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RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MEI STRUCTURED MATHEMATICS 4751

Introduction to Advanced Mathematics (C1)

Tuesday **6 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Question **13**.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

**WARNING**

**You are not allowed to use a calculator in this paper**

**This question paper consists of 4 printed pages and an insert.**

**Section A** (36 marks)

**1** The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$. Make $r$ the subject of this formula. [3]

**2** One root of the equation $x^3 + ax^2 + 7 = 0$ is $x = -2$. Find the value of $a$. [2]

**3** A line has equation $3x + 2y = 6$. Find the equation of the line parallel to this which passes through the point $(2, 10)$. [3]

**4** In each of the following cases choose one of the statements

$$\text{P} \Rightarrow \text{Q} \qquad\qquad \text{P} \Leftrightarrow \text{Q} \qquad\qquad \text{P} \Leftarrow \text{Q}$$

to describe the complete relationship between P and Q.

**(i)** P: $x^2 + x - 2 = 0$
Q: $x = 1$ [1]

**(ii)** P: $y^3 > 1$
Q: $y > 1$ [1]

**5** Find the coordinates of the point of intersection of the lines $y = 3x + 1$ and $x + 3y = 6$. [3]

**6** Solve the inequality $x^2 + 2x < 3$. [4]

**7** **(i)** Simplify $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$. [2]

**(ii)** Express $(2 - 3\sqrt{5})^2$ in the form $a + b\sqrt{5}$, where $a$ and $b$ are integers. [3]

**8** Calculate ${}^6\mathrm{C}_3$.

Find the coefficient of $x^3$ in the expansion of $(1 - 2x)^6$. [4]

**9** Simplify the following.

**(i)** $\dfrac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$ [2]

**(ii)** $\dfrac{12(a^3b^2c)^4}{4a^2c^6}$ [3]

**10** Find the coordinates of the points of intersection of the circle $x^2 + y^2 = 25$ and the line $y = 3x$. Give your answers in surd form. [5]

## Section B (36 marks)

**11** $A(9, 8)$, $B(5, 0)$ and $C(3, 1)$ are three points.

**(i)** Show that AB and BC are perpendicular. [3]

**(ii)** Find the equation of the circle with AC as diameter. You need not simplify your answer.

Show that B lies on this circle. [6]

**(iii)** BD is a diameter of the circle. Find the coordinates of D. [3]

**12** You are given that $f(x) = x^3 + 9x^2 + 20x + 12$.

**(i)** Show that $x = -2$ is a root of $f(x) = 0$. [2]

**(ii)** Divide $f(x)$ by $x + 6$. [2]

**(iii)** Express $f(x)$ in fully factorised form. [2]

**(iv)** Sketch the graph of $y = f(x)$. [3]

**(v)** Solve the equation $f(x) = 12$. [3]

**[Question 13 is printed overleaf.]**

**13 Answer the whole of this question on the insert provided.**

The insert shows the graph of $y = \frac{1}{x}$, $x \neq 0$.

**(i)** Use the graph to find approximate roots of the equation $\frac{1}{x} = 2x + 3$, showing your method clearly. [3]

**(ii)** Rearrange the equation $\frac{1}{x} = 2x + 3$ to form a quadratic equation. Solve the resulting equation, leaving your answers in the form $\frac{p \pm \sqrt{q}}{r}$. [5]

**(iii)** Draw the graph of $y = \frac{1}{x} + 2$, $x \neq 0$, on the grid used for part (**i**). [2]

**(iv)** Write down the values of $x$ which satisfy the equation $\frac{1}{x} + 2 = 2x + 3$. [2]

| Candidate Name | Centre Number | Candidate Number |
|---|---|---|
| | | |

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# MEI STRUCTURED MATHEMATICS 4751

Introduction to Advanced Mathematics (C1)

