k_1 t}$	
		M1	Integrate	
	Alternatively	*M1*	Auxiliary equation	
		A1	CF $A\mathrm{e}^{-k_1 t}$	
		M1	Constant PI (g/k_1)	
	$t = 0, v = 0 \Rightarrow A = g$	M1	Use condition	
	$v = \frac{g}{k_1}\left(1 - \mathrm{e}^{-k_1 t}\right)$	E1		7
(ii)	$x = \int v\,\mathrm{d}t = \frac{g}{k_1}\left(t + \frac{1}{k_1}\mathrm{e}^{-k_1 t} + B\right)$	M1	Integrate *v*	
		A1	cao (including constant)	
	$t = 0, x = 0 \Rightarrow B = -\frac{1}{k_1}$	M1	Use condition	
	$x = \frac{g}{k_1}\left(t + \frac{1}{k_1}\mathrm{e}^{-k_1 t} - \frac{1}{k_1}\right)$	A1	cao	4
(iii)	$mv\frac{\mathrm{d}v}{\mathrm{d}x} = mg - mk_2 v^2$	B1	N2L with $mk_2 v^2$ (accept ma or $m\frac{\mathrm{d}v}{\mathrm{d}t}$)	
	$\frac{v}{g - k_2 v^2}\frac{\mathrm{d}v}{\mathrm{d}x} = 1$	E1	Must follow from correct N2L	
	$\int \frac{v}{g - k_2 v^2}\,\mathrm{d}v = \int \mathrm{d}x$	M1	Integrate	
	$-\frac{1}{2k_2}\ln\lvert g - k_2 v^2\rvert = x + c_2$	A1	LHS	
	$g - k_2 v^2 = C\mathrm{e}^{-2k_2 x}$	M1	Rearrange (dealing properly with constant)	
	$x = 0, v = 0 \Rightarrow C = g$	M1	Use condition	
	$v = \sqrt{\frac{g}{k_2}\left(1 - \mathrm{e}^{-2k_2 x}\right)}$	A1	cao	7
(iv)	*t* *v* $\dot{v}$	B1	First line	
	0 0 9.8	M1	Use algorithm	
	0.1 0.98 8.61157	A1	0.98	
	0.2 1.84116	M1	Use algorithm	
		A1	1.84116 (accept 3sf or better)	5

(v)	$g - k_3 v^{\frac{3}{2}} = 0$ when $v = 4 \Rightarrow k_3 = \dfrac{g}{4^{\frac{3}{2}}} = 1.225$	E1	Deduce or verify value (must relate to resultant force or acceleration being zero)	1

4(i)	subtracting $\Rightarrow -5x+5=0$ $x=1$ $y=7$	M1 A1 A1	Solve simultaneously	3
(ii)	$\ddot{x}=-3\dot{x}-\dot{y}$ $=-3\dot{x}-(2x-y+5)$ $=-3\dot{x}-2x+(-\dot{x}-3x+10)-5$ $\ddot{x}+4\dot{x}+5x=5$	M1 M1 M1 M1 E1	Differentiate Substitute for $\dot{y}$ y in terms of $x,\dot{x}$ Substitute	5
(iii)	$\lambda^2+4\lambda+5=0$ $\lambda=-2\pm j$ CF $x=e^{-2t}(A\cos t+B\sin t)$ PI $x=\frac{5}{5}=1$ GS $x=e^{-2t}(A\cos t+B\sin t)+1$ $y=-\dot{x}-3x+10$ $=-e^{-2t}(-A\sin t+B\cos t)+2e^{-2t}(A\cos t+B\sin t)$ $-3e^{-2t}(A\cos t+B\sin t)-3+10$ $=e^{-2t}((-A-B)\cos t+(A-B)\sin t)+7$	M1 M1 A1 F1 B1 F1 M1 M1 M1 A1	Auxiliary equation Solve to get complex roots CF for their roots Their CF with 2 constants + their PI y in terms of $x,\dot{x}$ Differentiate their x Substitute cao	10
(iv)	$t=0, x=0 \Rightarrow A+1=0$ $t=0, y=0 \Rightarrow -A+B+7=0$ $A=-1, B=8$ $x=e^{-2t}(8\sin t-\cos t)+1$ $y=-e^{-2t}(7\cos t+9\sin t)+7$	M1 M1 A1	Use condition on x Use condition on y Both correct	3
(v)	 NB Oscillates about $x=1$, but not apparent at this scale due to small amplitude	B1 B1 B1	Through origin Positive gradient at t = 0 Asymptote x = 1, or their non-zero constant PI (accept oscillatory or non-oscillatory)	3

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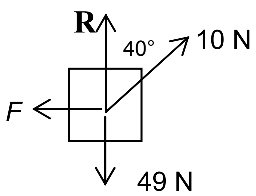
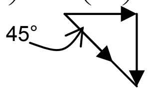
Q 1		mark		sub
	either			
		M1	Attempt at area. If not trapezium method at least one part area correct. Accept equivalent.	
	$70V$ obtained	A1	Or equivalent – need not be evaluated.	
	So $70V = 1400$	M1	Equate **their** $70V$ to 1400. Must have attempt at complete areas or equations.	
	and $V = 20$	A1	cao	
	or	M1	Attempt to find areas in terms of ratios (at least one correct)	
		A1	Correct total ratio – need not be evaluated. (Evidence may be 800 or 400 or 200 seen).	
		M1	Complete method. (Evidence may be 800/40 or 400/20 or 200/10 seen).	
	$V = 20$	A1	cao [Award 3/4 for 20 seen WWW]	
				4

Q 2		mark		sub
	$(v =)12 - 3t^2$	M1	Differentiating	
		A1	Allow confusion of notation, including x =	
	$v = 0 \Rightarrow 12 - 3t^2 = 0$	M1	Dep on 1st M1. Equating to zero.	
	so $t^2 = 4$ and $t = \pm 2$	A1	Accept one answer only but no extra answers. FT only if quadratic or higher degree.	
	$x = \pm 16$	A1	cao. Must have both and no extra answers.	
				5

Q 3		mark		sub
(i)	$R = mg$ so 49 N	B1	Equating to weight. Accept $5g$ (but not mg)	1
(ii)		B1	All except F correct (arrows and labels) (Accept mg, W etc and no angle). Accept cpts instead of 10N. No extra forces.	
		B1	F clearly marked and labelled	2
(iii)	↑ $R + 10\cos 40 - 49 = 0$	M1	Resolve vertically. All forces present and 10N resolved	
		B1	Resolution correct and seen in an equation. (Accept $R = \pm 10\cos 40$ as an equation)	
	R = 41.339… so 41.3 N (3 s. f.)	A1		
	$F = 10\sin 40$ = 6.4278… so 6.43 N (3 s. f.)	B1	Allow –ve if consistent with the diagram.	4
				7

Q 4		mark		sub
(i)	↓ $20 + 16\cos 60 = 28$	B1		1
(ii)	**either** → $16\sin 60$ Mag $\sqrt{28^2 + 192} = 31.2409...$ so 31.2 N (3 s.f.) **or** Cos rule $\text{mag}^2 = 16^2 + 20^2 - 2\times 16\times 20\times \cos 120$ 31.2 N (3 s. f.)	B1 M1 F1 M1 A1 A1	Any form. May be seen in (i). Accept any appropriate equivalent resolution. Use of Pythag with 2 distinct cpts (but not 16 and $\pm$ 20) Allow 34.788… only as FT Must be used with 20 N, 16 N and 60° or 120° Correct substitution	3
(iii)	Magnitude of accn is 15.620… m s^{-2} so 15.6 m s^{-2} (3 s. f.) angle with 20 N force is $\arctan\left(\frac{16\sin 60}{28}\right)$ so 26.3295… so 26.3° (3 s. f.)	B1 M1 A1	Award only for **their** *F* $\div 2$ Or equiv. May use force or acceleration. Allow use of sine or cosine rules. FT only $s \leftrightarrow c$ and sign errors. Accept reciprocal of the fraction. cao	3
				7

Q 5		mark		sub
(i)	sphere $19.6 - T = 2a$ block $T - 14.8 = 4a$	M1 A1 A1	N2L. All forces attempted in one equation. Allow sign errors. No extra forces. Don't condone *F* = *mga*. Accept 2*g* for 19.6	3
(ii)	Solving *T* = 18 *a* = 0.8	M1 A1 F1	Attempt to solve. Award only if two equations present both containing *a* and *T*. Either variable eliminated. Either found cao Other value. Allow wrong equation(s) and wrong working for 1st value [If combined equation used award: M1 as in (i) for the equation with mass of 6 kg; A1 for *a* = 0.8; M1 as in (i) for equation in *T* and *a* for either sphere or block; A1 equation correct; F1 for *T*, FT **their** *a*; B1 Second equation in *T* and *a*.]	3
				6