INSERT

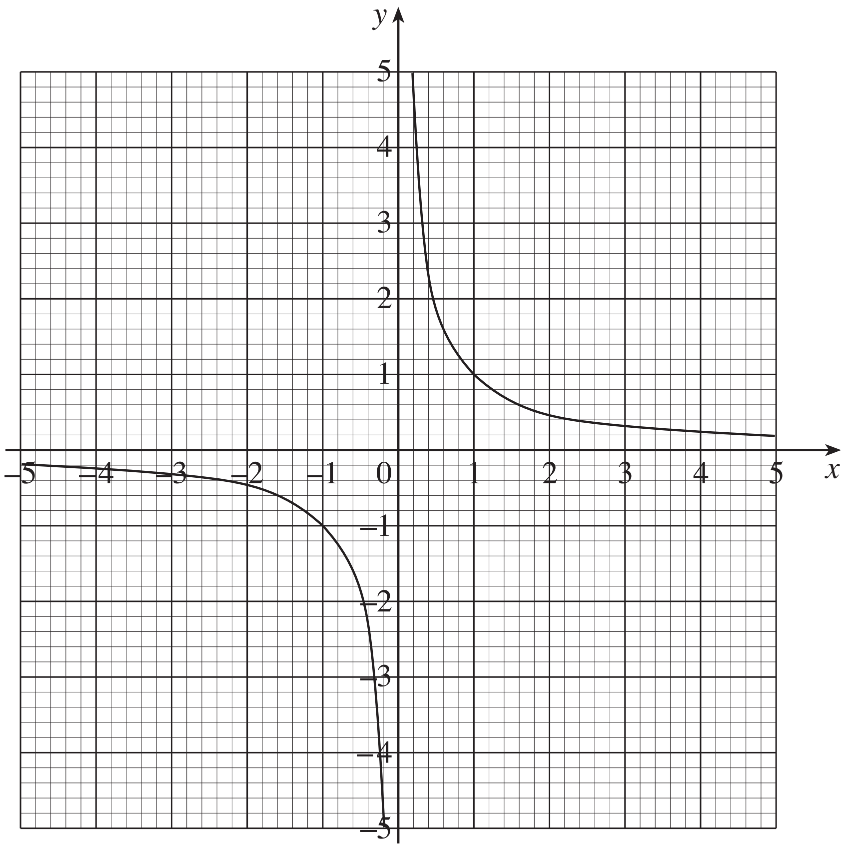
Tuesday **6 JUNE 2006** Afternoon 1 hours 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- This **insert** should be used in Question **13.**
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

**This insert consists of 2 printed pages.**

**13 (i) and (iii)**

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**(ii)** ..........................................................................................................................................

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**(iv)** ..........................................................................................................................................

# Mark Scheme 4751
# June 2006

Section A

| | | | | |
|---|---|---|---|---|
| **1** | $[r]=[\pm]\sqrt{\dfrac{3V}{\pi h}}$ o.e. 'double-decker' | 3 | 2 for $r^2=\dfrac{3V}{\pi h}$ or $r=\sqrt{\dfrac{V}{\frac{1}{3}\pi h}}$ o.e. or M1 for correct constructive first step or for $r=\sqrt{k}$ ft their $r^2=k$ | 3 |
| **2** | $a = ¼$ | 2 | M1 for subst of –2 or for −8 + 4*a* + 7 = 0 o.e. obtained eg by division by (*x* + 2) | 2 |
| **3** | 3*x* + 2*y* = 26 or *y* = –1.5*x* + 13 isw | 3 | M1 for 3*x* + 2*y* = *c* or *y* = –1.5*x* + *c*<br>M1 for subst (2, 10) to find *c* or for or for *y* − 10 = their gradient × (*x* − 2) | 3 |
| **4** | (i) P $\Leftarrow$ Q<br>(ii) P $\Leftrightarrow$ Q | 1<br>1 | condone omission of P and Q | 2 |
| **5** | *x* + 3(3*x* + 1) = 6 o.e.<br><br>10*x* = 3 or 10*y* = 19 o.e.<br>(0.3, 1.9) or *x* = 0.3 and *y* = 1.9 o.e. | M1<br><br>A1<br>A1 | for subst or for rearrangement and multn to make one pair of coefficients the same or for both eqns in form '*y* =' (condone one error)<br><br>graphical soln: (must be on graph paper) M1 for each line, A1 for (0.3, 1.9) o.e cao; allow B3 for (0.3, 1.9) o.e. | 3 |
| **6** | –3 < *x* < 1<br>[condone *x* < 1, *x* > −3] | 4 | B3 for –3 and 1 or<br>M1 for $x^2 + 2x - 3$ [< 0]or $(x + 1)^2$ < / = 4 and M1 for (*x* + 3)(*x* – 1) or *x*= (−2 ±4)/2 or for (*x* + 1) and ±2 on opp. sides of eqn or inequality;<br>if 0, then SC1 for one of *x* < 1, *x* > −3 | 4 |
| **7** | (i) 28√6<br><br>(ii) 49 – 12√5 isw | 2<br><br>3 | 1 for 30√6 or 2√6 or 2√2√3 or 28√2√3<br><br>2 for 49 and 1 for – 12√5 or M1 for 3 correct terms from 4 − 6√5 − 6√5 + 45 | 5 |
| **8** | 20<br><br>–160 or ft for –8 × their 20 | 2<br><br>2 | 0 for just 20 seen in second part; M1 for 6!/(3!3!) or better<br>condone $-160x^3$; M1 for [–]$2^3$ × [their] 20 seen or for [their] 20 × $(-2x)^3$; allow B1 for 160 | 4 |
| **9** | (i) 4/27<br><br>(ii) $3a^{10}b^8c^{-2}$ or $\dfrac{3a^{10}b^8}{c^2}$ | 2<br><br>3 | 1 for 4 or 27<br><br>2 for 3 'elements' correct, 1 for 2 elements correct, −1 for any adding of elements; mark final answer; condone correct but unnecessary brackets | 5 |
| **10** | $x^2 + 9x^2 = 25$<br>$10x^2 = 25$<br><br>*x*= ±(√10)/2 or.±√(5/2) or ±5/√10 oe<br>*y* = [±] 3√(5/2) o.e. eg *y* = [±] √22.5 | M1<br>M1<br><br>A2<br>B1 | for subst for *x* or *y* attempted<br>or $x^2$ = 2.5 o.e.; condone one error from start [allow $10x^2$ − 25 = 0 + correct substn in correct formula]<br>allow ±√2.5; A1 for one value<br>ft 3 × their *x* value(s) if irrational; condone not written as coords. | 5 |