Q 6		mark		sub
(i)	$t = 2.5 \Rightarrow \mathbf{v} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} + 2.5\begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$	B1	Need not be in vector form	
		E1	Accept diag and/or correct derivation of just $\pm 45°$	
	speed is $\sqrt{10^2 + 10^2} = 14.14...$ so 14.1 m s^{-1} (3 s. f.)	F1	FT **their v**	3
(ii)	$\mathbf{s} = 2.5\begin{pmatrix} -5 \\ 10 \end{pmatrix} + \frac{1}{2} \times 2.5^2 \times \begin{pmatrix} 6 \\ -8 \end{pmatrix}$	M1	Consideration of **s** (const accn or integration)	
		A1	Correct sub into *uvast* with **u** and **a.** (If integration used it must be correct but allow no arb constant)	
	$= \begin{pmatrix} 6.25 \\ 0 \end{pmatrix}$	A1		
	so 090°	A1	cao. CWO.	4
				7

Q 7		mark		sub
(i)	acceleration is $\frac{24}{12}$ so 2 m s^{-2}	B1		1
(ii)	$24 - 15 = 12a$	M1	Use of N2L. Both forces present. Must be *F* = *ma*. No extra forces.	
	a = 0.75 m s^{-2}	A1		
	1st distance is $0.5 \times 2 \times 16 = 16$	M1	Appropriate *uvast* applied at least once.	
	2nd distance is $0.5 \times 0.75 \times 16 = 6$	A1	Need not evaluate. Both found. May be implied. FT (i)	
	Difference is 10 m	A1	cao	5
(iii)	$12g\sin 5 - 15 = 12a$	M1	Use of *F* = *ma*, allow 15 N missing *or* weight not resolved. No extra forces. Allow use of $12\sin 5$.	
		M1	Attempt at weight cpt. Allow sin ↔ cos. Accept seen on diagram. Accept the use of 12 instead of 12*g*.	
		A1	Weight cpt correct. Accept seen on diagram. Allow not used.	
	$a = -0.39587...$ so – 0.396 m s^{-2} (3 s. f.)	A1	Correct direction must be made clear	4
(iv)	time $0 = 1.5 + at \Rightarrow t = 3.789...$	M1	Correct *uvast* . Use of 0, 1.5 and **their** *a* from (iii) or **their** *s* from (iv). Allow sign errors. Condone $u \leftrightarrow v$.	
	so 3.79 s (3 s. f.)	A1		
	distance $s = 0.5 \times (1.5 + 0) \times 3.789...$ (or…)	M1	Correct *uvast* . Use of 0, 1.5 and **their** *a* from (iii) or **their** *t* from (iv). Allow sign errors. Condone $u \leftrightarrow v$.	
	giving *s* = 2.8418… so 2.84 m (3 s. f.)	A1	[The first A1 awarded for *t* or *s* has FT **their** *a* if signs correct; the second awarded is cao]	4
(v)	accn is given by $0 = 1.5 + 3.5a \Rightarrow a = -\frac{3}{7} = -0.42857...$	M1	Use of 0, 1.5 and 3.5 in correct *uvast.* Condone $u \leftrightarrow v$.	
		A1	Allow ±	
	$12g\sin 5 - R = 12 \times -0.42857...$	M1	N2L. Must use **their** *new* accn. Allow only sign errors.	
	so *R* = 15.39… so 15.4 N (3 s. f.)	A1	cao	4
				18

Q 8		mark		sub
(i)	Using $s = ut + 0.5at^2$ with u = 10 and a = -10	E1	Must be clear evidence of derivation of – 5. Accept one calculation and no statement about the other.	1
(ii)	**either** s = 0 gives $10t - 5t^2 = 0$ so $5t(2-t) = 0$ so t = 0 or 2. Clearly need t = 2 **or** Time to highest point is given by 0 = 10 – 10t Time of flight is 2×1 = 2 s horizontal range is 40 m as 40 < 70, hits the ground	 B1 M1 A1 M1 M1 A1 B1 E1	 Factorising Award 3 marks for t = 2 seen WWW Dep on 1st M1. Doubling **their** t. Properly obtained FT 20 × **their** t Must be clear. FT **their** range.	5
(iii)	need $10t - 5t^2 = -15$ Solving $t^2 - 2t - 3 = 0$ so $(t-3)(t+1) = 0$ and t = 3 range is 60 m	M1 M1 A1 M1 A1	[May divide flight into two parts] Equate s = -15 or equivalent. Allow use of ± 15. Method leading to solution of a quadratic. Equivalent form will do. Obtaining t = 3. Allow no reference to the other root. [Award SC3 if t = 3 seen WWW] Range is 20 × **their** t (provided t > 0) cao. CWO.	5
(iv)	Using (ii) & (iii), since 40 + 60 > 70, paths cross (For $0 < t \le 2$) both have same vertical motion so B is always 15 m above A	E1 E1	Must be convincing. Accept sketches. Do not accept evaluation at one or more points alone. That B is *always* above A must be clear.	2
(v)	Need x components summing to 70 $20 \times 0.75 + 20 \times 2.75 = 15 + 55 = 70$ so true Need y components the same $10 \times 2.75 - 5 \times 2.75^2 + 15 = 4.6875$ $10 \times 0.75 - 5 \times 0.75^2 = 4.6875$	M1 E1 M1 B1 E1	May be implied. Or correct derivation of 0.75 s or 2.75 s Attempt to use 0.75 and 2.75 in two vertical height equations (accept same one or wrong one) 0.75 and 2.75 each substituted in the appropriate equn Both values correct. [Using cartesian equation: B1, B1 each equation: M1 solving: A1 correct point of intersection: E1 Verify times]	5
				18

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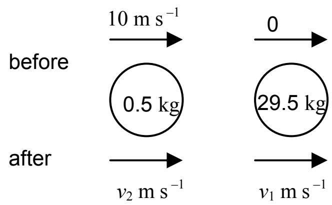
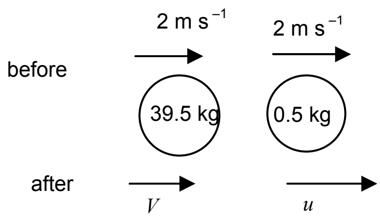
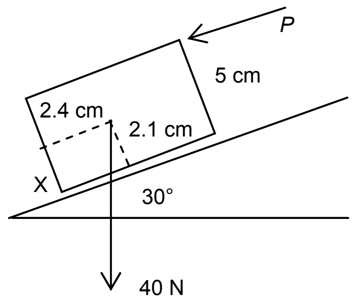
Q 1		mark		sub
(i)				
	$10 \times 0.5 = 0.5v_2 + 29.5v_1$	M1 A1	PCLM and two terms on RHS All correct. Any form.	
	$\frac{v_1 - v_2}{0 - 10} = -0.8$	M1	NEL	
		A1	Any form	
	$v_1 = 0.3$ so $V_1 = 0.3$	A1		
	$v_2 = -7.7$ so $V_2 = 7.7$ m s^{-1}	A1	Speed. Accept ± .	
	in opposite to original direction	F1	Must be correct interpretation of clear working	7
(ii) (A)	$10 \times 0.5 = 30V$	M1 A1	PCLM and coalescence All correct. Any form.	
	so $V = \frac{1}{6}$	A1	Clearly shown. Accept decimal equivalence. Accept no direction.	3
(B)	Same velocity No force on sledge in direction of motion	E1 E1	Accept speed	2
(iii)		B1		
	$2 \times 40 = 0.5u + 39.5V$	M1 A1	PCLM, masses correct Any form	
	$u - V = 10$	B1	May be seen on the diagram.	
	Hence $V = 1.875$	A1	Accept no reference to direction.	5
		17		

Q 2		mark	comment	sub
(i)	$X = R\cos 30$ (1) $Y + R\sin 30 = L$ (2)	B1 M1 A1	 Attempt at resolution	3
(ii)	ac moments about A $R - 2L = 0$ Subst in (1) and (2) $X = 2L\frac{\sqrt{3}}{2}$ so $X = \sqrt{3}L$ $Y + 2L \times \frac{1}{2} = L$ so $Y + L = L$ and $Y = 0$	B1 M1 E1 E1	 Subst **their** $R = 2L$ into **their** (1) or (2) Clearly shown Clearly shown	4
(iii)	(Below all are taken as tensions e. g. T_{AB} in AB)	B1 B1	Attempt at all forces (allow one omitted) Correct. Accept internal forces set as tensions or thrusts or a mix	2
(iv)	$\downarrow$ A $T_{AD}\cos 30\ (-Y) = 0$ so $T_{AD} = 0$	M1 E1	Vert equilibrium at A attempted. $Y = 0$ need not be explicit	2
(v)	Consider the equilibrium at pin-joints A $\rightarrow$ $T_{AB} - X = 0$ so $T_{AB} = \sqrt{3}L$ (T) C $\downarrow$ $L + T_{CE}\cos 30 = 0$ so $T_{CE} = \frac{-2L}{\sqrt{3}}$ so $\frac{2L}{\sqrt{3}}\left(= \frac{2L\sqrt{3}}{3}\right)$ (C) C $\leftarrow$ $T_{BC} + T_{CE}\cos 60 = 0$ so $T_{BC} = -\left(-\frac{2\sqrt{3}L}{3}\right) \times \frac{1}{2} = \frac{\sqrt{3}L}{3}$ (T)	M1 B1 B1 B1 B1 B1 F1	At least one relevant equilib attempted (T) not required Or equiv from **their** diagram Accept any form following from **their** equation. (C) not required. Or equiv from **their** diagram FT **their** T_{CE} or equiv but do not condone inconsistent signs even if right answer obtained. (T) not required. T and C consistent with **their** answers and **their** diagram	7
(vi)	$\downarrow$ B $T_{BD}\cos 30 + T_{BE}\cos 30 = 0$ so $T_{BD} = -T_{BE}$ so mag equal and opp sense	M1 E1	Resolve vert at B A statement required	2
		20		

Q 3		mark		sub
(i)	(10, 2, 2.5)	B1		1
(ii)	By symmetry $\bar{x} = 10$, $\bar{y} = 2$ $(240+80)\bar{z} = 80 \times 0 + 240 \times 2.5$ so $\bar{z} = 1.875$	B1 B1 B1 M1 A1	 Total mass correct Method for c.m. Clearly shown	5
(iii)	$\bar{x} = 10$ by symmetry $(320+80)\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = 320\begin{pmatrix}10\\ 2\\ 1.875\end{pmatrix} + 80\begin{pmatrix}10\\ 4\\ 3\end{pmatrix}$ $\bar{y} = 2.4$ $\bar{z} = 2.1$	E1 M1 B1 B1 E1 E1	Could be derived Method for c.m. *y* coord c.m. of lid *z* coord c.m. of lid shown shown	6
(iv)	 c.w moments about X $40 \times 0.024\cos 30 - 40 \times 0.021\sin 30$ = 0.41138… so 0.411 N m (3 s. f.)	B1 B1 B1 E1	Award for correct use of dimensions 2.1 and 2.4 or equivalent 1st term o.e. (allow use of 2.4 and 2.1) 2nd term o.e. (allow use of 2.4 and 2.1) Shown [Perpendicular method: M1 Complete method: A1 Correct lengths and angles E1 Shown]	4
(v)	0.41138… - 0.05*P* = 0 *P* = 8.22768…… so 8.23 (3 s. f.)	M1 A1	Allow use of 5 Allow if cm used consistently	2
		18		

Q 4		mark		sub
(i)	$F_{max} = \mu R$ $R = 2g \cos 30$ so $F_{max} = 0.75 \times 2 \times 9.8 \times \cos 30 = 12.730...$ so 12.7 N (3 s. f.) **either** Weight cpt down plane is $2g\sin 30 = 9.8$ N so no as 9.8 < 12.7 **or** Slides if $\mu < \tan 30$ But 0.75 > 0.577… so no	M1 B1 A1 B1 E1 B1 E1	Must have attempt at *R* with *mg* resolved [Award 2/3 retrospectively for limiting friction seen below] The inequality must be properly justified The inequality must be properly justified	5
(ii) (A)	Increase in GPE is $2 \times 9.8 \times (6 + 4\sin 30) = 156.8$ J	M1 B1 A1	Use of *mgh* 6 + 4 sin 30	3
(B)	WD against friction is $4 \times 0.75 \times 2 \times 9.8 \times \cos 30 = 50.9222...$ J	M1 A1	Use of WD = *Fd*	2
(C)	Power is $10 \times (156.8 + 50.9222\ldots)/60$ = 34.620… so 34.6 W (3 s. f.)	M1 A1	Use *P* = WD/*t*	2
(iii)	$0.5 \times 2 \times 9^2$ $= 2 \times 9.8 \times (6 + x\sin 30)$ $+ \ 0.5 \times 2 \times 4^2$ -90 so $x = 3.8163\ldots$ so 3.82 (3 s. f.)	M1 B1 A1 A1 A1	Equating KE to GPE and WD term. Allow sign errors and one KE term omitted. Allow ‘old’ friction as well. Both KE terms. Allow wrong signs. All correct but allow sign errors All correct, including signs. cao	5
		17		

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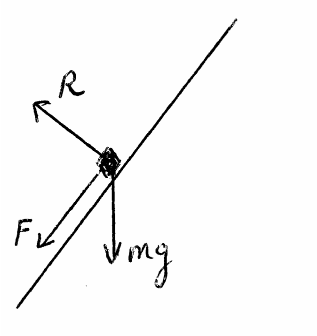
1 (i)	[Velocity] $=\mathrm{L\,T^{-1}}$ [Acceleration] $=\mathrm{L\,T^{-2}}$ [Force] $=\mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*Deduct 1 mark if answers given as* $\mathrm{m\,s^{-1}},\ \mathrm{m\,s^{-2}},\ \mathrm{kg\,m\,s^{-2}}$
(ii)	$[G]=\dfrac{[F][r^2]}{[m_1][m_2]}=\dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L^2})}{\mathrm{M^2}}$ $=\mathrm{M^{-1}\,L^3\,T^{-2}}$	M1 E1 **2**	
(iii)	$G=6.67\times10^{-11}\times0.4536\times\dfrac{1}{(0.3048)^3}$ $=1.07\times10^{-9}\quad(\mathrm{lb^{-1}\,ft^3\,s^{-2}})$	M1M1 A1 **3**	For $\times0.4536$ and $\times\dfrac{1}{(0.3048)^3}$ *SC* Give M1 for $6.67\times10^{-11}\times\dfrac{1}{0.4536}\times(0.3048)^3$ $(=4.16\times10^{-12})$
(iv)	$[\mathrm{RHS}]=\sqrt{\dfrac{(\mathrm{M^{-1}\,L^3\,T^{-2}})(\mathrm{M})}{\mathrm{L}}}$ $=\sqrt{\mathrm{L^2\,T^{-2}}}=\mathrm{L\,T^{-1}}$ which is the same as [LHS]	M1A1 E1 **3**	
(v)	$\mathrm{T}=(\mathrm{M^{-1}\,L^3\,T^{-2}})^{\alpha}\,\mathrm{M}^{\beta}\,\mathrm{L}^{\gamma}$ Powers of M: $-\alpha+\beta=0$ of L: $3\alpha+\gamma=0$ of T: $-2\alpha=1$ $\alpha=-\frac{1}{2},\quad\beta=-\frac{1}{2},\quad\gamma=\frac{3}{2}$	M1 M1 A1 M1 A1 **5**	At least two equations Three correct equations Obtaining at least one of $\alpha,\ \beta,\ \gamma$

2(a)(i)	At the highest point, $T+5\times 9.8=5\times\frac{v^2}{1.8}$ For least speed, $T=0$, $v^2=1.8\times 9.8$ Speed is at least 4.2 m s^{-1}	M1 A1 E1 **3**	Using acceleration $v^2/1.8$ *T* may be omitted
(ii)	For least tension, speed at top is 4.2 m s^{-1} By conservation of energy, $\frac{1}{2}\times 5\times(w^2-4.2^2)=5\times 9.8\times 3.6$ $w^2=88.2\quad(w=9.39)$ $T-5\times 9.8=5\times\frac{88.2}{1.8}$ Tension is at least 294 N	M1 A1 M1 A1 ft A1 **5**	Energy equation with 3 terms Equation of motion with 3 terms
(b)(i)	$R\sin\theta=0.02\times 9.8$ $R\cos\theta=0.02\times 0.32\times 8.75^2$ $\tan\theta=\frac{0.02\times 9.8}{0.02\times 0.32\times 8.75^2}=0.4$	B1 M1 A1 E1 **4**	Using acceleration 0.32×8.75^2 *SC* If $\sin\theta$ and $\cos\theta$ interchanged, award B0M1A1E0
(ii)		B1 B1 **2**	For *R* and *mg* For *F* acting down the slope
(iii)	$R\sin\theta=0.02\times 9.8+F\cos\theta$ $R\cos\theta+F\sin\theta=0.02\times 0.32\,\omega^2$ For maximum ω, $F=\mu R$ $R(\sin\theta-\mu\cos\theta)=0.02\times 9.8$ $R(\cos\theta+\mu\sin\theta)=0.02\times 0.32\,\omega^2$ $\omega^2=\frac{9.8(\cos\theta+\mu\sin\theta)}{0.32(\sin\theta-\mu\cos\theta)}=\frac{9.8(1+\mu\tan\theta)}{0.32(\tan\theta-\mu)}$ $=\frac{9.8(1+0.11\times 0.4)}{0.32(0.4-0.11)}$ $\omega=10.5$	M1 A1 A1 M1 M1 A1 cao **6**	Resolving *F* and *R* [or *mg* and accn] *Can give A1A1 for sin / cos interchanged consistent with (i)* *Dependent on first M1* Obtaining a numerical value for ω^2 *Dependent on M1M1*

3 (i)	$k \times 0.8 = 60 \times 9.8$ Stiffness is 735 N m^{-1}	M1 A1 **2**	
(ii)	Loss of PE is $60 \times 9.8(32 + x)$ Gain in EE is $\frac{1}{2} \times 735x^2$ $\frac{1}{2} \times 735x^2 = 60 \times 9.8(32 + x)$ $x^2 = 1.6(32 + x)$ $x^2 - 1.6x - 51.2 = 0$ $(x - 8)(x + 6.4) = 0$ $x = 8$ Length of rope is 40 m	B1 B1 M1 E1 M1 A1 **6**	*If x is measured from equilibrium position, treat as MR* Obtaining a value of *x*
(iii)	Tension $T = 735x$ $mg - T = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $60 \times 9.8 - 735x = 60\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 12.25x = 9.8$	B1 M1 A1 E1 **4**	 Equation of motion with 3 terms
(iv)	SHM with $\omega^2 = 12.25$ ($\omega = 3.5$) Time taken is $\frac{1}{4} \times \frac{2\pi}{\omega}$ $= \frac{1}{7}\pi = 0.449 \text{ s}$	M1 M1 A1 **3**	 or $\omega t = \frac{1}{2}\pi$
(v)	When $x = 8$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 9.8 - 12.25 \times 8$ $= -88.2$ Acceleration is 88.2 m s^{-2} (upwards) This acceleration (9*g*) is too large for comfort	M1 A1 B1 **3**	or $735 \times 8 - 60 \times 9.8 = 60a$