Section B

| | | | | | |
|---|---|---|---|---|---|
| **11** | **i** | grad AB = 8/4 or 2 or $y = 2x - 10$<br>grad BC = 1/–2 or – ½ or $y = -\frac{1}{2}x + 2.5$<br>product of grads = –1 [so perp] (allow seen or used) | 1<br>1<br><br>1 | or M1 for $AB^2 = 4^2 + 8^2$ or 80 and $BC^2 = 2^2 + 1^2$ or 5 and $AC^2 = 6^2 + 7^2$ or 85; M1 for $AC^2 = AB^2 + BC^2$ and 1 for [Pythag.] true so AB perp to BC;<br>if 0, allow G1 for graph of A, B, C | 3 |
| | **ii** | midpt E of AC = (6, 4.5)<br>$AC^2 = (9 – 3)^2 + (8 – 1)^2$ or 85<br><br>rad = ½ √85 o.e.<br>$(x – 6)^2 + (y – 4.5)^2 = 85/4$ o.e.<br><br>$(5–6)^2 + (0–4.5)^2 = 1 + 81/4$ [= 85/4] | 1<br>M1<br><br>A1<br>B2<br><br>1 | allow seen in (i) only if used in (ii); or $AE^2 = (9 – \text{their } 6)^2 + (8 – \text{their } 4.5)^2$ or rad.$^2$ = 85/4 o.e. e.g. in circle eqn<br>M1 for $(x – a)^2 + (y – b)^2 = r^2$ soi or for lhs correct<br>some working shown; or 'angle in semicircle [=90°]' | 6 |
| | **iii** | $\overrightarrow{BE} = \overrightarrow{ED} = \begin{pmatrix} 1 \\ 4.5 \end{pmatrix}$<br>D has coords (6 + 1, 4.5 + 4.5) ft or<br>(5 + 2, 0 + 9)<br>= (7, 9) | M1<br><br>M1<br><br>A1 | o.e. ft their centre; or for $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$<br>or (9 − 2, 8 + 1); condone mixtures of vectors and coords. throughout part iii<br>allow B3 for (7,9) | 3 |
| **12** | **i** | f(–2) used<br>–8 + 36 – 40 + 12 = 0 | M1<br>A1 | or M1 for division by $(x + 2)$ attempted as far as $x^3 + 2x^2$ then A1 for $x^2 + 7x + 6$ with no remainder | 2 |
| | **ii** | divn attempted as far as $x^2 + 3x$<br>$x^2 + 3x + 2$ or $(x + 2)(x + 1)$ | M1<br>A1 | or inspection with $b = 3$ or $c = 2$ found;<br>B2 for correct answer | 2 |
| | **iii** | $(x + 2)(x + 6)(x + 1)$ | 2 | allow seen earlier;<br>M1 for $(x + 2)(x + 1)$ | 2 |
| | **iv** | sketch of cubic the right way up<br><u>through</u> 12 marked on y axis<br>intercepts –6, –2, –1 on $x$ axis | G1<br>G1<br>G1 | with 2 turning pts; no 3rd tp<br>curve must extend to $x > 0$<br>condone no graph for $x < -6$ | 3 |
| | **v** | $[x](x^2 + 9x + 20)$<br>$[x](x + 4)(x + 5)$<br>$x = 0, –4, –5$ | M1<br>M1<br>A1 | or other partial factorisation<br><br>or B1 for each root found e.g. using factor theorem | 3 |
| **13** | **i** | $y = 2x + 3$ drawn on graph<br>$x$ = 0.2 to 0.4 and – 1.7 to –1.9 | M1<br>A2 | <br>1 each; condone coords; must have line drawn | 3 |
| | **ii** | $1 = 2x^2 + 3x$<br>$2x^2 + 3x – 1$ [= 0]<br><br>attempt at formula or completing square<br>$x = \frac{-3 \pm \sqrt{17}}{4}$ | M1<br>M1<br><br>M1<br><br>A2 | for multiplying by $x$ correctly<br>for correctly rearranging to zero (may be earned first) or suitable step re completing square if they go on<br>ft, but no ft for factorising<br><br>A1 for one soln | 5 |
| | **iii** | branch through (1,3),<br>branch through (−1,1),approaching $y = 2$ from below | 1<br><br>1 | and approaching $y = 2$ from above<br><br>and extending below $x$ axis | 2 |
| | **iv** | –1 and ½ or ft intersection of their curve and line [tolerance 1 mm] | 2 | 1 each; may be found algebraically; ignore $y$ coords. | 2 |

## 4751 - Introduction to Advanced Mathematics (C1)

### General Comments

A full spread of marks was seen, but it was perhaps slightly harder to obtain 65+ marks on this paper than on some past C1 papers.

There were excellent scripts seen from the candidates who were able to approach this examination with confidence in their mastery of the content and the skills required for this paper. However, examiners were concerned at the long tail of weak candidates seen this session. Centres are also reminded that calculators are not allowed in this paper. Examiners found some instances of calculators being used, which were reported as suspected malpractice.

There were some very sketchy attempts at question 13 and some did not attempt it, suggesting that perhaps some candidates had time problems and a few may have failed to turn the page. However, some candidates had several goes at parts of other questions where they had had difficulty, indicating that unfamiliarity with the graph of a reciprocal function may also have been a cause.

### Comments on Individual Questions

### Section A

1) This rearrangement of the formula was well done, but many candidates left their answers as a triple-decker fraction.

2) Most candidates knew that they had to substitute $x = -2$ and many successfully found the value of $a$.

3) Many candidates did not identify the gradient of the line correctly, although most then went on to use the correct method for a line through (2, 10).

4) There was often no working by the candidates to support their decisions, but examiners suspected that many had only a limited understanding of the meaning of the implication symbols being tested. Few candidates gained both marks here.

5) Relatively few candidates chose the most straightforward method of substitution. There were many errors, both algebraic and numeric, especially when elimination was attempted or fractions were used.

6) Most candidates could solve the quadratic equation but far fewer could go on to solve the inequality correctly.

7) Only a minority completed part (i) successfully and many showed several lines of ineffective working. The second part was better, but there were many errors in squaring $3\sqrt{5}$.