4 (i)	Area is $\int_1^a \frac{1}{x^2}\,dx = \left[-\frac{1}{x} \right]_1^a$	M1	
	$= 1 - \frac{1}{a}$	A1	
	$\int x\,y\,dx = \int_1^a \frac{1}{x}\,dx \quad (= \ln a)$	M1	
	$\bar{x} = \frac{\int x\,y\,dx}{\int y\,dx}$	M1	
	$= \frac{\ln a}{1 - \frac{1}{a}} \quad \left(= \frac{a \ln a}{a-1} \right)$	A1	
	$\int \frac{1}{2} y^2\,dx = \int_1^a \frac{1}{2x^4}\,dx = \left[-\frac{1}{6x^3} \right]_1^a$ $= \frac{1}{6}\left(1 - \frac{1}{a^3}\right)$	M1	Condone omission of $\frac{1}{2}$
	$\bar{y} = \frac{\int \frac{1}{2} y^2\,dx}{\int y\,dx}$	M1	($\frac{1}{2}$ needed for this mark)
	$= \frac{\frac{1}{6}\left(1 - \frac{1}{a^3}\right)}{1 - \frac{1}{a}} = \frac{a^3 - 1}{6(a^3 - a^2)}$	E1 **8**	
(ii)	When $a = 2,\ \bar{x} = 2\ln 2,\ \bar{y} = \frac{7}{24}$		
	$\tan\theta = \frac{\bar{x} - 1}{1 - \bar{y}}$	M1	CM vertically below A
	$= \frac{2\ln 2 - 1}{1 - \frac{7}{24}}$	A1	Correct expression for $\tan\theta$ or $\tan(90-\theta)$
	$\theta = 28.6°$	A1 **3**	

(iii)	Volume is $\int \pi y^2 \, dx \;= \pi \int_1^a \frac{1}{x^4} dx$ $= \pi \left[-\frac{1}{3x^3} \right]_1^a = \frac{\pi}{3}\left(1 - \frac{1}{a^3}\right)$ $\int \pi x y^2 \, dx \;= \pi \int_1^a \frac{1}{x^3} dx = \pi \left[-\frac{1}{2x^2} \right]_1^a$ $= \frac{\pi}{2}\left(1 - \frac{1}{a^2}\right)$ $\bar{x} = \frac{\int \pi x y^2 \, dx}{\int \pi y^2 \, dx}$ $= \frac{\frac{\pi}{2}\left(1-\frac{1}{a^2}\right)}{\frac{\pi}{3}\left(1-\frac{1}{a^3}\right)} = \frac{3(a^3 - a)}{2(a^3 - 1)}$ Since $a > 1, \;\; a^3 - a < a^3 - 1$ Hence $\bar{x} < \frac{3}{2}$, i.e. $\bar{x} < 1.5$	M1 A1 M1 M1 A1 M1 E1 **7**	π may be omitted throughout Any correct form or $\bar{x} \to 1.5$ as $a \to \infty$ Fully convincing argument

Mark Scheme 4766
January 2007

GENERAL INSTRUCTIONS

Marks in the mark scheme are explicitly designated as **M**, **A**, **B**, **E** or **G**.

M marks ("method") are for an attempt to use a correct method (not merely for stating the method).

A marks ("accuracy") are for accurate answers and can only be earned if corresponding **M** mark(s) have been earned. Candidates are expected to give answers to a sensible level of accuracy in the context of the problem in hand. The level of accuracy quoted in the mark scheme will sometimes deliberately be greater than is required, when this facilitates marking.

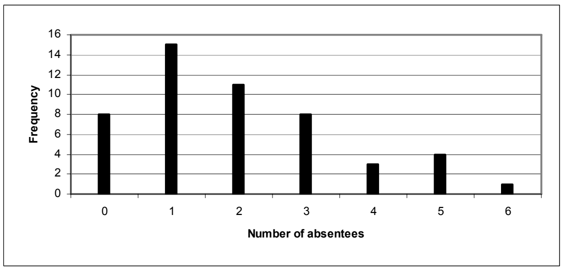
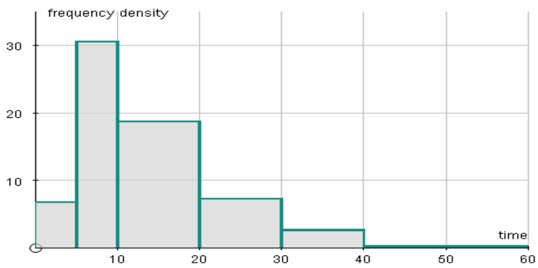
B marks are independent of all others. They are usually awarded for a single correct answer.

E marks ("explanation") are for explanation and/or interpretation. These will frequently be sub divisible depending on the thoroughness of the candidate's answer.

G marks ("graph") are for completing a graph or diagram correctly.

- Insert part marks in **right-hand** margin in line with the mark scheme. For fully correct parts tick the answer. For partially complete parts indicate clearly in the body of the script where the marks have been gained or lost, in line with the mark scheme.
- Please indicate incorrect working by ringing or underlining as appropriate.
- Insert total in **right-hand** margin, ringed, at end of question, in line with the mark scheme.
- Numerical answers which are not exact should be given to at least the accuracy shown. Approximate answers to a greater accuracy *may* be condoned.
- Probabilities should be given as fractions, decimals or percentages.
- FOLLOW-THROUGH MARKING SHOULD NORMALLY BE USED WHEREVER POSSIBLE. There will, however, be an occasional designation of '**c.a.o.**' for "correct answer only".
- Full credit MUST be given when correct alternative methods of solution are used. If errors occur in such methods, the marks awarded should correspond as nearly as possible to equivalent work using the method in the mark scheme.
- The following notation should be used where applicable:

FT	Follow-through marking
BOD	Benefit of doubt
ISW	Ignore subsequent working

Q	Answer	Marks	Total
Q 1 (i)	Mean = 127.6/13 = 9.8 Median = 8.6 Midrange = 14.5	M1 for 127.6/13 soi A1 CAO B1 CAO B1 CAO	**4**
(ii)	Mean slightly inflated due to the outlier Median good since it is not affected by the outlier Midrange poor as it is highly inflated due to the outlier	B1 B1 B1	**3**
		TOTAL	**7**
Q 2 (i)		G1 labelled linear scales on both axes G1 heights	**2**
(ii)	Mean = $\frac{99}{50}$ = 1.98 S_{xx} = $315 - \frac{99^2}{50}$ (= 118.98) *rmsd* = $\sqrt{\frac{118.98}{50}}$ = 1.54 *NB full marks for correct results from recommended method which is use of calculator functions*	B1 for mean M1 for attempt at S_{xx} A1 CAO	**3**
(iii)	New mean = 30 – 1.98 = 28.02 New rmsd = 1.54 (unchanged)	B1 FT their mean B1 FT their rmsd	**2**
		TOTAL	**7**
Q 3 (i)	time / freq / width / f dens 0- / 34 / 5 / 6.8 5- / 153 / 5 / 30.6 10- / 188 / 10 / 18.8 20- / 73 / 10 / 7.3 30- / 27 / 10 / 2.7 40- / 5 / 20 / 0.25 	M1 for fds A1 CAO Accept any suitable unit for fd such as eg freq per 5 mins. G1 linear scales on both axes and label G1 width of bars G1 height of bars	**5**
(ii)	Positive skewness	B1 CAO (indep)	**1**
		TOTAL	**6**

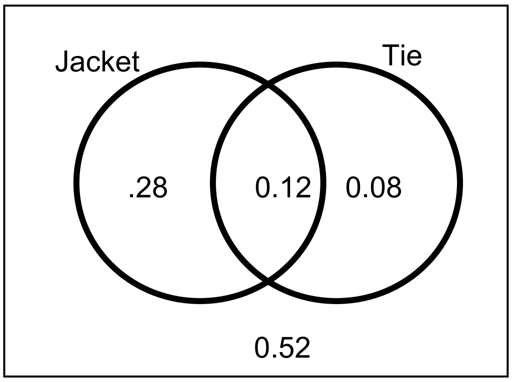
<table>
<tr><td>Q 4(i)</td><td>
<table>
<tr><td>r</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>P($X = r$)</td><td>k</td><td>$3k$</td><td>$5k$</td><td>$7k$</td><td>$9k$</td><td>$11k$</td></tr>
</table>

$36k = 1$, so $k = \frac{1}{36}$
</td><td>B1 for $3k$, $5k$, $7k$, $9k$

M1 for sum of six multiples of k = 1

A1 CAO MUST BE FRACTION IN SIMPLEST FORM</td><td>3</td></tr>
<tr><td>(ii)</td><td>E(X) =
$1 \times \frac{1}{36} + 2 \times \frac{3}{36} + 3 \times \frac{5}{36} + 4 \times \frac{7}{36} + 5 \times \frac{9}{36} + 6 \times \frac{11}{36} = \frac{161}{36} = 4.47$</td><td>M1 for $\Sigma\, rp$
A1 CAO</td><td>2</td></tr>
<tr><td>(iii)</td><td>P(X=16) = $6 \times \left(\frac{1}{6}\right)^3$

$= \frac{6}{216} = \frac{1}{36}$</td><td>M1 for $6 \times$
M1 indep for $\left(\frac{1}{6}\right)^3$

A1 CAO</td><td>3</td></tr>
<tr><td></td><td></td><td>TOTAL</td><td>8</td></tr>
<tr><td>Q 5(i)</td><td>P(jacket and tie) = $0.4 \times 0.3 = 0.12$</td><td>M1 for multiplying
A1 CAO</td><td>2</td></tr>
<tr><td>(ii)</td><td>
</td><td>G1 for two intersecting circles labelled

G1 for 0.12 and either 0.28 or 0.08

G1 for remaining probabilities

<u>Note</u> FT their 0.12 provided < 0.2</td><td>3</td></tr>
<tr><td>(iii)</td><td>(A) P(jacket or tie) =P(J) + P(T) – P(J∩T)
= 0.4 + 0.2 – 0.12 = 0.48
OR = 0.28 + 0.12 + 0.08 = 0.48

(B) P(no jacket or no tie) = 0.52 + 0.28 + 0.08 = 0.88
OR 0.6 + 0.8 – 0.52 = 0.88
OR 1 – 0.12 = 0.88</td><td>B1 FT

B2 FT
<u>Note</u> FT their 0.12 provided < 0.2</td><td>3</td></tr>
<tr><td></td><td></td><td>TOTAL</td><td>8</td></tr>
</table>

Q 6			
(i)	Median = 3370 Q_1 = 3050 Q_3 = 3700 Inter-quartile range = 3700 – 3050 = 650	B1 B1 for Q_3 or Q_1 B1 for IQR	**3**
(ii)	Lower limit 3050 – 1.5 × 650 = 2075 Upper limit 3700 + 1.5 × 650 = 4675 Approx 40 babies below 2075 and 5 above 4675 so total 45	B1 B1 M1 (for either) A1	**4**
(iii)	Decision based on convincing argument: eg 'no, because there is nothing to suggest that they are not genuine data items and these data may influence health care provision'	E2 for convincing argument	**2**
(iv)	All babies below 2600 grams in weight	B2 CAO	**2**
(v)	(*A*) $X \sim B(17, 0.12)$ $P(X = 2) = \binom{17}{2} \times 0.12^2 \times 0.88^{15} = 0.2878$ (*B*) $P(X > 2)$ $= 1 - (0.2878 + \binom{17}{1} \times 0.12 \times 0.88^{16} + 0.88^{17})$ $= 1 - (0.2878 + 0.2638 + 0.1138) = 0.335$	M1 $\binom{17}{2} \times p^2 \times q^{15}$ M1 indep $0.12^2 \times 0.88^{15}$ A1 CAO M1 for $P(X=1) + P(X=0)$ M1 for $1 - P(X \leq 2)$ A1 CAO	**3** **3**
(vi)	Expected number of occasions is 33.5	B1 FT	**1**
		TOTAL	**18**

Q			
Q 7 (i)	(*A*) P(both) = $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ (*B*) P(one) = $2 \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$ (*C*) P(neither) = $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$	B1 CAO B1 CAO B1 CAO	**3**
(ii)	Independence necessary because otherwise, the probability of one seed germinating would change according to whether or not the other one germinates. May not be valid as the two seeds would have similar growing conditions eg temperature, moisture, etc. *NB Allow valid alternatives*	E1 E1	**2**
(iii)	Expected number = 2 $\times \frac{2}{3} = \frac{4}{3}$ (= 1.33) E(X^2) = $0 \times \frac{1}{9} + 1 \times \frac{4}{9} + 4 \times \frac{4}{9} = \frac{20}{9}$ Var(X) = $\frac{20}{9} - \left(\frac{4}{3}\right)^2 = \frac{4}{9}$ = 0.444 *NB use of npq scores M1 for product, A1CAO*	B1 FT M1 for E(X^2) A1 CAO	**3**
(iv)	Expect $200 \times \frac{8}{9} = 177.8$ plants So expect 0.85 × 177.8 = 151 onions	M1 for 200 × $\frac{8}{9}$ M1 dep for × 0.85 A1 CAO	**3**
(v)	Let $X \sim B(18, p)$ Let p = probability of germination (for population) H_0: $p = 0.90$ H_1: $p < 0.90$ P($X \leq 14$) = 0.0982 > 5% So not enough evidence to reject H_0 Conclude that there is not enough evidence to indicate that the germination rate is below 90%. Note: use of critical region method scores M1 for region {0,1,2,…, 13} M1 for 14 does not lie in critical region then A1 E1 as per scheme	B1 for definition of p B1 for H_0 B1 for H_1 M1 for probability M1 dep for comparison A1 E1 for conclusion in context	**7**
		TOTAL	**18**

Mark Scheme 4767
January 2007

Question 1

(i)	$\bar{t}$ = 112.8, $\bar{v}$ = 0.6 $b = \frac{Svt}{Svv} = \frac{405.2 - 3 \times 564/5}{2.20 - 3^2/5} = \frac{66.8}{0.4}$ = 167 OR $b = \frac{405.2/5 - 0.6 \times 112.8}{2.20/5 - 0.6^2} = \frac{13.36}{0.08}$ = 167 hence least squares regression line is: $t - \bar{t} = b(v - \bar{v})$ $\Rightarrow t - 112.8 = 167(v - 0.6)$ $\Rightarrow t = 167v + 12.6$	B1 for $\bar{t}$ and $\bar{v}$ used (SOI) M1 for attempt at gradient (*b*) A1 for 167 CAO M1 for equation of line A1 FT	**5**
(ii)	(*A*) For 0.5 litres, predicted time = = 167 × 0.5 + 12.6 = 96.1 seconds (*B*) For 1.5 litres, predicted time = = 167 ×1.5 + 12.6 = 263.1 seconds Any valid relevant comment relating to each prediction such as eg: 'First prediction is fairly reliable as it is interpolation and the data is a good fit' 'Second prediction is less certain as it is an extrapolation'	M1 for at least one prediction attempted A1 for both answers (FT their equation if *b*>0) NB for reading predictions off the graph only award A1 if accurate to nearest whole number E1 (first prediction) E1 (second prediction)	**4**
(iii)	The *v*-coefficient is the number of additional seconds required for each extra litre of water	E1 for indication of rate wrt *v* E1 *dep* for specifying ito units	**2**
(iv)	*v* = 0.8 ⇒ predicted *t* = 167 × 0.8 + 12.6 = 146.2 Residual = 156 – 146.2 = 9.8 *v* = 1.0 ⇒ predicted *t* = 167 × 1.0 + 12.6 = 179.6 Residual = 172 – 179.6 = –7.6	M1 for either prediction M1 for either subtraction A1 *CAO* for absolute value of both residuals B1 for both signs correct.	**4**
(v)	The residuals can be measured by finding the vertical distance between the plotted point and the regression line. The sign will be negative if the point is below the regression line (and positive if above).	E1 for distance E1 for vertical E1 for sign	**3**
			18

Question 2

(a) (i)	$X \sim$ N(28,16) P(24 < X < 33) = $\mathrm{P}\left(\frac{24-28}{4} < Z < \frac{33-28}{4}\right)$ = P(–1 < Z < 1. 25) = Φ(1.25) – (1 – Φ(1)) = 0.8944 – (1 –0.8413) = 0.8944 – 0.1587 = 0.7357 (4 s.f.) *or* 0.736 (to 3 s.f.)	M1 for standardizing A1 for 1. 25 and -1 M1 for prob. with tables and correct structure A1 CAO (min 3 s.f., to include use of difference column)	**4**
(ii)	25000 ×0.7357 ×0.1 = £1839 25000 ×0.1587 ×0.05 = £198 Total = £1839 + £198 = £2037	M1 for either product, (with or without price) M1 for sum of both products with price A1 CAO awrt £2040	**3**
(iii)	$X \sim$ N(k, 16) From tables Φ^{-1} (0.95) = 1.645 $\frac{33-k}{4} = 1.645$ 33 – k = 1.645 × 4 k = 33 – 6.58 k = 26.42 (4 s.f.) *or* 26.4 (to 3 s.f.)	B1 for ±1.645 seen M1 for correct equation in k with positive z-value A1 *CAO*	**3**
(b) (i)	H_0: μ = 0.155; H_1: μ > 0.155 Where μ denotes the mean weight in kilograms of the population of onions of the new variety	B1 for both correct & ito μ B1 for definition of μ	**2**
(ii)	Mean weight = 4.77/25 = 0.1908 Test statistic = $\frac{0.1908-0.155}{\sqrt{0.005}/\sqrt{25}} = \frac{0.0358}{0.01414}$ = 2.531 1% level 1-tailed critical value of z = 2.326 2.531 > 2.236 so significant. There is sufficient evidence to reject H_0 It is reasonable to conclude that the new variety has a higher mean weight.	B1 M1 must include √25 A1FT B1 for 2.326 M1 For sensible comparison leading to a conclusion A1 for correct, consistent conclusion in words and in context	**6**
			18