8) This was the first C1 paper on which the ${}^nC_r$ notation has been explicitly tested and many candidates were clearly unfamiliar with it, although knowledge of it is a statement in the specification. Many candidates started again in finding the coefficient of $x^3$ in the binomial expansion, using Pascal's triangle and not realising that any result for ${}^6C_3$ that they had found already had any relevance.

9) There were many errors in coping with the indices here, although weaker candidates who knew the rules of indices could pick up marks in the second part very easily, and some of them did. In the first part, most obtained $16^{1/2} = 4$, but then often did not know how to proceed. Some tried to cube 81 and then despaired at finding the fourth root of that, whilst correctly finding $3^3$ but evaluating that as 9 was a common error. Better candidates usually sailed through with no problems.

10) Most candidates knew they had to substitute for $y$ and solve the resulting equation, but common errors were $3x^2$ instead of $(3x)^2$, or thinking that $x^2 + y^2 = 25$ leads to $x + y = 5$. Later in the question it was common to omit the negative root.

**Section B**

11 (i) Overall, question 11 was the question which caused the greatest problems to candidates in section B. However, most candidates had a reasonable attempt at this first part, usually understanding that it was necessary to find the gradients and show that the product was -1. Some made arithmetic slips in their calculations (usually with signs) yet managed to believe that the product was -1. A few calculated the gradient as change in x ÷ change in y, but were still "successful" in showing the product to be -1. A few used the alternative method of Pythagoras' theorem instead of gradients.

(ii) Part (ii) was done correctly and elegantly by some candidates, but in general there were many errors. Expressions like $(x - a)^2+(y - b)^2 = r^2$ showed that many of the candidates know the form of equation required for a circle. However, some candidates failed to recognise that the midpoint of AC was the centre of the required circle, and many failed to calculate the radius of the circle correctly, often finding $AC^2$, but concluding that $r = \sqrt{(85/2)}$ rather than $\sqrt{85}/2$. Few candidates gained the final mark in part (ii) either because they made no attempt or because their figures were not going to work out. A few showed no working in the belief that their calculation must be right! Only a handful used the geometric property to gain this mark, which could have been gained even if the equation of the circle had been wrong.

(iii) By this stage some candidates had had enough of this question. There were surprisingly few correct answers in evidence for part (iii) - many candidates appeared to have made no attempt. However, those candidates who drew a diagram often saw quite quickly how easy this part really was. Some weak candidates who floundered in part (ii) were able to succeed here. Many candidates wasted time on this question doing unnecessary and involved algebra.

12 (i) -(iii) Question 12 was much more to the candidates' liking than question 11 and the majority showed a good understanding of the factor theorem, as well as the technique of algebraic division. As a consequence the first three parts were often correct.

(iv) The curve sketching was not so well done though most arrived at a curve that was basically a cubic, even if it was a 'mirror image'. The labelling of the axes was not always done fully; it was not uncommon for the intercept on the $y$-axis to be missed.

(v) This final part was not done as well as the rest of the question. Only the better candidates simplified and then fully factorised the expression in order to yield the full solution. Many continued to try to determine the roots using the factor theorem – usually being satisfied once they had found one solution. Candidates often realised the connection with the previous part to obtain $x = 0$. Some candidates who did factorise, cancelled out the $x$ factor and so lost the solution $x = 0$.

13 (i) Many candidates did not seem to realise the need to draw the line $y = 2x + 3$ on the graph. Those who did know the technique usually used it correctly and obtained full marks here and in part (v). A few drew the line correctly but did not give the solutions to the equation in part (i).

(ii) Many candidates did well on (ii), usually generating a quadratic equation and making a good attempt to apply the formula. However, a number of careless slips were in evidence. Some did not know how to cope with the $1/x$ term and attempted to apply the quadratic formula to a non-quadratic equation.

(iii) The curve drawing here was not done too well, the lower part of the graph causing more problems than the top, often being translated 2 down instead of 2 up. Many candidates were content with rough sketches, in contrast to some who showed considerable care in translating the graph of $y = 1/x$ that was already drawn for them on the grid.

(iv) This part was done independently of part (i), with many candidates finding the solution algebraically or by inspection and often arriving at just one of the roots.