Question 3

(i)	Mean = $\frac{\Sigma xf}{n} = \frac{0+20+12+3}{80} = \frac{35}{80}$ (= 0.4375)	B1 for mean ***NB answer given***	**1**
(ii)	Variance = 0.6907^2 = 0.4771 So Poisson distribution may be appropriate, since mean is close to variance	B1 for variance E1*dep on squaring s*	**2**
(iii)	P(*X* = 1) = $e^{-0.4375}\frac{0.4375^1}{1!}$ = 0.282 (3 s.f.) *Either:* Thus the expected number of 1's is 22.6 which is reasonably close to the observed value of 20. *Or*: This probability compares reasonably well with the relative frequency 0.25	M1 for probability calc. M0 for tables unless interpolated (0.2813) A1 B1 for expectation of 22.6 or r.f. of 0.25 E1 for comparison	**4**
(iv)	$\lambda = 8\times0.4375 = 3.5$ Using tables: P(*X* ≥ 12) = 1 – P(*X* ≤ 11) = 1 – 0.9997 = 0.0003	B1 for mean (SOI) M1 for using tables to find 1 – P(*X* ≤ 11) A1 FT	**3**
(v)	The probability of at least 12 free repairs is very low, so the model is not appropriate. This is probably because the mean number of free repairs in the launderette will be much higher since the machines will get much more use than usual.	E1 for 'at least 12' E1 for very low E1	**3**
(vi)	*(A)* λ = 0.4375 + 0.15 = 0.5875 P(*X* = 3) = $e^{-0.5875}\frac{0.5875^3}{3!}$ = 0.0188 (3 s.f.)	B1 for mean (SOI) M1 A1	**3**
	(B) P(Drier needs 1) = $e^{-0.15}\frac{0.15^1}{1!}$ = 0.129 P(Each needs just 1) = 0.282 ×0.129 = 0.036	B1 for 0.129 (SOI) B1FT for 0.036	**2**
			18

Question 4

(i)	H_0: no association between ambition and home location; H_1: some association between ambition and home location;	B1 in context	**1**
	(tables below)	M1 A1 for attempt at expected values M1 for valid attempt at $(O-E)^2/E$ A1CAO for X^2	**4**
	$X^2 = 3.716$ Refer to χ_1^2 Critical value at 5% level = 3.841 Result is not significant There is insufficient evidence to conclude that there is any association between home location and ambition. NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1or final B1 or final E1	B1 for 1 dof SOI B1 CAO for cv B1 *dep on attempt at cv* E1 conclusion in context	**4**
(ii) (*A*)	Expected Country, Results = 249 * 156 / 480 = 80.93 Expected Country, Other = 231 * 156 / 480 = 75.08	B1 B1	**2**
(*B*)	Refer to χ_2^2 Critical value at 5% level = 5.991 Result is significant There is evidence to conclude that there is association between home location and ambition.	B1 for 2 dof SOI B1 CAO for cv E1 for conclusion in context	**3**
(*C*)	'Country' students are much less likely than city or town to have 'Results' as their main ambition. Low contributions show that city and town students do not appear to differ markedly in their ambitions.	E1 for correct obs^n for 'Country' E1 for additional correct observation (must refer to contributions)	**2**
(iii)	Conclusion in (i) is valid if only categorizing home location into city and non-city. However if non-city is subdivided into town and country this additional subdivision gives the data more precision and allows the relationship in part (ii) (*C*) to be revealed.	E1 E1	**2**
			18

Observed		Home location	
		City	Non-city
Ambition	Good results	102	147
	Other	75	156

Expected		Home location	
		City	Non-city
Ambition	Good results	91.82	157.18
	Other	85.18	145.82

Contribution to the test statistic		Home location	
		City	Non-city
Ambition	Good results	1.129	0.659
	Other	1.217	0.711

Mark Scheme 4768
January 2007

Q1	$\mathrm{f}(x) = k(1-x) \qquad 0 \le x \le 1$			
(i)	$\int_0^1 k(1-x)\mathrm{d}x = 1$ $\therefore k[x - \frac{1}{2}x^2]_0^1 = 1$ $\therefore k(1-\frac{1}{2}) - 0 = 1$ $\therefore k = 2$ Labelled sketch: straight line segment from (0,2) to (1,0).	M1 E1 G1 G1	Integral of f(x), including limits (possibly implied later), equated to 1. Convincingly shown. Beware printed answer. Correct shape. Intercepts labelled.	 4
(ii)	$\mathrm{E}(X) = \int_0^1 2x(1-x)\mathrm{d}x$ $= [x^2 - \frac{2}{3}x^3]_0^1 = (1-\frac{2}{3}) - 0 = \frac{1}{3}$ $\mathrm{E}(X^2) = \int_0^1 2x^2(1-x)\mathrm{d}x$ $= [\frac{2}{3}x^3 - \frac{2}{4}x^4]_0^1 = (\frac{2}{3} - \frac{1}{2}) - 0 = \frac{1}{6}$ $\mathrm{Var}(X) = \frac{1}{6} - (\frac{1}{3})^2$ $= \frac{1}{18}$	M1 A1 M1 M1 A1	Integral for E(X) including limits (which may appear later). Integral for E(X^2) including limits (which may appear later). Convincingly shown. Beware printed answer.	 5
(iii)	$\mathrm{F}(x) = \int_0^x 2(1-t)\mathrm{d}t$ $= [2t - t^2]_0^x = (2x - x^2) - 0 = 2x - x^2$ $\mathrm{P}(X > \mu) = \mathrm{P}(X > \frac{1}{3}) = 1 - \mathrm{F}(\frac{1}{3})$ $= 1 - (2 \times \frac{1}{3} - (\frac{1}{3})^2) = 1 - \frac{5}{9} = \frac{4}{9}$	M1 A1 M1 A1	Definition of cdf, including limits, possibly implied later. Some valid method must be seen. [for 0 ≤ x ≤ 1; do not insist on this.] For 1 – c's F(μ). ft c's E(X) and F(x). If answer only seen in decimal expect 3 d.p. or better.	 4
(iv)	$\mathrm{F}\left(1 - \frac{1}{\sqrt{2}}\right) = 2\left(1 - \frac{1}{\sqrt{2}}\right) - \left(1 - \frac{1}{\sqrt{2}}\right)^2$ $= 2 - \frac{2}{\sqrt{2}} - 1 + \frac{2}{\sqrt{2}} - \frac{1}{2} = \frac{1}{2}$ **Alternatively:** $2m - m^2 = \frac{1}{2}$ $\therefore m^2 - 2m + \frac{1}{2} = 0$ $\therefore m = 1 \pm \frac{1}{\sqrt{2}}$ so $m = 1 - \frac{1}{\sqrt{2}}$	M1 E1 M1 E1	Substitute $m = 1 - \frac{1}{\sqrt{2}}$ in c's cdf. Convincingly shown. Beware printed answer. Form a quadratic equation $\mathrm{F}(m) = \frac{1}{2}$ and attempt to solve it. ft c's cdf provided it leads to a quadratic. Convincingly shown. Beware printed answer.	 2
(v)	$\bar{X} \sim \mathrm{N}(\frac{1}{3}, \frac{1}{1800})$	B1 B1 B1	Normal distribution. Mean. ft c's E(X). Correct variance.	 3
				18

Q2				
(i)	$H_0 : \mu = 0{\cdot}6$ $H_1 : \mu < 0{\cdot}6$ Where μ is the (population) mean height of the saplings.	B1 B1 B1	 Allow absence of "population" if correct notation μ is used, but do NOT allow "$\bar{X} = \ldots$" or similar unless $\bar{X}$ is clearly and explicitly stated to be a population mean. Hypotheses in words only must include "population".	
	$\bar{x} = 0{\cdot}5883$, $s_{n-1} = 0{\cdot}03664$ ($s_{n-1}^2 = 0{\cdot}00134$)	B1	Do not allow $s_n = 0{\cdot}03507$ ($s_n^2 = 0{\cdot}00123$).	
	Test statistic is $\frac{0{\cdot}5883 - 0{\cdot}6}{\left(\frac{0{\cdot}03664}{\sqrt{12}}\right)}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 0·6 ± (c's –1·796) × $\frac{0{\cdot}03664}{\sqrt{12}}$ (=0·5810, 0·6190) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ ± (c's –1·796) × $\frac{0{\cdot}03664}{\sqrt{12}}$ (=0·5693, 0·6073) for comparison with 0·6.)	
	= –1·103	A1	c.a.o. but ft from here in any case if wrong. Use of 0·6 – $\bar{x}$ scores M1A0, but ft.	
	Refer to t_{11}. Lower 5% point is –1·796.	M1 A1	No ft from here if wrong. No ft from here if wrong. Must be –1·796 unless it is clear that absolute values are being used.	
	–1·103 > –1·796, ∴ Result is not significant.	E1	ft only c's test statistic.	
	Seems mean height of saplings meets the manager's requirements.	E1	ft only c's test statistic.	11
	Underlying population is Normal.	B1		
(ii)	CI is given by 0·5883 ± 2·201 $\times \frac{0{\cdot}03664}{\sqrt{12}}$	M1 B1 M1	ft c's $\bar{x}$ ±. ft c's s_{n-1}.	
	= 0·5883 ± 0·0233 = (0·565(0), 0·611(6))	A1	c.a.o. Must be expressed as an interval. ZERO if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_{11} is OK.	

	In repeated sampling, 95% of intervals constructed in this way will contain the true population mean.	E1		5
(iii)	Could use the Wilcoxon test. Null hypothesis is "Median = 0.6".	E1 E1		2
				18

Q3	$M \sim N(44,\ 4{\cdot}8^2)$ $H \sim N(32,\ 2{\cdot}6^2)$ $P \sim N(21,\ 3{\cdot}7^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables, penalise the first occurrence only.	
(i)	$P(M < 50) = P(Z < \frac{50-44}{4\cdot8} = 1{\cdot}25)$ $= 0{\cdot}8944$	M1 A1 A1	For standardising. Award once, here or elsewhere.	3
(ii)	$H + P \sim N(32 + 21 = 53,$ $2{\cdot}6^2 + 3{\cdot}7^2 = 20{\cdot}45)$ $P(H + P < 50) = P(Z < \frac{50-53}{\sqrt{20\cdot45}} = -0{\cdot}6634)$ $= 1 - 0{\cdot}7465 = 0{\cdot}2535$	B1 B1 A1	Mean. Variance. Accept sd = $\sqrt{20{\cdot}45}$ = 4·522... c.a.o.	3
(iii)	Want $P(M > H + P)$ i.e. $P(M - (H + P) > 0)$ $M - (H + P) \sim N(44 - (32 + 21) = -9,$ $4{\cdot}8^2 + 2{\cdot}6^2 + 3{\cdot}7^2 = 43{\cdot}49)$ $P(\text{this} > 0) = P(Z > \frac{0-(-9)}{\sqrt{43\cdot49}} = 1{\cdot}365)$ $= 1 - 0{\cdot}9139 = 0{\cdot}0861$	M1 B1 B1 A1	Allow $H + P - M$ provided subsequent work is consistent. Mean. Variance. Accept sd = $\sqrt{43{\cdot}49}$ = 6·594... c.a.o.	4
(iv)	Mean = 44 + 44 + 32 + 32 + 21 + 21 = 194 Variance = $4{\cdot}8^2 + 4{\cdot}8^2 + 2{\cdot}6^2 + 2{\cdot}6^2 + 3{\cdot}7^2 + 3{\cdot}7^2$ = 86·98	 B1 B1	 (sd = 9·3263…)	2
(v)	$C \sim N(194 \times 0\cdot15 + 10 = 39\cdot10,$ $86\cdot98 \times 0\cdot15^2 = 1\cdot957)$ $P(C \le 40) = P(Z \le \frac{40-39\cdot10}{\sqrt{1\cdot957}} = 0{\cdot}6433)$ $= 0{\cdot}7400$ **Alternatively:** $P(C \le 40) = P(\text{total time} \le \frac{40-10}{0.15} = 200$ minutes) $= P(Z \le \frac{200-194}{\sqrt{86\cdot98}} = 0{\cdot}6433)$	M1 M1 A1 M1 A1 A1 M1 M1 A1 M1 A1	c's mean in (iv) × 0·15 + 10 (or subtract 10 from 40 below) ft c's mean in (iv). c's variance in (iv) × $0{\cdot}15^2$ ft c's variance in (iv). c.a.o. – 10 ÷ 0.15 c.a.o. Correct use of c's variance in (iv). ft c's mean and variance in (iv).	 6

	= 0·7400	A1	c.a.o.	
				18

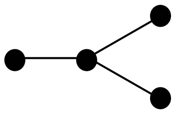
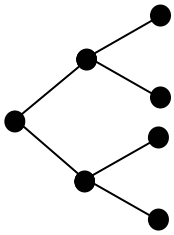
Q4				
(a)	Obs: 10, Exp: 6·68	M1	Combine first two rows.	
	$\therefore X^2 = \frac{(10-6\cdot68)^2}{6\cdot68} + \text{etc}$	M1		
	= 1·6501 + 1·7740 + 3·3203 + 4·5018 + 0·4015 + 0·8135			
	= 12·46(12)	A1		
	d.o.f. = 6 – 3 = 3		Require d.o.f. = No. cells used – 3.	
	Refer to χ_3^2.	M1	No ft from here if wrong.	
	Upper 5% point is 7·815	A1	No ft from here if wrong.	
	12·46 > 7·815 $\therefore$ Result is significant.	E1	ft only c's test statistic.	
	Seems the Normal model does not fit the data at the 5% level.	E1	ft only c's test statistic.	
	E.g. • The biggest discrepancy is in the class 1·01 < *a* ≤ 1·02	E1		
	• The model overestimates in classes …, but underestimates in classes …	E1	Any two suitable comments.	9
(b)				

Old – New:	0·007	0·002	–0·001	–0·003	0·004	–0·008	–0·010	0·009	–0·005	–0·016
Rank of |diff|	6	2	1	3	4	7	9	8	5	10

		M1	For differences. ZERO in this section if differences not used.	
		M1	For ranks of |difference|.	
		A1	All correct.	
			ft from here if ranks wrong.	
	$W_+ = 6 + 2 + 4 + 8 = 20$	B1	Or $W_- = 1 + 3 + 7 + 9 + 5 + 10 = 35$	
	Refer to Wilcoxon single sample (/paired) tables for *n* = 10.	M1	No ft from here if wrong.	
	Lower two-tail 10% point is …	M1	Or, if 35 used, upper point is 45.	
	… 10.	A1	No ft from here if wrong.	
	20 > 10 $\therefore$ Result is not significant.	E1	Or 35 < 45. ft only c's test statistic.	
	Seems there is no reason to suppose the barometers differ.	E1	ft only c's test statistic.	9
				18

Mark Scheme 4771
January 2007

1.

(i) 	B1
(ii) Any two of 1 or 2 or 3 or 5 or 7	B1 B1
(iii) 	M1 branching tree A1
(iv) 	M1 branching tree A1
(v) A tree	B1

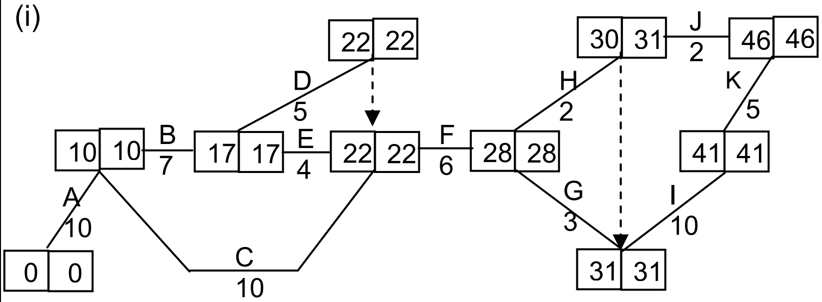
2.

(i) 109; 32; 3; 523; 58 32; 3; 109; 58; 523 4 comparisons and 3 swaps 3; 32; 58; 109; 523 3 and 2 3; 32; 58; 109; 523 2 and 0 3; 32; 58; 109; 523 1 and 0 10 and 5 in total	M1 A1 only if all iterations completed B1 B1
(ii) 523; 109; 58; 32; 3 10 swaps	B1 B1
(iii) $1.5 \times 100^2 = 15000$ seconds = 4 hrs 10 mins	M1 A1 hours and minutes

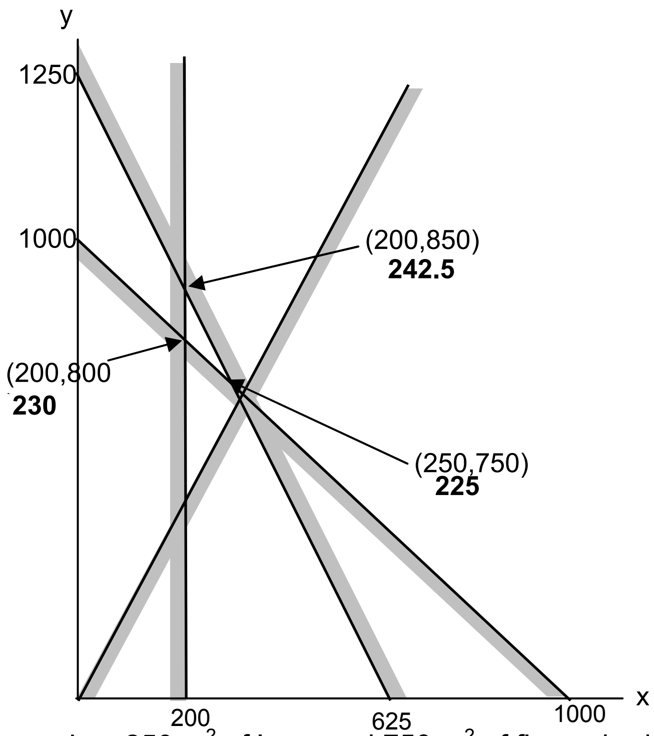
3.

(i) e.g. 0, 1 → A 2, 3 → B 4, 5 → C 6, 7 → D 8, 9 → E	M1 A1 proportions OK B1 efficient
(ii) e.g: 3, 4, 4, 4, 1	M1 A1
(iii) In the above simulation mean = 3.2 (Correct expectation is 2.5 – geometric rand variable)	M1 A1
(iv) More repetitions	B1

4.

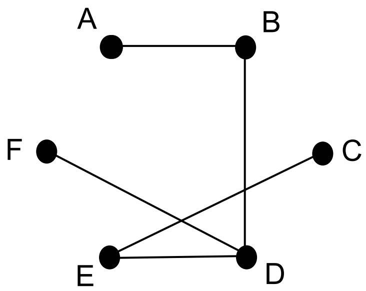
(i) 	M1 activity-on-arc A1 single start and end A1 dummy 1 A1 dummy 2 A1 rest
(ii) See above Critical activities: A; B; D; F; G; I; K Duration = 46	M1 A1 forward pass M1 A1 backward pass B1 critical activities B1 duration
(iii) E: total float = 1; independent float = 1 H: 1 and 0 J: 14 and 13 C: 2 and 2	B1 total floats B1 independent floats
(iv) Tiler (I) – 2 days – £500 Electrician (D) – 1 day – £300 Bricklayer (B) – 1 day – £350	B1 tiler B1 electrician B1 bricklayer

5.

(i) Let x be the number of m^2 of lawn. Let y be the number of m^2 of flower beds.	B1
$x + y \geq 1000$	B1
$0.80x + 0.40y \leq 500$, i.e. $2x + y \leq 1250$	B1
$y \geq 2x$	B1
$x \geq 200$	B1
Minimise $0.15x + 0.25y$	B1 B1
(ii) & (iii)	
	B1 axes labelled + scaled B4 lines B1 shading
Lay 250 m^2 of lawn and 750 m^2 of flower beds.	M1
Annual maintenance = £225.	A1
(iv) Intersection of $y \geq 2x$ & area constraint is at (333.33,666.67) so max useful capital is £533.33. So £33.33.	B1 (allow £533.33)

6.

(i) DtoE; BtoD; CtoE; DtoF; AtoB — M1; A1 no BC nor BE; A1



B1

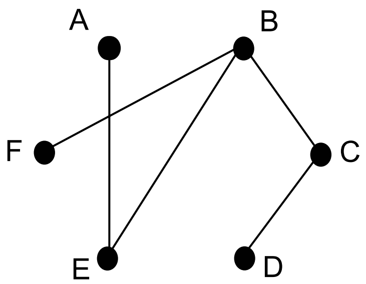
Total length = 20 — B1

(iii) e.g.

	1	3	4	6	2	5
	A	B	C	D	E	F
A	–	–	–	–	12	–
B	–	–	5	–	(6)	6
C	–	(5)	–	8	–	7
D	–	–	(8)	–	–	–
E	(12)	6	–	–	–	7
F	–	(6)	7	–	7	–

B1 reduced table

M1 delete/select/delete
A1 first 2 rows
A1 rest of table
A1 order



B1

Total length = 37 — B1

(iii) Lengths are 27 and 28. — B1 B1
Shorter and more nearly equal. — B1 B1

Mark Scheme 4776
January 2007

MEI Numerical Methods (4776) January 2007 Mark scheme

1

mpe:	0.000 000 05 x 10^{98}	= 5 x 10^{90}		**[M1A1]**
mpre:	0.000 000 05 / 1.7112245 =		2.92 x 10^{-8}	**[M1A1]**

Extra digits are used internally so that rounding errors will not (usually) show in the displayed answer **[E1]**

[TOTAL 5]

2

(i)	tan 0.2=	0.20271 0	approx =	0.20266 7		**[A1A1]**
	error:	-4.3E-05	rel error:	-0.00021		**[A1A1]**
						[subtotal 4]
(ii)	k 0.2^5=	4.34E-05	hence k=	0.13552 8	accept 0.13 or 0.14	**[M1A1A1]**
						[subtotal 3]

[TOTAL 7]

3

r	0	1	2	3	
x_r	0.35	0.354767	0.35646 2	0.35706 7	
Differences		0.004767	0.00169 5	0.00060 5	**[M1A1]**
Ratio of differences			0.35557 0	0.35693 2	**[M1A1]**

root = 0.35706 7 +0.000605 (0.356932 + 0.356932^2 + …) **[M1A1]**

= 0.35740 3 **[A1]**

0.3574 seems justified **[A1]**

[TOTAL 8]

---

4 Graph of y = cos x and y = x^2 showing one intersection for x > 0. (Or equivalent.) **[G2]**

x	0.7	0.9	change of sign so root	
cos x -x^2	0.27484 2	-0.18839		**[M1A1]**

r	0	1	2	3	4	root	
x_r	0.7	0.9	0.81866 3	0.82390 9	0.82413 3	0.824	**[M1A1A1]**
f(x)	0.27484 2	-0.18839	0.01298 9	0.00053 1	-1.6E-06	to 3dp	**[A1]**

[TOTAL 8]

5

x	0	0.25	0.5
f(x)	1.1105	1.2446	1.4065

h	0.5	0.25	
f '(0)	0.5920	0.5364	**[M1A1A1]**

poor accuracy: estimates very different, at most 1 dp reliable **[E1]**

[subtotal 4]

h	0.25	
f '(0.25)	0.5920	**[M1A1]**

accuracy likely to be better because of the use of central difference formula; **[E1]**

nothing more than 1 dp because there is nothing to compare the answer with. **[E1]**
[subtotal 4]
[TOTAL 8]

6

x	0.9	1.1	1.2	1.4	1.5
f(x)	-0.43	-0.09	0.15	0.78	1.15

(i) y = -0.09 (x - 1.2) / (1.1 - 1.2) + 0.15 (x - 1.1) / (1.2 - 1.1) = 2.4 x - 2.73 **[M1A1A1A1]**
Estimate of α: 1.1375 **[A1]**

Using values -0.085 and 0.155 gives α as 1.1354 **[M1A1]**
Using values -0.095 and 0.145 gives α as 1.1396 **[M1A1]**
Hence quote 1.14 **[A1]**
[subtotal 10]

(ii) y = -0.09 (x - 1.2) (x - 1.4) / (1.1 - 1.2) (1.1 - 1.4) + two similar terms **[M1A1A1A1]**
y = -3 (x^2 - 2.6x + 1.68) - 7.5 (x^2 - 2.5x +1.54) + 13 (x^2 - 2.3x + 1.32) **[A1]**
= 2.5 x^2 - 3.35x + 0.57 **[A1]**
y = 0 gives α = 1.14 (reject other root) **[M1A1]**
[subtotal 8]

[TOTAL 18]

7

(i)

x	x^-x	M	T	S		
1	1					
2	0.25		0.625			
	0.54433		0.58466	**0.57122**		
1.5	1	0.544331	6	**1**	(h=0.5)	**[M1A1A1]**
	0.75659					
1.25	**3**					
	0.37556		0.57537	**0.57227**	(h=0.25)	
1.75	**4**	0.566078	2	**4**		**[M1A1A1A1]**
						[subtotal 7]

(ii)

a =	0.57122					
	1					
b =	0.57227		0.00105			
	4	b - a =	3			
c =	0.57234			ratio =	0.06647	
	4	c - b =	7E-05		7	**[M1A1A1]**

Theoretically 1/16 (= 0.0625): good agreement with theory. **[E1E1]**
[subtotal 5]

(iii) 0.572344 + 0.0000699 (1/16 + $1/16^2$ + …) **[M1A1]**
= 0.572349 **[A1]**

0.57235 appears completely secure from the rate of convergence **[A1E1]**
but there may be rounding errors in the 6th dp **[E1]**
[subtotal 6]

[TOTAL 18]

7895-8,3895-3898 AS and A2 MEI Mathematics
January 2007 Assessment Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	50	43	36	29	23	0
4752	Raw	72	52	45	38	31	25	0
4753	Raw	72	61	54	47	39	31	0
4753/02	Raw	18	14	12	10	9	8	0
4754	Raw	90	68	60	52	44	37	0
4755	Raw	72	59	51	43	35	27	0
4756	Raw	72	53	46	39	32	25	0
4758	Raw	72	58	50	42	33	24	0
4758/02	Raw	18	14	12	10	9	8	0
4761	Raw	72	56	48	40	33	26	0
4762	Raw	72	58	50	43	36	29	0
4763	Raw	72	53	46	39	32	25	0
4766	Raw	72	51	44	38	32	26	0
4767	Raw	72	59	52	45	38	31	0
4768	Raw	72	59	51	43	35	28	0
4771	Raw	72	55	47	40	33	26	0
4776	Raw	72	52	46	40	33	27	0
4776/02	Raw	18	13	11	9	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	28.9	59.8	83.5	95.9	96.9	100	97
7896	30.8	69.2	100	100	100	100	13
7897	100	100	100	100	100	100	1
7898							0
3895	18.0	39.1	61.6	78.4	94.4	100	445
3896	33.3	66.7	83.3	100	100	100	6
3897	100	100	100	100	100	100	2
3898	84.6	92.3	92.3	100	100	100	13

For a description of how UMS marks are calculated see;
http://www.ocr.org.uk/exam_system/understand_ums.html

Statistics are correct at the time of publication

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